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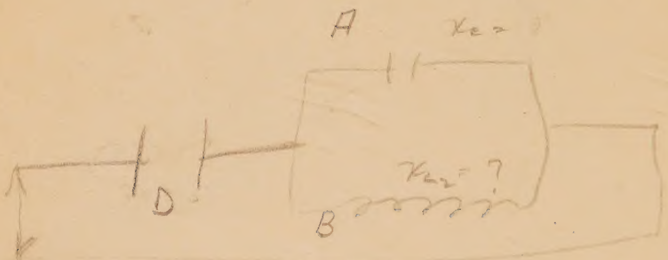
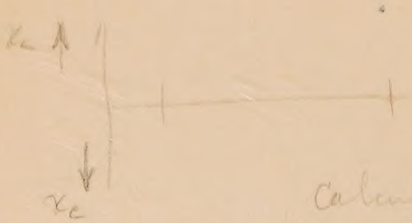


voltage resonance at 60 cycles.
A = 40 m.f.
B = .2 henry
D = 20 m.f., .. to give voltage resonance

calculate frequency at current resonance
I is minimum

plot reactance against frequency.



Calculate Z for circuit
and solve for I

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$5\frac{1}{32}$ in beam

4.7 between punch marks

$$\log_{10} \frac{d}{r} = 2.1$$

$$\frac{d}{r} = 10^{2.1}$$

ALTERNATING CURRENTS

BOOKS BY

C. E. MAGNUSSON

ELECTRIC TRANSIENTS
Second Edition
237 pages, 6 × 9, 216 illustrations

ALTERNATING CURRENTS
Third Edition
611 pages, 6 × 9, 522 illustrations

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495 pages, 6 × 9, 400 illustrations

ALTERNATING CURRENTS

BY

CARL EDWARD MAGNUSSON, M. S., Ph. D., E. E.

Author of "Direct Currents" and "Electric Transients"; Professor of Electrical Engineering and Director Engineering Experiment Station, University of Washington; Fellow, American Institute of Electrical Engineers; American Physical Society; Member, American Society of Civil Engineers; Society of American Military Engineers; American Mathematical Society

Fourth Edition
Second Impression

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK AND LONDON
1931

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE TO THE FOURTH EDITION

In several respects this volume is superior to the preceding third edition. The chapter on Instruments has been rewritten and greatly extended. New chapters on Systems of Units and Symbols, Mercury-arc Rectifiers, Hot-cathode Rectifiers and Oscillators and several sectional additions to the text are included. Many new oscillograms, line drawings and photographs have been added. In order not to increase the size of the book the Laboratory Experiments, introduced in the second edition, have been omitted.

The author is indebted to Professor Gordon R. Shuck, Professor George L. Hoard, Professor Roy E. Lindblom, Professor George S. Smith, Professor Austin V. Eastman, Mr. L. B. Robinson and Mr. George W. DeSellem for valuable assistance and many constructive suggestions.

C. EDWARD MAGNUSSON.

UNIVERSITY OF WASHINGTON,
SEATTLE, WASH.,
June, 1931.

PREFACE TO THE THIRD EDITION

The book has been thoroughly revised and still the preface for the first edition may well be used as an introduction to the third edition. The fundamental laws of alternating currents are well established and a textbook must deal mainly with basic principles. Much progress has been made during the decade since the appearance of the first edition, in the application of alternating currents to industrial requirements. Many of the illustrations used in the first edition have been replaced by later designs and many new cuts illustrating present practice have been introduced. The author is indebted to Professor Stewart Anderson, Professor E. A. Loew, Mr. George S. Smith, Professor F. K. Kirsten and others for valuable assistance and helpful criticism.

C. EDWARD MAGNUSSON.

UNIVERSITY OF WASHINGTON,
SEATTLE, WASH.,
March, 1926.

PREFACE TO THE SECOND EDITION

All the material in the first edition has been retained essentially in its original form. The principal addition is an outline of laboratory experiments on alternating current machinery, correlated with the presentation of principles in the text. A number of new problems have been added, but it must not be forgotten that, in order to secure good results in the teaching of alternating currents, an abundance of problems with quantitative data from power systems and machines with which the student comes in direct contact must be provided by the instructor.

Helpful criticism and suggestions for improvement have been received from many sources. Special credit is due my colleague, Assistant Professor L. F. Curtis, for the computations for Line C, Chap. XXVII, and for preparing the laboratory experiments.

C. EDWARD MAGNUSSON.

UNIVERSITY OF WASHINGTON,
SEATTLE, WASH.,
November 22, 1920.

PREFACE TO THE FIRST EDITION

This book is an outline of lectures and class-room discussions on alternating-current phenomena given by the author during the past ten years to students in the University of Washington. A textbook for undergraduates, in so extensive a field, must be limited to fundamental principles with a few illustrations of their application to industrial problems.

Energy transformation and energy transmission, with due emphasis on the storage features of the magnetic and dielectric fields, are the fundamental concepts in the study of all alternating-current phenomena. On the energy basis the apparently widely divergent phenomena of the magnetic, electric and dielectric circuits can be coördinated under the same general laws.

The topics discussed are arranged in the sequence that has proved advantageous to the student. The elemental principles are first applied to the transformer, as this is the pivotal apparatus of alternating-current systems and also the simplest of all alternating-current machines. The treatment of long-distance transmission lines in the last chapter is more extensive than is ordinarily given in undergraduate courses. The author believes that after the student has completed a general survey of the main characteristics of alternating-current machinery, as given in Chaps. XII to XXVI inclusive, it is highly desirable that at least one corner of the field be examined more thoroughly. For this purpose long-distance transmission lines offer special advantages, in that the problem is complicated, the solution is particularly elegant and the theoretical equations must be used in the computations of commercial transmission lines.

The purpose of the book is to aid the student in gaining clear concepts of what actually takes place in alternating-current machinery, to explain the relations between the factors involved and to express the physical facts in mathematical forms in such manner that the student shall understand the equations and be able to use them rationally in the solution of everyday industrial problems. Graphic diagrams are used extensively to show the relations between the physical concepts and the algebraic equa-

tions. For convenience, a few typical problems are included, but it is essential that the teacher supply an abundance of problems with quantitative data in accord with local conditions. The use of problems relating to nearby power systems or to machines in the laboratory is of great importance, for the center of interest must be on the real machines and on the alternating-current phenomena as they appear in industrial problems, otherwise the student merely recites from a textbook, which would be of little or no value.

In the preparation of the book material has been drawn from technical journals and other sources, while space limitations make it impossible to give the references except in a few special cases. Even a casual examination of this volume will show the author's indebtedness to the matchless works of Dr. C. P. Steinmetz and to Dr. A. S. McAllister's treatise on alternating-current motors. Credit should be given to my colleague, Assistant Professor F. K. Kirsten, for the computations on the long-distance transmission lines *A* and *B*, Chap. XXVIII. The author also desires to acknowledge indebtedness to Dean A. S. Langsdorf, Dr. A. S. McAllister, Mr. L. F. Curtis, Mr. S. R. Burbank and Professor E. A. Loew for helpful criticism and for many valuable suggestions.

C. Edward Magnusson.

University of Washington,
Seattle, Wash.,
June 28, 1916.

CONTENTS

INTRODUCTION

A logical analysis of cause and effect in electrical phenomena shows that the *Principle of the Conservation of Energy* is the fundamental assumption, the basis upon which the argument rests. Electrical engineering deals with energy, not only in the electric, magnetic and dielectric forms and their interrelated phenomena, but also with its transformation into light, heat, mechanical work and chemical reactions. Moreover, during the many transformations which the energy can undergo while in the electric form, it is essential for a clear understanding of the processes involved that the student should take observations from the *energy point of view.* In general, the problem of the electrical engineer consists of receiving energy in the mechanical form from a prime mover, transforming it into electric energy, transmitting in this form to the desired locality, reconverting the energy and delivering to the customer light, heat and power. Electric energy is the connecting link or chain by which energy can be economically transmitted and effectively controlled. To study readily the laws for the generation, transmission and control of electric energy it has been found expedient to group the phenomena into three divisions:

1. Direct currents.
2. Alternating currents.
3. Electric transients.

This grouping is of special advantage to the student, as it follows the natural order of first analyzing the simple problem and then proceeding to more complex phenomena.

Direct Currents.—In the field covered by direct currents the fundamental laws are fully established and the phenomena can be calculated with mathematical exactness. In direct currents the flow of energy is continually in one direction. The energy delivered is changed into heat, light and mechanical work, or stored chemically. The laws for direct currents state the relations of the several factors involved when the current and voltage have reached constant value and cannot be correctly applied while the current or the voltage is increasing or decreasing.

Electric Transients.—While the current or voltage increases, as when the circuit is formed, energy is being stored in the magnetic and dielectric circuits; and part or all of this energy will be returned to the electric circuit when the current or voltage decreases. Phenomena that occur during the change of circuit conditions are of comparatively short duration and hence form part of Group III.

The chief characteristic of transients, as implied by the term, is their limited and usually short duration between two periods of stable conditions. Often the meaning given to the term *transients* is broadened so as to include other phenomena, such as unstable electric equilibrium, permanent instability and cumulative oscillations, that would not be included by the above definition.

Short-circuits, lightning discharges, switching under load, opening field circuits and similar sudden changes with their resultant effects on the system are included in this division. Any disturbance, even a small change in load or voltage, causes a readjustment of the energy content in the system and hence produces a transient condition. The interdependence of the impedance, dielectric stresses and other factors with the periodicity, voltage, current, etc., forms the vast and extremely important field of transient phenomena. This field is rapidly increasing in commercial importance and a better understanding of transients and related phenomena is now necessary for the successful operation of large power systems.

Alternating Currents.—In this volume are discussed the laws for energy transfer and transformation when the current and voltage periodically change directions; that is, for alternating currents. In the direct-current generator the voltage induced in the armature reverses in direction when passing from one pole to the next. In the armature the voltage, and hence the resulting current, is alternating, and it requires a continuous commutation to produce unidirectional voltage and current at the brushes. By eliminating the commutator and substituting collector rings the alternating voltage generated in the armature causes alternating currents to flow through the circuit. With the rapidly changing magnitude of current and voltage many new factors must be taken into consideration. The laws as developed for direct currents must be modified so as to include these additional factors before application can be made to an alternating-

current system. The phenomena of magnetic and dielectric induction are of fundamental importance and become essential parts of the calculations. The flow of energy need not be continuous in one direction as for direct currents, but by virtue of the storage possibilities in the magnetic and dielectric fields it may change in direction during each cycle. The field covered by alternating currents has been thoroughly investigated and the fundamental laws are now well known and can be expressed in the form of quantitative equations. By the use of *effective values* and *complex quantities* alternating currents may be expressed like continuous phenomena and the laws stated as completely and concisely as for direct currents.

ALTERNATING CURRENTS

CHAPTER I

THE ELECTRIC FIELD

In 1831 Joseph Henry in America and Michael Faraday in England showed that a conductor carrying an electric current was surrounded by magnetic and dielectric fields, and determined the fundamental relations existing between the current in the conductor and the electric field in the surrounding space. Any variation of the current in the conductor produces a corresponding change in the magnetic field. While the current is increasing, energy is stored in the magnetic field and while the current decreases, the magnetically stored energy is returned to the electric circuit.

Similarly energy is stored dielectrically (electrostatically) in the space between conductors differing in electric potential. With increasing difference of potential, energy is stored in the dielectric, and while the potential decreases, the dielectrically stored energy is returned to the electric circuit. These effects of the electric current and voltage on the surrounding space are termed *magnetic induction* and *dielectric* (*electrostatic*) *induction;* and the laws of the energy changes causing these magnetic and dielectric phenomena are of fundamental importance in the study of alternating currents.

Lines of Force.—In the description of his experiments Faraday developed the idea of *magnetic curves* and *lines of magnetic force.* Although *lines of force* had been used in earlier publications, it was Faraday who applied the concept to magnetic and dielectric phenomena. By means of a system of lines of force he mapped the properties of the magnetic and dielectric fields. The space surrounding a magnet as well as the electric field surrounding a conductor carrying electric power was pictured as containing a definite number of lines of force possessing clearly defined properties. This method has proved extremely useful for both practical and theoretical purposes. The designer

of electrical machinery bases his quantitative calculations on the number of lines of force; theoretical investigations are based on the distribution and interaction of the lines of force; and even superficial electricians make use of the lines of force as a guide. The phenomena of magnetic and dielectric induction are closely related and both can be expressed by similar systems of lines of force having almost identical properties.

Magnetic Lines of Force.—(*a*) All magnetic lines of force, or lines of induction, are continuous and closed upon themselves.

(*b*) The direction of a magnetic line of force at any point follows the resultant of the magnetic forces acting on a north pole placed at the point.

(*c*) Magnetic lines of force in the same direction repel and in opposite directions attract one another.

(*d*) The strength of a magnetic field is measured by the density of the lines of force.

Dielectric Lines of Force.—(*a*) All dielectric lines of force are continuous and terminate at conductors.

(*b*) The direction of a dielectric line of force at any point follows the resultant of the dielectric forces acting on a conductor of positive potential, or a positive charge, placed at the point.

(*c*) Dielectric lines of force in the same direction repel and in opposite directions attract one another.

(*d*) The strength of a dielectric field is measured by the density of the lines of force.

Fig. 1.1.—Electric circuit.

Energy Stored in an Electric Field.—Let Fig. 1.1 represent an electric circuit, in which a generator G supplies power to a load M. Inside the conductors A and B, some of the electric energy is converted into heat, as represented by the ri^2 losses. In the space surrounding the conductors magnetic and dielectric forces exist, and these can best be studied by means of magnetic and dielectric lines of force. In a single conductor A, the magnetic properties of the electric field can be pictured by lines of force as in Fig. 2.1.

The conductor is surrounded by a magnetic field as shown by the continuous lines drawn concentric with the conductor. The field is strongest at the surface of the conductor and rapidly decreases with increasing distance from the conductor, as is indicated by the spacing of the lines.

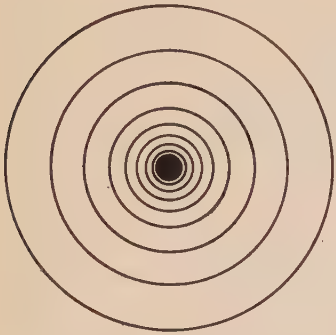


Fig. 2.1.—Magnetic field of single conductor.

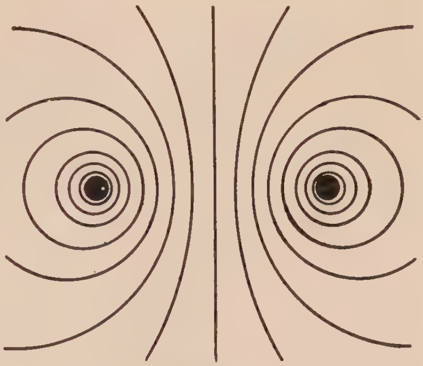


Fig. 3.1.—Magnetic field of circuit.

Likewise in Fig. 4.1, the dielectric stresses surrounding the conductor are represented by the lines drawn radially from the conductor. The strength of the dielectric field likewise decreases

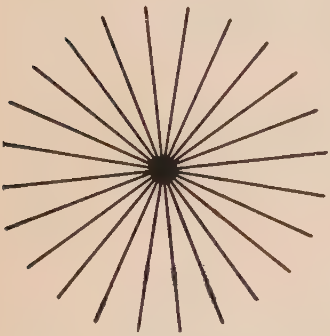


Fig. 4.1.—Dielectric field of single conductor.

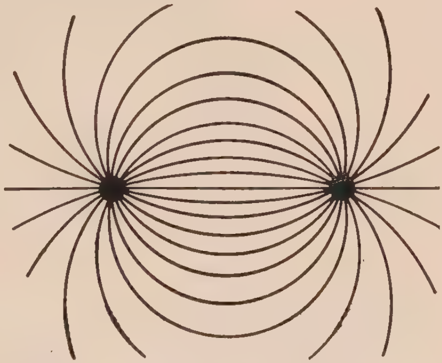


Fig. 5.1.—Dielectric field of circuit.

with the distance from the conductor, as indicated by the widening of the space between the lines.

If the conductors are brought close together, both the magnetic and dielectric stresses in the electric field will be modified, as represented by the distribution of the lines of force in Figs.

3.1 and 5.1. Superimposing the magnetic and dielectric lines a map of the *complete electric field* for the circuit is shown in Fig. 6.1.

Magnetic Energy.—The magnetic flux ϕ[1] surrounding a conductor is proportional to the current as expressed by equation (1.1).

$$\phi = Li \tag{1.1}$$

The proportionality factor L is called the *inductance* of the circuit. The magnetic field represents energy, stored in the space surrounding the conductor, and hence requires energy from the electric circuit to produce it. Expressing power in terms of current and voltage we have equation (2.1) in which $_Le$ is called the inductance voltage.

$$p = {}_Lei \tag{2.1}$$

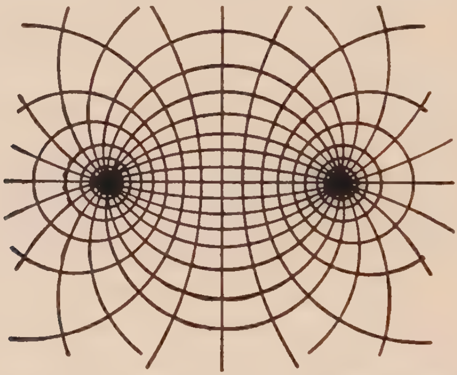


FIG. 6.1.—Dielectric and magnetic fields, or electric field of circuit.

The voltage $_Le$ required to form the field is directly proportional to the rate of change of the flux.

$$_Le = \frac{d\Phi}{dt} \tag{3.1}$$

Hence by equation (1.1)

$$_Le = L\frac{di}{dt} \quad = N\, d\phi \tag{4.1}$$

[1] A constant multiplying factor $N \cdot 10^{-8}$ must be inserted with ϕ in equations (1.1) and (3.1) if the units used are L in henrys, e in volts, i in amperes, N number of turns of the electric circuit interlinking the flux lines, in order to have ϕ expressed in lines of force or maxwells.

$$\frac{N(\text{turns})\,\phi(\text{maxwells})}{10^8} = L(\text{henrys})\,i(\text{amp.}) \tag{5.1}$$

The energy supplied by the power p in forming the magnetic field is:

For ploting ener curve

$$dW = pdt \tag{6.1}$$

From equations (2.1) and (4.1):

$$W = L\int_0^i idi \tag{7.1}$$

or

$$W = \frac{Li^2}{2} \tag{8.1}$$

The energy stored magnetically in the electric field surrounding a conductor is proportional to the square of the current. When the current decreases, the energy is returned to the circuit, for if i and therefore ϕ decrease, then $\frac{di}{dt}$ and hence ${}_Le$ are negative, which means that the energy is returned to the electric circuit.

The practical unit of inductance, L, is the *henry*. In any consistent system of units a circuit possesses one unit of inductance if *a unit rate of change* of current in the circuit generates or consumes one unit of voltage. If the current changes *at the rate of one ampere per second* and the voltage generated or consumed is *one volt* then the inductance in the circuit is *one henry*.

Dielectric Energy.—For the dielectric field exactly similar relations exist. The dielectric flux Ψ[1] is proportional to the voltage.

$$\Psi = Ce \tag{9.1}$$

The proportionality factor C is called the *condensance* (capacitance) of the circuit. The dielectric field represents energy stored in the dielectric. This energy must be supplied by the electric circuit when the dielectric field is established.

$$p = {}_cie \tag{10.1}$$

In equation (10.1) the power required to produce the dielectric field surrounding the conductor is expressed in terms of the voltage e and the condensance current ${}_ci$. The condensance current

[1] A multiplying factor $\frac{1}{3.77 \cdot 10^{10}}$ must be inserted with Ψ in equations (9.1) and (12.1) if the units used are C in farads, e in volts, i in amperes, in order to have Ψ expressed in dielectric lines of force.

$$\frac{\Psi(\text{lines of force})}{3.77 \cdot 10^{10}} = C(\text{farads})\, e(\text{volts}) \tag{11.1}$$

is directly proportional to the rate of change in the dielectric flux.

$$ {}_ci = \frac{d\Psi}{dt} \tag{12.1} $$

From equations (9.1) and (12.1):

$$ {}_ci = C\frac{de}{dt} \tag{13.1} $$

$$ dW = pdt \tag{14.1} $$

$$ W = C\int_o^e ede \tag{15.1} $$

$$ W = \frac{Ce^2}{2} \tag{16.1} $$

Hence the energy stored dielectrically in the electric field surrounding a conductor is proportional to the square of the voltage. When the voltage decreases the energy is returned to the electric circuit, for if e and therefore Ψ decrease, then $\frac{de}{dt}$ and hence ${}_ci$ are negative, which means that the energy is returned to the electric circuit.

The unit of condensance (capacity, capacitance), C, is the *farad.* In any consistent system of units a circuit possesses one unit of condensance if *a unit of change* of voltage produces (or absorbs) one unit of current. If the voltage changes *at the rate of one volt per second* and the current produced (or absorbed) is *one ampere,* the condensance of the circuit is *one farad.* The farad is too large a unit for practical use and hence in most commercial problems the condensance is measured in microfarads.

$$ 1 \text{ farad} = 10^6 \text{ mf.} \tag{17.1} $$

Kinetic Energy.—In dealing with new phenomena it is well to compare the derived expressions with the laws of similar phenomena previously studied.

The storing of energy in a moving body follows relations exactly analogous to those just derived for dielectric and magnetic fields. Using the customary notation for mass (M), force (F), velocity (v), acceleration (a), power (p) and energy (W), we have the relations given in equations (18.1) to (24.1).

$$ F = Ma \tag{18.1} $$

$$ a = \frac{dv}{dt} \tag{19.1} $$

$$p = Fv \tag{20.1}$$

$$dW = pdt \tag{21.1}$$

$$dW = Mvdv \tag{22.1}$$

$$W = M\int_o^v vdv \tag{23.1}$$

$$W = \frac{Mv^2}{2} \tag{24.1}$$

The energy stored kinetically in a moving mass is proportional to the square of the velocity. If the moving body is connected to a machine, as the reciprocating parts of a steam engine, when the velocity is reduced the energy is returned to the system. In such reciprocating parts energy is stored when the velocity of the mass increases, and returned when the velocity decreases.

By comparing equations (8.1), (16.1) and (24.1) it is readily seen that inductance L, condensance C and mass M are similar coefficients representing the amount of energy that can be stored in the system for any given current, voltage or velocity.

In order to use these equations for numerical calculations the proper units and reduction factors must be used. For commercial work the following units are in general use:

Energy in a Magnetic Field:

$$W(\text{joules}) = \frac{L(\text{henrys})i^2(\text{amp.})}{2} \tag{25.1}$$

Energy in a Dielectric Field:

$$W(\text{joules}) = \frac{C(\text{microfarads})e^2(\text{volts})}{2 \times 10^6} \tag{26.1}$$

Energy in a Moving Body:

$$W(\text{ergs}) = \frac{M(\text{grams})v^2(\text{cm. per sec.})}{2} \tag{27.1}$$

$$W(\text{joules}) = \frac{M(\text{kg.})v^2(\text{meters per sec.})}{2} \tag{28.1}$$

$$W(\text{ft.-lb.}) = \frac{M(\text{lb.})v^2(\text{ft. per sec.})}{2 \times 32.2} \tag{29.1}$$

1 joule = 1 watt-sec. = 10^7 ergs = 0.7376 ft.-lb.
= 0.2389 g.-cal. = 0.102 kg.-m. = 0.000948 B.t.u.
1 ft.-lb. = 1.356 joules = 0.3239 g.-cal. = 0.1383 kg.-m.
= 0.001285 B.t.u. = 0.0003766 watt-hr.
1 B.t.u. = 1,055 joules = 778.1 ft.-lb. = 252 g.-cal.
= 0.2930 watt-hr.

The close analogy existing between the magnetic and dielectric (electrostatic) fields can best be shown by arranging the corresponding quantities in tabular form as in Table I. The equations will also serve as definitions for terms used when discussing the properties of the magnetic and dielectric fields.

The magnetic and dielectric phenomena are caused by energy changes. The laws for storing and returning energy in the

TABLE I[1]

Magnetic field	Dielectric field
Magnetic flux: $\Phi = Li\dfrac{10^8}{N}$ lines of magnetic force	*Dielectric flux:* $\Psi = 3.77 \cdot 10^{10}\, Ce$ lines of dielectric force
Inductance voltage: ${}_Le = N\dfrac{d\Phi}{dt}10^{-8} = L\dfrac{di}{dt}$ volts	*Condensance, permittance (capacity) current:* ${}_Ci = \dfrac{1}{3.77 \cdot 10^{10}}\dfrac{d\Psi}{dt} = C\dfrac{de}{dt}$ amp.
Magnetic energy: $W = \dfrac{Li^2}{2}$ joules	*Dielectric energy:* $W = \dfrac{Ce^2}{2}$ joules
Magnetomotive force: $\mathfrak{F} = Ni$ amp.-turns	*Electromotive force:* $e =$ volts
Magnetizing force: $f = \dfrac{\mathfrak{F}}{l}$ amp.-turns per cm.	*Electrifying force or voltage gradient:* $G = \dfrac{e}{l}$ volts per cm.
Magnetic-field intensity: $\mathfrak{H} = 4\pi f 10^{-1}$ lines of magnetic force per $\overline{\text{cm.}}^2$	*Dielectric-field intensity:* $K = \dfrac{G}{4\pi v^2}$ lines of dielectric force per $\overline{\text{cm.}}^2$
Magnetic density: $\mathfrak{B} = \mu\mathfrak{H}$ lines of magnetic force per $\overline{\text{cm.}}^2$	*Dielectric density:* $D = \kappa K$ lines of dielectric force per $\overline{\text{cm.}}^2$
Permeability: μ	*Condensivity, permittivity or specific capacity:* κ
Magnetic flux: $\Phi = A\mathfrak{B}$ lines of magnetic force	*Dielectric flux:* $\Psi = AD$ lines of dielectric force

$v = 3 \times 10^{10}$ = velocity of electromagnetic field in space = velocity of light

[1] STEINMETZ: "Electric Discharges, Waves and Impulses and Other Transients," p. 17 (slightly modified). McGraw-Hill Book Company, Inc.

magnetic field are exactly analogous to the laws for a like energy transformation in the dielectric field. Much confusion has been caused by erroneous metaphysical concepts of the nature of the so-called electrostatic (dielectric) phenomena. It should be

kept clearly in mind that the energy or *charge* in all dielectric (electrostatic) phenomena is distributed in the dielectric outside the conductor, and does not consist of something smeared on the surface of the conductor. A serious difficulty in studying dielectric phenomena is found in the misleading terminology still in common use. Such terms as *electrostatic electricity*, *capacity*, *specific inductive capacity* and *charging current* lead the student astray by giving the impression that dielectric (electrostatic) phenomena are radically different from those in the magnetic or electric circuits. By referring to Table I it is evident that by using the terms dielectric, condensance, condensive reactance, condensance current and permittivity or condensivity, the close analogy between magnetic and dielectric phenomena may be more easily kept in mind, and the study of the complete electric field greatly simplified.

Magnetic, Dielectric and Electric Circuits.—Since all magnetic lines of force are closed upon themselves, forming a complete circuit, it is customary to speak of the *magnetic circuit* in a manner analogous to the *electric circuit.* The magnetic field in a dynamo or motor completes a circuit through the field poles, the yoke, air gap and armature iron, thus forming a magnetic circuit. Likewise all dielectric lines of force are continuous and terminate in conductors. The path taken by the dielectric line of force is called the *dielectric circuit.* The close analogy existing between magnetic, dielectric and electric circuits can best be shown by arranging the corresponding quantities in tabular form as in Table II. The three circuits are interdependent and, in general, exist simultaneously in all electromagnetic-dielectric phenomena. The relative magnitude may vary greatly and in many cases one or even two of the circuits may be negligibly small and may be omitted from the calculations, but intrinsically all three are present. For a unidirectional constant current the magnetic field remains constant; and similarly for a unidirectional constant voltage the dielectric field is constant. With both the current and the voltage unidirectional and constant, the electric circuit alone transfers energy and hence is the only circuit that enters in the problem. But in alternating currents both the voltage and the current are continually varying in magnitude and, moreover, for each successive half cycle reversing in direction. Therefore, in the study of alternating currents energy changes occur continuously

TABLE II

Electric circuit	Dielectric circuit	Magnetic circuit
Electromotive force, voltage: e, volts	*Electromotive force:* e, volts 1 stat volt = 300 volts	*Magnetomotive force:* $\mathfrak{F} = 0.4\pi NI$ gilberts
Resistance: $R = \frac{e}{i}$, ohms	*Elastance:* $S = \frac{1}{C} = \frac{3.77 \cdot 10^{10}e}{\psi}$ darafs	*Reluctance:* $\mathfrak{R} = \frac{\mathfrak{F}}{\phi}$ oersteds
Conductance: $G = \frac{i}{e}$, mhos	*Condensance, capacitance, permittance or capacity:* $C = \frac{\psi}{3 \cdot 77 \cdot 10^{10}e}$ farads	*Permanence:* $P = \frac{1}{\mathfrak{R}}$ *Inductance:* $L = \frac{\phi}{i}10^{-8} = \frac{N\phi}{I}10^{-8}$ henrys
Electric current: $i = Ge = \frac{e}{R}$ amp.	*Dielectric flux (dielectric current):* $\psi = 4\pi \cdot 3 \cdot 10^9 Ce$ $= 3.77 \cdot 10^{10}Ce$ $= 3.77 \cdot 10^{10}\frac{e}{S}$ lines of dielectric force	*Magnetic flux (magnetic current):* $\phi = Li10^{-8}$ lines of magnetic force, maxwells. One line = 1 maxwell One weber = 10^8 maxwells
Electric power: $P = ei = Ri^2 = Ge^2$ watts	*Dielectric energy:* $W = \frac{\psi e}{7.54 \cdot 10^{10}} = \frac{Ce^2}{2}$ joules	*Magnetic energy:* $W = \frac{\phi i}{2}10^{-8} = \frac{Li^2}{2}$ joules
Electric-current density: $I_d = \frac{i}{A}$ amp. per $\overline{\text{cm.}}^2$	*Dielectric-flux density:* $D = \frac{\psi}{A} = \kappa K$ lines per $\overline{\text{cm.}}^2$	*Magnetic-flux density:* $\mathfrak{B} = \frac{\phi}{A}$ lines per $\overline{\text{cm.}}^2$ One maxwell per $\overline{\text{cm.}}^2$ = 1 gauss
Electric gradient: $G' = \frac{e}{l}$ volts per cm.	*Dielectric gradient:* $G' = \frac{e}{l}$ volts per cm.	*Magnetic gradient:* $H = \frac{\mathfrak{F}}{l}$ amp.-turns per cm.
Conductivity: $Y = \frac{I_d}{G'}$ mhos per $\overline{\text{cm.}}^3$	*Condensivity, permittivity, or specific capacity:* $\kappa = \frac{D}{K}$	*Permeability:* $\mu = \frac{B}{H}$
Resistivity: $\rho = \frac{1}{Y} = \frac{G'}{I_d}$ ohms per $\overline{\text{cm.}}^3$	*Elastivity:* $\sigma = \frac{1}{\kappa} = \frac{K}{D}$	*Reluctivity:* $\nu = \frac{H}{\mathfrak{B}}$

and simultaneously in the interlinked magnetic, dielectric and electric circuits.

PROBLEMS

1.1. A bullet weighing 8 oz. is projected from a gun with a velocity of 2,250 ft. per sec. An incandescent lamp takes 25 watts. How long must the

lamp burn to consume an amount of energy equal to the kinetic energy of the bullet?

2.1. Find the height to which 1 ton must be raised in order to have the same amount of potential energy as is consumed by a 40-watt Mazda lamp in 1 hr.

3.1. An air-cored solenoid has an inductance of 0.052 henry. How much energy is stored in the magnetic field when 30 amp. flow through the coil? Compare the amount of magnetic energy in the coil to the amount consumed by a 25-watt incandescent lamp in 3 sec. In $\frac{1}{20}$ sec.

4.1. Let the current flowing through the solenoid in problem 3.1 vary from 0 to 100 amp. Plot a curve with amperes as abscissæ and joules as ordinates.

5.1. A condenser has 280-mf. condensance. How much energy is stored in the condenser when 625 volts are impressed across its terminals? 100 volts? 2,250 volts? Express the answers in joules, gram-calories and foot-pounds.

6.1. Let the voltage impressed on the condenser in problem 5.1 vary from 0 to 3,000 volts. Plot a curve with volts as abscissæ and joules as ordinates.

7.1. In a water-cooled transformer a system of pipes is placed in the oil, and water passed through to remove the heat produced by the copper and iron losses. Given a 300-kv.a. transformer operating at full load with an efficiency of 98.2 per cent. The temperature of the water on entering the transformer is 16°C. and on leaving 54°C. How much water is flowing through the transformer pipes? Give answer in pounds per hr.

8.1. What must be the velocity of a body weighing 6 oz. if its kinetic energy is equal to the energy stored in a condenser of 750 mf. when 850 volts are impressed on its terminals? For 2,500 volts?

9.1. Find the condensance of a condenser that, at 1,250 volts, would store the same amount of energy as is consumed by a 40-watt incandescent lamp in 7 sec. In $\frac{1}{60}$ sec.

10.1. The field of a certain dynamo has 50 henrys inductance. Find the energy stored magnetically when a current of 3.2 amp. is flowing in the field winding.

11.1. An electromagnet has a self-inductance of 180 henrys and takes 53.2 amp. What energy is stored in the field? Find the condensance of a condenser that would store the same amount of energy at 1,200 volts. What must be the velocity of a mass of 15 grams in order to have an equal amount of kinetic energy?

12.1. How much heat expressed in B.t.u.'s would be generated per min. in a flatiron having 28.2 ohms resistance if connected to a 124-volt circuit? If the electric power cost $5\frac{1}{2}$ cts. per kw.h., what is the cost per B.t.u. of heat?

13.1. Find the work in ergs and the power expended in kilowatts, in raising (velocity constant) a mass of 2,840 lb. to a height of 480 ft. in 1 min. What would be the condensance of a condenser that would store the same amount of energy, if connected to a circuit having 2,400 volts?

14.1. The field of a motor has 114 henrys inductance and takes 4.2 amp. at full load. Find the energy stored magnetically in the field. Give the answer in terms of joules, foot-pounds, B.t.u.'s and gram-calories.

CHAPTER II

GENERATION OF ELECTROMOTIVE FORCE

WAVE FORMS

When the magnetic lines of force are cut by an electrical conductor an *electromotive force* is generated in the conductor. The voltage produced is proportional to the length of the conductor, the intensity of the magnetic field and the speed of the motion perpendicular to the magnetic field and the direction of the conductor, or to the rate of cutting lines of force.

$$e \propto \frac{d\phi}{dt} \tag{1.2}$$

$$1 \text{ volt} \approx 10^8 \text{ lines of force per sec.} \tag{2.2}$$

If the conductor forms a closed circuit the voltage produces a current. Faraday showed that the *relative* motion of the field and the conductor is the determining factor. In small alternators the field is usually stationary as in direct-current generators while the conductor on the rotating armature cuts the lines of force, thus generating voltage. In larger generators more economical designs are obtained by placing the fields on the rotating spider, while the armature conductors are stationary, surrounding the moving part. In the transformer the field moves, due to the periodic increase and decrease of the current, while both the primary and secondary conductors are stationary.

Direction of Generated Voltage.—In determining the direction of the generated voltage several rules or devices have been suggested.

Fleming's Rule.—If the thumb, forefinger and middle finger of the right hand are all set perpendicularly to each other so as to represent the three coördinates in space, the thumb pointed in the direction of the motion of the conductor relative to the magnetic field, and the forefinger in the direction of the lines of force, then

the middle finger will point in the direction in which the generated e.m.f. tends to send the current of electricity, Fig. 1.2.

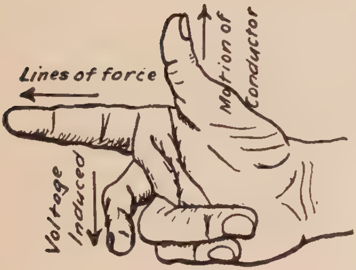


Fig. 1.2.

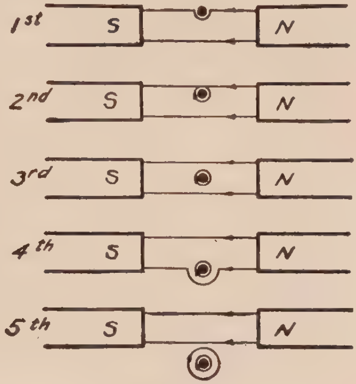


Fig. 2.2.

Lenz's Law.—The generated e.m.f. always tends to send a current in such a direction as to oppose the change in flux which produces it.

An easily remembered method is found in visualizing the relative motion of the conductor and the field and referring to the properties already ascribed to the lines of force. In Fig. 2.2 let the conductor relative to the field assume successively the first to the fifth positions. When the conductor moves across the lines of force these bend and form small circles around the conductor indicated in the figure. Given the direction of the field, it is evident that the conductor reaches the second and following positions encircled by lines of force whose direction indicates that the voltage generated tends to cause a current to flow downward in the conductor. It is apparent that if either the direction of the field or the relative direction of the motion of the conductors and the flux were reversed, the circular lines around the conductor would also be in the opposite direction, and hence indicate a voltage tending to send a current upward in the conductor.

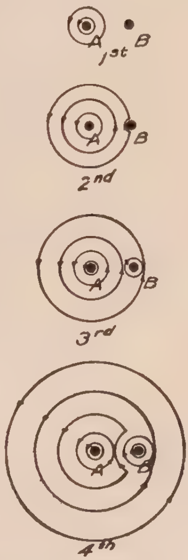


Fig. 3.2.

By increasing or decreasing the current in a conductor the field expands and contracts. In Fig. 3.2 let *A* and *B* represent the

parallel conductors, and let a current of increasing strength be sent through A, in the upward direction. Then the direction of the field around A is indicated by the arrows, and the intensity increases with the strength of the current by the number of lines in the four successive time intervals. With increasing current the field around A expands and the lines of force cut conductor B. The direction of the arrows in the lines around B shows that the induced voltage is in the opposite direction to the voltage in A. With decreasing current in A the lines of force contract and move toward the conductor. Hence lines will encircle B in the opposite direction, indicating an induced voltage in B in the same direction as the voltage in A. By visualizing the direction and relative motion of the lines of force and the conductors, the direction of the induced voltage can readily be determined.

Wave Forms.—Since the e.m.f. at any instant depends directly on the rate of cutting lines of force, this determines the shape of

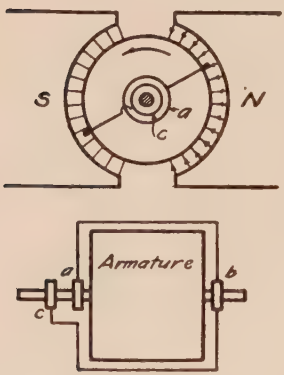


Fig. 4.2.

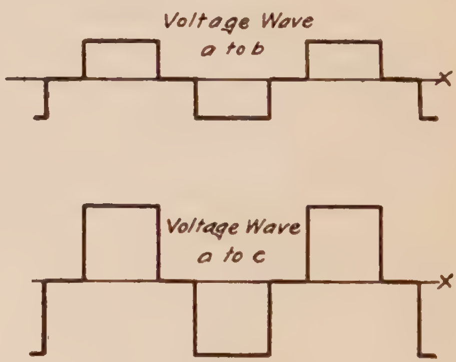


Fig. 5.2.

the wave from an alternator. The rate of cutting will depend upon the distribution and direction of the magnetic flux, the arrangement and the number of conductors in series, and the speed. From equation (1.2) it is evident that while a conductor moves at a uniform speed across a uniform field the voltage will be constant. If the field is reversed for half the revolution, as in Fig. 4.2, the voltage also reverses. In Fig. 5.2 is shown the form of the wave produced by a single conductor moving at right angles in a uniform field as illustrated in Fig. 4.2 between

rings a and b. Evidently the voltage wave between a and c will have the same rectangular shape but of double amplitude as shown in Fig. 5.2 since the two conductors are in symmetrical positions and connected in series. In those portions not covered by the poles no lines appear and hence no voltage is produced.

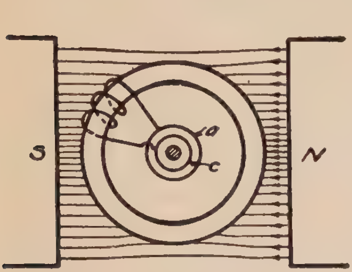


FIG. 6.2.

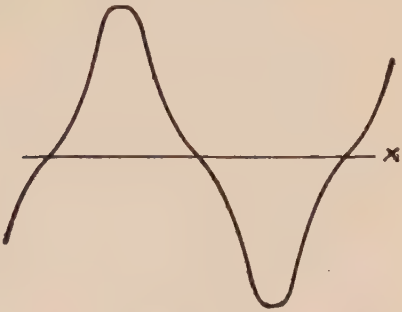


FIG. 7.2.

If the distribution of the flux be not uniform but concentrated at the center of the poles as in Fig. 6.2, the resulting wave for a concentrated winding becomes peaked, as shown in Fig. 7.2.

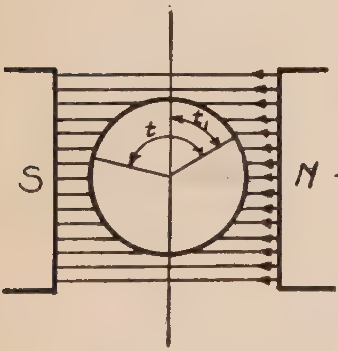


FIG. 8.2.

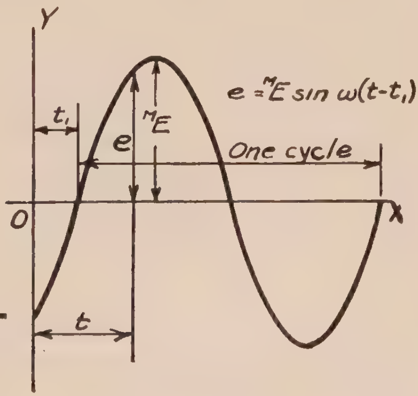


FIG. 9.2.

The shape of the wave is affected by the number and spacing of the conductors between the rings as well as by the density and direction of the lines of force. In a uniform field, as in Fig. 8.2, the rate of cutting magnetic lines by a single conductor will depend upon the angular position of the moving conductor. The voltage wave will under these conditions be a simple sine wave,

Fig. 9.2, and the relation between the instantaneous and maximum values is given by equation (3.2):

$$e = {}^{M}E \sin 2\pi f(t - t_1) \tag{3.2}$$

e = instantaneous voltage.
${}^{M}E$ = maximum voltage.
f = frequency or number of complete cycles in 1 sec.
t_1 = epoch or phase of voltage wave.
t = time in seconds.

It may be well to note that if a sine voltage wave, as expressed by equation (3.2), be impressed on a circuit of constant ohmic resistance the resulting current in the circuit will likewise be of sine-wave form and will be in phase with the voltage. This necessarily follows from Ohm's law since at any instant the current i must be equal to the quotient of the voltage, e, and the constant resistance, R. The phase relation is illustrated by the oscillogram in Fig. 10.2.

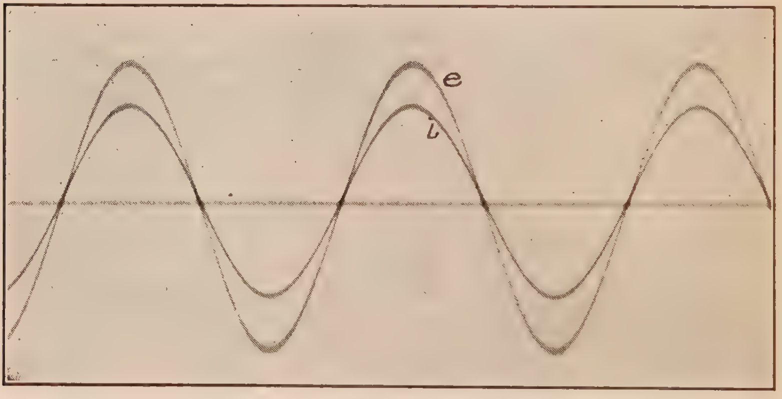


FIG. 10.2.

The current curve i and the voltage curve e are of zero value at the same instant and likewise reach their maximum values at the same time. If equation (3.2) gives the impressed voltage wave, then the current is expressed by equation (4.2).

$$i = {}^{M}I \sin 2\pi f(t - t_1) \tag{4.2}$$

i = instantaneous value of current.
${}^{M}I$ = maximum value of current.
f = frequency in cycles per second.
t_1 = epoch or phase of current wave.
t = time in seconds.

On small systems of low voltage and few machines, the shape of the generated voltage wave may be of little importance. Thus in Fig. 11.2 is shown an oscillogram of a voltage wave from a 60-kw., 1,100-volt, 1-ϕ, 60-cycle alternator supplying an independent lighting load in a satisfactory manner. In larger systems with several generators operating in parallel and with higher potential on the system a better wave form is necessary. With the increase in the size of the system the shape of the generated voltage wave becomes of increasing importance. For several reasons, as will be explained in detail[1] later, the ideal wave

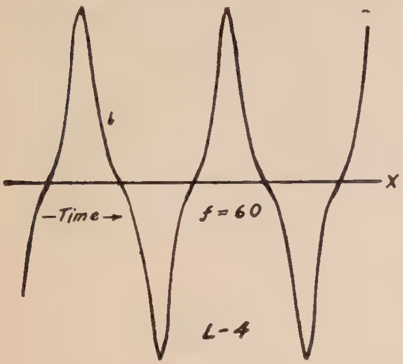


Fig. 11.2

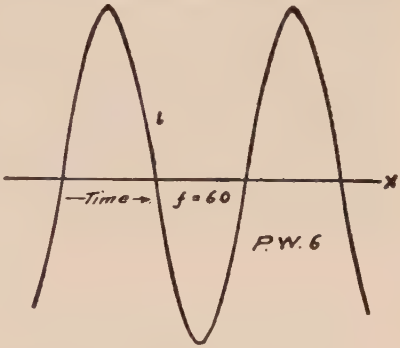


Fig. 12.2.

form is simple harmonic or sinusoidal, as expressed by equations (3.2) and (4.2). Modern alternators, especially for large systems, closely approach the ideal form. In Fig. 12.2 is shown an oscillogram of a 10,000-kw., 6,600-volt, three-phase, 60-cycle alternator. The shape of the wave is very nearly simple harmonic.

In commercial systems the voltage and current waves are modified by many factors in the system outside of the machine itself. The relative amounts, and their location in the system, of inductance, condensance and resistance; the iron in transformers and other apparatus; the location, size and design of synchronous and induction motors; reaction of leading and lagging currents; load variations and many other factors modify and distort the original wave supplied from the generator into an endless variety of forms. As an illustration of distorted waves, Fig. 13.2 shows an oscillogram of busbar voltage, curve v_1, and the no-load current, curve v_2, of a 500-kv.a., 2,300-volt, 60-cycle, synchronous motor floating on the line. While the

[1] See Chap. XXIV.

voltage is nearly of the sine form the current wave is so greatly distorted as to bear no resemblance to the simple sine wave. Under full load the current wave, curve v_2, Fig. 14.2, approaches

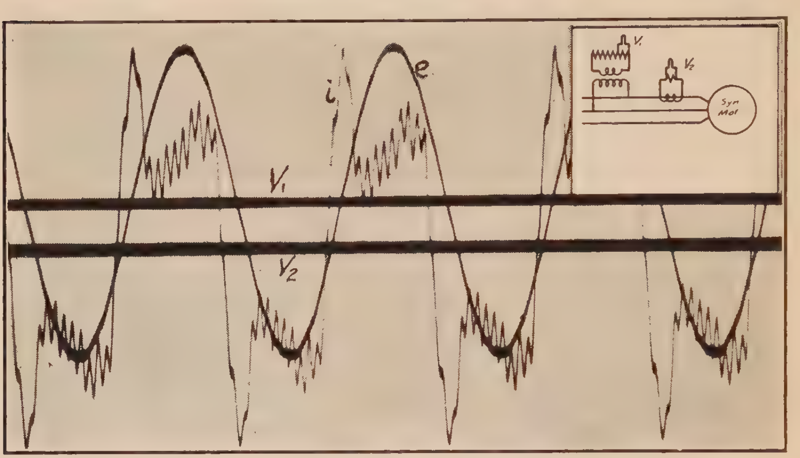


FIG. 13.2.

the form of the voltage wave, curve v_1. While the voltage or current waves may be greatly distorted in certain parts of the system, as illustrated by the no-load current, on the synchro-

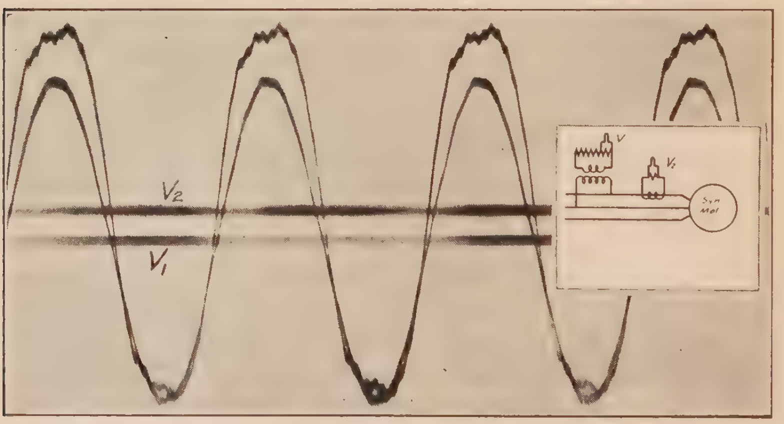


FIG. 14.2.

nous motor the power involved is relatively small compared to the total power of the system. The busbar voltage wave should closely approach the ideal sine form as shown by curve v_1, Fig. 13.2.

As a rule, operating difficulties as well as losses from cross-currents are reduced to a minimum when the voltage wave in all parts of the system closely approaches the simple sine form. To gain this much desired end a number of factors must be given careful consideration both in the design of the generators and motors as well as in every part of the transmission line and distribution network. The more important factors are discussed in later chapters.

To appreciate the need for eliminating these distortions it is first necessary to understand fully the laws that govern alternating-current phenomena, and to be thoroughly familiar with their application of simple sine waves.

Instantaneous and Maximum Values.—An alternating current or voltage wave passes through an infinite number of values in each cycle. In engineering problems the *instantaneous*, *maximum*, *average* and *effective* values are of importance.

Let:

i, e = instantaneous values.
${}^{M}I, {}^{M}E$ = maximum values.
${}^{av}I, {}^{av}E$ = average values.
I, E = effective values.

The relation between the instantaneous and maximum value in *simple sine waves* is given from the equations shown in (5.2) and (6.2).

$$i = {}^{M}I \sin (\omega t \pm \gamma_1) \tag{5.2}$$
$$e = {}^{M}E \sin (\omega t \pm \gamma_2) \tag{6.2}$$

Average Value.—The average value, Fig. 15.2, is found by integrating the area for one-half wave and dividing by the base. Letting ωt be the variable,

$${}^{av}I = \frac{{}^{M}I}{\pi} \int_0^{\pi} \sin (\omega t)\, d(\omega t) = 0.636\, {}^{M}I \tag{7.2}$$

Effective Value.—In 1841 Joule proved that *the energy converted into heat when an electric current flows in a conductor is proportional to the square of the current.* Stated in the form of an algebraic equation Joule's law is expressed by equation (8.2) for direct currents.

$$J = RI^2T \tag{8.2}$$

J, in joules, is the energy expended by I amp. flowing through a circuit having R ohms resistance, in T sec.

In watts,

$$P = RI^2 \tag{9.2}$$

The quantity of energy converted into heat, expressed in calories, for the time T in seconds, is given by equation (10.2) for direct currents.

$$H = 0.24RI^2T \tag{10.2}$$

In alternating currents, the instantaneous current is continually varying throughout the cycle. Since the complete cycle repeats again and again, the average heating value may be

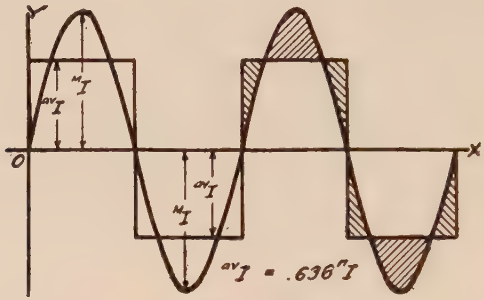


FIG. 15.2.

found by taking the sum of the instantaneous heating values through one or more complete cycles and dividing by the time for the same number of complete waves.

$$J = R\int_0^T i^2dt \tag{11.2}$$

$$H = 0.24\,R\int_0^T i^2dt \tag{12.2}$$

Equation (11.2) gives the heat in joules generated in one cycle, if T is the time for one complete cycle, and equation (12.2) gives the heat generated, in calories, during the cycle. If T represents time for several cycles, the equations give the heat generated during the given number of cycles; that is, for the length of time stated by the limits.

The power expended in heating the conductor is therefore proportional to the average of the squared instantaneous currents, as shown in equation (13.2)

$$P = \frac{R}{T}\int_0^T i^2dt \tag{13.2}$$

The direct current that would produce the same heating effect in a circuit having the same resistance would be equivalent to the *square root of the mean square of the instantaneous currents.*

The value is termed the *effective* alternating current, or the *root mean square.*

$$RI^2T = R\int_0^T i^2dt \qquad (14.2)$$

$$I = {}^{M}I\sqrt{\frac{1}{2\pi}\int_0^{2\pi}\sin^2(\omega t)d(\omega t)} = \frac{{}^{M}I}{\sqrt{2}} = 0.707\,{}^{M}I \qquad (15.2)$$

Hence by using *effective* values, Joule's law applies to alternating currents in the same form as for direct currents.

In Fig. 16.2 the squared and effective values are compared to the current wave. It is of importance to note that the effective

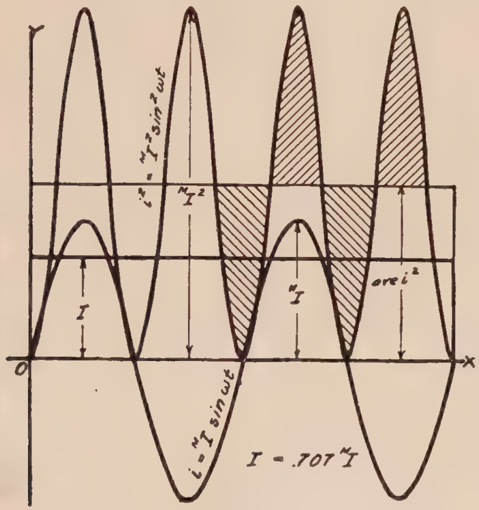


FIG. 16.2.

values appear as continuous positive values and not half positive and half negative as in Fig. 15.2 for the average values.

Alternating-current indicating voltmeters and ammeters give effective values, and in commercial work, unless specifically stated otherwise, volts and amperes always refer to effective values.

Form Factor.—The ratio between the effective and average values is called the form factor, as this gives an indication of the shape of the wave. For sine waves the form factor is 1.11.

Fundamental Equation.—Since voltage is generated by cutting lines of force, electrical generators are arranged so that the coil in which the e.m.f. is produced alternates in position relatively to a magnetic field, or flux, ϕ. Either the coil revolves in a field

or the flux passes in and out of the turns. In a complete cycle the total flux would be cut four times by each turn of wire in the coil. Hence, if f is the number of complete cycles per second, Φ the total flux and n the number of turns in series, the average voltage generated would be

equ. $$ {}^{av}E = 4n\Phi f 10^{-8} \text{ volts} \qquad (16.2) $$

This is the fundamental equation of electrical engineering.

In the preceding paragraphs the ratios between the average, maximum and effective values have been derived.

$$ {}^{M}E = \frac{\pi}{2}\,{}^{av}E = 2\pi n\Phi f 10^{-8} \text{ volts} \qquad (17.2) $$

$$ E = \frac{{}^{M}E}{\sqrt{2}} = \sqrt{2}\pi n f\Phi 10^{-8} = 4.44 n f\Phi 10^{-8} \text{ volts} \qquad (18.2) $$

Equation (18.2) is the fundamental relation for alternating-current induction for sine waves.

Frequency.—In alternators the armature conductors pass alternately under north and south poles and hence the generated voltage wave completes a cycle for each pair of poles. The number of cycles per sec. is termed the frequency of the alternating current. In the United States 60 and 25 cycles per sec. are standard frequencies for light and power. In Europe 50 and 25 cycles per sec. are in general use. Other frequencies are in commercial use both in America and Europe, but the advantages that come from standardization in frequency are very great so that by far the greater portion of industrial power plants operate at frequencies of 60, 50 or 25 cycles.

PROBLEMS

1.2. An alternator has 36 poles. At what speed must it run to give a voltage of 25 cycles frequency? 60 cycles?

2.2. A 10-pole alternator has a normal speed of 1,650 r.p.m. What is the frequency of the generated voltage?

3.2. What is the normal speed of a 60-cycle, eight-pole alternator?

4.2. A 110-volt battery is connected to a circuit containing resistance only. The direction of the voltage is reversed 50 times per second. The resistance is 15 ohms. Plot the voltage and current waves to scale with time as abscissæ.

5.2. The maximum value of a voltage wave of sinusoidal shape is 127 volts.

(*a*) What is the effective value?

(*b*) What is the average value?

6.2. In a circuit having 15 ohms resistance 2,174 cal. of heat are generated in 5 min. by a 25-cycle sinusoidal current.

(*a*) Find the average, effective and maximum values of the current in the circuit.

(*b*) Find the average, effective and maximum values of the voltage impressed on the circuit.

7.2. A 60-cycle sine wave whose average value is 245 volts is impressed on a circuit having 32 ohms resistance.

(*a*) Find the effective, average and maximum values of the current.

(*b*) Find the amount of heat (calories) generated in 30 min.

8.2. The normal speed for a 24-pole alternator is 300 r.p.m. What is the frequency of the generated e.m.f.? How many poles would a 25-cycle alternator have for the same speed? A 50-cycle machine? How many poles has a turbo-alternator that gives 60 cycles at 3,600 r.p.m.?

9.2. In a testing laboratory a direct-current motor whose speed may be varied from 200 to 2,100 r.p.m. is connected to the shaft of an eight-pole alternator. Find the range in frequencies that can be produced in the volt-age generated by the alternator?

10.2. A 60-cycle alternator provides 50 amp. at 230 volts (effective values) for heating an oven (resistor type). What current would be required from a 25-cycle alternator at 224 volts, to give the same amount of heat per min.?

Conduc

Conductor Mo

$P\phi$ lines/N

$= \frac{RPM\ P\Phi}{60}$

$= 120 f P\Phi$

$N = 120f$

CHAPTER III

SYSTEMS OF UNITS. SYMBOLS

The elemental units in mechanics for length, mass and time, as defined in the metric system, are used in systems of units for electric quantities. The *centimeter* is the unit of *length*, the *gram* the unit of *mass*, and the *second* the unit of *time*, and for this reason the metric system is frequently referred to as the centimeter-gram-second system or the *c.g.s. system*. The decimal plan of the c.g.s. system is also adhered to in forming secondary electric units. Larger or smaller secondary units are obtained by multiplying the fundamental unit in the system by the factor 10^x, in which x may be any positive or negative integer. The names for the secondary units are obtained by using prefixes that indicate the relative magnitude of the unit with respect to the standard for the system.

The basic units in mechanics for both the c.g.s. and English systems used in electrical engineering are listed with conversion factors in Table III.

Several systems of units for electric quantities have been developed and are more or less in general use. Three are of outstanding importance—the *electrostatic or stat* system, the *electromagnetic or absolute or ab* system, and the *international or practical* system.

Electrostatic or Stat System.—The electrostatic system of units is the oldest and, while the units are to some extent still used, it is chiefly of interest as the first systematic coördination of quantitative measurement of electric quantities.

The system centers on the properties of the electric charge and the structure is based on:

1. Coulomb's law of attraction and repulsion between electrically charged bodies.
2. The definition of a unit electric charge.
3. The assumption that the unit charge produces 4π lines of dielectric flux.
4. The assumption that the permittivity of space is unity (essentially unity for air under atmospheric conditions).

The dimensions of the system are the *centimeter*, *gram*, *second*, and the *permittivity* factor.

TABLE III.—FUNDAMENTAL AND MECHANICAL UNITS. CONVERSION FACTORS

Quantity	Symbol	Defining equation	Metric system, c.g.s. units	Abbreviation	English system, foot-pound-second units	Abbreviation	Equivalent values
Length	l	Fundamental	Centimeter	cm.	Foot Inch	ft. in.	1 foot = 30.48 centimeters 1 inch = 2.54 centimeters
Mass	m	Fundamental	Gram	g.	Pound	lb.	1 pound = 453.59 grams
Time	t	Fundamental	Second	sec.	Minute Second	min. sec.	1 minute = 60 seconds
Area	A	$A = l_1l_2$	Square centimeter	sq. cm.; $\overline{\text{cm.}}^2$	Square foot Square inch	sq. ft. sq. in.	1 square foot = 929.03 $\overline{\text{cm.}}^2$ 1 square inch = 6.45 $\overline{\text{cm.}}^2$
Volume	V	$V = l_1l_2l_3$	Cubic centimeter	cu. cm.; cm.^{3}	Cubic foot Cubic inch	cu. ft. cu. in.	1 cubic foot = 28,317 $\overline{\text{cm.}}^3$ 1 cubic inch = 16.39 $\overline{\text{cm.}}^3$
Angle	α β γ	$\alpha = \frac{jl_1}{l_2}$	Radian Degree Minute Second	rad. deg.; ° min.; ′ sec.; ″	.... Degree Minute Second	.... deg.; ° min.; ′ sec.; ″	1 radian = 57° 17′ 45″ 1 degree = 60 minutes 1 minute = 60 seconds 1° = 60′ 1′ = 60″
Velocity (linear)	v	$v = \frac{dl}{dt}$	Centimeter per second	cm. per sec.	Feet per minute Feet per second	ft. per min. ft. per sec.	1 foot per minute = 0.508 centimeter per second 1 foot per second = 30.48 centimeters per second
Velocity (angular)	ω	$\omega = \frac{d\alpha}{dt}$	Radians per second	rad. per sec.			
Acceleration	a	$a = \frac{dv}{dt}$	Centimeter per second per second	cm. per sec. per sec.	Feet per sec. per sec.	ft. per sec. per sec.	1 foot per second per second = 30.48 centimeters per second per second
Force	F	$F = ma$	Dyne	dyne	Pound (av.)	lb.	1 pound = 4.448 × 10^5 dynes
Torque	T	$T = \frac{W}{\alpha}$	Dyne perpendicular-centimeter	dyne-perp.-cm.	Pound perp. foot	lb. perp. ft.	1 pound perp. foot = 1.357 × 10^7 dynes perp. centimeters
Power	P	$P = Fv$ $= EI$	Erg per second Watt	erg per sec. watt	Foot-pound per minute	ft.-lb. per min.	1 foot-pound per minute = 230.4 gram centimeters per second 1 foot-pound per minute = 0.0226 joule or watt second; 1 watt = 10^7 ergs per second
Energy	W	$W = Fvt = Fl$ $= EIt$ $= Pt$	Erg Joule	erg joule	Foot-pound	ft.-lb.	1 joule = 10^7 ergs 1 foot-pound = 1.357 × 10^7 ergs.
Modulus of elasticity	..	$\frac{F}{A}$	Dyne per square centimeter	Dyne-per cm.^{2}	Pound per square inch	lb. per sq. in.	1 pound per square inch = 2.8697 × 10^6 dynes per cm.^{2}
Temperature	$T°$	....	Centigrade	C., cent.	Fahrenheit scale	F.	1°F. = $\frac{5}{9}$°C. 0°C. = 32°F. on scale.

Coulomb's Law.—In 1785, Coulomb, a French engineer, determined experimentally the law of repulsion and attraction between electrically charged bodies. By means of a simple torsion balance he measured the force existing between pairs of small bodies electrically charged and found that: *The force of repulsion between two charges of the same kind of electricity is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between the charged bodies.* If Q and Q' represent the magnitude of the charges, d the distance, and F the force of repulsion, Coulomb's law is expressed by equation (1.3):

$$F \propto \frac{QQ'}{d^2} \tag{1.3}$$

If the charges Q and Q' are of the same magnitude, the distance d, 1 cm., and the force F, 1 dyne, then each charge would be of unit value. The *unit electrostatic charge* is that quantity of electricity with which a small body must be charged, so that if placed in air at a distance of 1 cm. from a similar body charged with an equal amount of the same kind of electricity, it produces an electrostatic force of repulsion of one *dyne* between the two bodies.

It is also assumed that each electric charge produces or consists of 4π lines of dielectric flux; that is, one line of force per $\overline{\text{cm.}}^2$ on the surface of the sphere of 1 cm. radius surrounding the unit charge. The unit electrostatic charge may also be defined in terms of the *electronic charge*, a natural quantitative unit. One electronic charge equals $4.774 \cdot 10^{-10}$ unit of electrostatic charge; or the unit electrostatic charge equals $2.09342 \cdot 10^9$ electronic charges.

Coulomb's investigations preceded by 35 years Ampere's discovery (1820) of the magnetic field surrounding electric currents; and the basis of quantitative units in the dielectric field was definitely established before electromagnetic phenomena were known to exist. The electrostatic system was later extended to include quantities relating to electric currents and magnetic fields. The quantitative relations of units in the electrostatic, electromagnetic, and international or practical systems are given in Table IV.

It is somewhat difficult, if not impossible, to gain clear concepts of the quantitative relations existing between voltage, permeance,

and electrostatic lines of force in the dielectric circuit from the basic definition that a unit electric charge on a sphere of unit radius produces one line of force per $\overline{\text{cm}}.^2$ of surface, that is, 4π lines of force. This important concept can be gained with little effort, however, by the same method as is generally used for finding the similar relations between voltage, resistance and current in the electric circuit. Consider two plane metal surfaces each 1 $\overline{\text{cm}}.^2$ placed parallel to each other, 1 cm. apart and let the space between them be either empty or filled with some material whose permittivity is unity. To establish an *electrostatic field represented by one dielectric line of force between the two surfaces, passing through the* $\overline{\text{cm}}.^3$ *between the plates, requires one statvolt or* 300 *volts.*

If the $\overline{\text{cm}}.^3$ volume between the two plates were filled with some material, as glass for example, whose permittivity is not unity but 4.8 (for the sample used), then one statvolt, that is 300 volts, would cause 4.8 dielectric lines of force to pass between the plates through the glass cube.

Electromagnetic, Absolute or Ab System.—The electromagnetic or ab system is centered on the properties of the magnetic pole and was developed independently of the electrostatic system. The quantitative ratio between the units in the two systems, that is, the number of electrostatic units in one electromagnetic unit of electric charge, as shown in Table IV, has been proved to be the velocity v of an electromagnetic-dielectric field in free space, which is the same as the velocity of light.

$$v = 3 \cdot 10^{10} \text{ cm. per sec.} \tag{2.3}$$

The electromagnetic system is based on:

1. The law of repulsion or attraction between magnetic fields.
2. The definition of the unit magnetic pole.
3. The assumption that the unit pole has 4π lines of magnetic flux.
4. The assumption that the permeability for space is unity (practically unity for air under atmospheric conditions).

The dimensions of the system are the *centimeter, gram, second,* and the *permeability factor.*

Consider two long slender magnets placed on the same straight line with the north poles of strength m and m' a distance d apart. If the magnets be of sufficient length the south poles would be

so far apart that any force produced by them would be negligible in comparison to the force between the two adjacent north poles. Under these conditions the *force of repulsion between the north poles would be directly proportional to the product of the strength of each pole and inversely proportional to the square of the distance between them, as expressed by equation* (3.3)

$$F \propto \frac{mm'}{d^2} \tag{3.3}$$

The law for the interaction between magnetic poles is therefore of the same form as Coulomb's law for the forces existing between electrically charged bodies.

Each of the above north poles would have unit strength or would be defined as *unit poles* if m equals m', the distance d, 1 cm., and the force of repulsion F, *one dyne.* It is also assumed that the unit magnetic pole has 4π lines of magnetic flux; that is, one line of force passes through or emanates from each $\overline{\text{cm}}.^2$ on the surface of a sphere of 1 cm. radius surrounding the unit pole.

On the basis of the definitions of a unit pole and the units of length, mass, time and permeability in combination with the established laws of electromagnetic phenomena, the electromagnetic system of units is constructed. To make the process clear it may be well to state briefly the derivation of units for (*a*) *magnetic-field strength*, (*b*) *electric current*, and (*c*) *electromotive force.*

(*a*) *Unit of Field Strength.*—Any region in which a magnetic pole, if placed in the given space, would be acted upon by a magnetic force is a magnetic field. From the definition of a unit magnetic pole and the law of inverse squares, equation (3.3), the number of lines of force per $\overline{\text{cm}}.^2$ or field strength H is at a distance d in air equal to $\frac{m'}{d^2}$. Substituting in equation (3.3), the mechanical force acting between the magnetic field and the magnetic pole is directly proportional to strength of the field H and the strength of the magnetic pole m.

$$F = Hm \tag{4.3}$$

If the magnetic field exerts a force of one dyne on a unit magnetic pole the field has unit strength.

(*b*) *Unit of Current.*—Electric currents are surrounded by magnetic fields. The strength of the magnetic field at any given point produced by a current in an element of the conductor is

directly proportional to the magnitude of the current and to the length of conductor element and inversely proportional to the square of the distance of the point from the conductor element.

$$H \propto \frac{Idl}{r^2} \tag{5.3}$$

Let the conductor be bent into an arc of 1 cm. radius. The current flowing in the conductor will be of unit value if for each centimeter of conductor length a magnetic field of unit strength is produced at the center of the arc; that is, the unit current flowing through the conductor forming a complete circumference of the circle of unit radius produces a field strength 2π units at the center of the circle and in direction at right angles to the plane of the circle.

The unit of current as specified in the preceding sentences is the unit of current in the electromagnetic or ab system and is called the *ab-ampere.*

$$1 \text{ ab-amp.} = 10 \text{ amp.} \tag{6.3}$$

(*c*) *Electromotive Force.*—When an electric conductor moves relatively to a magnetic field the voltage generated in the conductor is directly proportional to the time rate of cutting lines of force, or, to the rate of change in the number of interlinkages of the magnetic flux lines with the electric current.

$$e \propto \frac{d\phi}{dt} \tag{7.3}$$

If the lines of force are cut at the rate of one line per sec. the e.m.f. generated is *one ab-volt,* the unit of voltage in the electromagnetic system.

$$1 \text{ ab-volt} = 10^{-8} \text{ volt} \tag{8.3}$$

The quantitative relations of the electromagnetic units to the corresponding units of the electrostatic and the international practical units are given in Table IV.

International or Practical System.[1]—The definition for the unit

[1] For more extended information on systems of units the student is referred to the following bulletins of the U. S. Bureau of Standards:

Wolff, F. A., "The So-called International Units," *Bull.* 1, Vol. I, 1904.

Wolff, F. A., "Selection and Definition of the Fundamental Electrical Units to Be Proposed for International Adoption," *Bull.* 2, Vol. V, 1908.

Stratton, S. W., "Announcement of a Change in the Value of the International Volt," *Circ.* 29, 1910.

Dellinger, J. H., "International System of Electric and Magnetic Units," *Sci. Paper* 292, 1916.

TABLE IV.—ELECTRICAL UNITS, CONVERSION FACTORS

International c.g.s. system of practical units				Conversion factors $v = 3 \cdot 10^{10}$			Quantitative relations
Quantity	Symbol	Name of unit	Equations	Practical to electromagnetic	Electromagnetic to electrostatic	Practical to electrostatic	The International c.g.s. system of practical units; The electromagnetic absolute c.g.s. system of ab units; The electrostatic absolute c.g.s. system of stat units
Capacitance........ Condensance.......	C	farad	$C(\text{farads}) = \dfrac{1}{S(\text{darafs})} = \dfrac{Q(\text{coulombs})}{E(\text{volts})}$ $= \dfrac{kA(\text{cm.}^2)10^9}{4\pi v^2(\text{cm.}^2)l(\text{cm.})} = \dfrac{kA(\text{cm.}^2)}{113.1 \cdot 10^{11}\, l(\text{cm.})}$	10^{-9}	v^2	$9 \cdot 10^{11}$	1 farad = 10^{-9} abfarad = $9 \cdot 10^{11}$ statfarads
Charge..............	Q	coulomb	$Q(\text{coulombs}) = C(\text{farads})E(\text{wels})$				
Conductance.........	G	mho	$G(\text{mhos}) = \dfrac{1}{R(\text{ohms})} = \dfrac{I(\text{amp.})}{E(\text{volts})}$	10^{-9}	v^2	$9 \cdot 10^{11}$	1 mho = 10^{-9} abmho = $9 \cdot 10^{11}$ statmhos
Conductivity.........	γ	....	$\gamma = \dfrac{1}{\rho}$				
Current.............	i, I	ampere	$I(\text{amperes}) = \dfrac{E(\text{volts})}{R(\text{ohms})}$	10^{-1}	v	$3 \cdot 10^9$	1 ampere = 10^{-1} abampere = $3 \cdot 10^9$ stat-amperes
Dielectric flux........	ψ, Ψ	line of force	$\psi(\text{lines of force}) = 4\pi Q'(\text{stat coulombs})$ $= 3.77 \cdot 10^{10} Q(\text{coulombs})$ $= 3.77 \cdot 10^{10} C(\text{farads})E(\text{volts})$ $= \dfrac{kA(\text{cm.}^2)E(\text{volts})}{300 l(\text{cm.})}$				
Elastance...........	S	daraf	$S(\text{darafs}) = \dfrac{1}{C(\text{farads})} = \dfrac{E(\text{volts})}{Q(\text{coulombs})}$ $= \dfrac{4\pi v^2 l(\text{cm.})}{kA(\text{cm.}^2)10^9} = \dfrac{113.1 \cdot 10^{11}\, l(\text{cm.})}{kA(\text{cm.}^2)}$	10^9	$\dfrac{1}{v^2}$	$\dfrac{1}{9 \cdot 10^{11}}$	1 daraf = 10^9 abdarafs = $\dfrac{1}{9 \cdot 10^{11}}$ statdaraf

Elasticity	σ	....	$\sigma = \frac{1}{k}$				
Electromotive force / Electric potential	e, E	volt	$E(\text{volts}) = R(\text{ohms})I(\text{amperes}) = \frac{Q(\text{coulombs})}{C(\text{farads})}$ $= \frac{d\phi(\text{maxwells})10^{-8}}{dt(\text{seconds})}$	10^8	......	$\frac{1}{300}$	1 volt = 10^8 abvolts = $\frac{1}{300}$ statvolt
Energy	W	joule	$W(\text{joules}) = E(\text{volts})I(\text{amperes})t(\text{seconds})$ 1 joule = 1 watt second = 10^7 ergs = 0.000948 B.t.u. = 0.2389 gram calorie = 0.102 kg.-meter = 0.7376 ft.-lb.	10^7	......	10^7	1 joule = 10^7 abjoules = 10^7 statjoules
Impedance	z, Z	ohm	$Z(\text{ohms}) = \sqrt{r^2(\text{ohms}) + x^2(\text{ohms})}$ $z = r + jx = r + j({}_Lx - {}_cx)$				
Inductance	L	henry	$L(\text{henrys}) = \frac{4\pi\mu N^2(\text{turns})A(\text{cm.}^2)}{10^9 l(\text{cm.})}$	10^9	$\frac{1}{v^2}$	$\frac{1}{9 \cdot 10^{11}}$	1 henry = 10^9 abhenry = $\frac{1}{9 \cdot 10^{11}}$ stathenry
Magnetic flux	ϕ, Φ	line of force; tube of force, line of induction, maxwell; weber	1 maxwell = 1 line of force = 1 line of induction. 1 weber = 10^8 maxwells. $\phi(\text{maxwells}) = \frac{\mathfrak{F}(\text{gilberts})}{\mathcal{R}(\text{oersteds})}$ $= \frac{0.4\pi N(\text{turns})I(\text{amperes})}{\frac{l(\text{cm.})}{\mu A(\text{cm.}^2)}}$ $\phi\ (\text{webers}) = \frac{L(\text{henrys})I(\text{amperes})}{N(\text{turns})}$				
Magnetic-flux density	$\mathcal{B}$	gauss	1 gauss = 1 maxwell per cm.^{2} $\mathcal{B}(\text{gausses}) = \frac{\phi(\text{maxwells})}{A(\text{cm.}^2)}$ $= \mu H(\text{gilberts per cm.})$				
Magnetic intensity	H	gilberts per centimeter	$H(\text{gilberts per cm.}) = \frac{\mathcal{B}(\text{gausses})}{\mu}$				
Magnetomotive force	$\mathfrak{F}$ m.m.f.	ampere-turn gilbert	1 gilbert = $\frac{1}{0.4\pi}$ ampere-turn $\mathfrak{F}(\text{gilberts}) = 0.4\pi N(\text{turns})I(\text{amperes})$ $= \mathcal{R}(\text{oersteds})\phi(\text{maxwells})$				

TABLE IV.—ELECTRICAL UNITS, CONVERSION FACTORS.—(*Continued*)

International c.g.s. system of practical units				Conversion factors $v = 3 \cdot 10^{10}$			Quantitative relations
Quantity	Symbol	Name of unit	Equations	Practical to electromagnetic	Electromagnetic to electrostatic	Practical to electrostatic	The International c.g.s. system of practical units; The electromagnetic absolute c.g.s. system of ab units; The electrostatic absolute c.g.s. system of stat units
Permeability.........	μ	*a* number	$\mu = \frac{1}{v} = \frac{\mathcal{B}(\text{gausses})}{H(\text{gilberts per cm.})}$				
Permittivity.........	κ	*a* number	$\kappa = \frac{1}{\sigma}$				
Power...............	p, P	watt	$P(\text{watts}) = E(\text{volts})I(\text{amperes}) = R(\text{ohms})I^2(\text{amperes})$	......	......	......	1 watt = 10^7 abwatts = 10^7 statwatts
Reactance...........	x, $_Lx$, $_cx$	ohm	$x(\text{ohms}) = {_Lx} - {_cx}(\text{ohms})$	10^7	......	10^7	
			$= 2\pi fL(\text{henrys}) - \frac{1}{2\pi fC(\text{farads})}$	10^9	$\frac{1}{v^2}$	$\frac{1}{9 \cdot 10^{11}}$	
Reluctance..........	$\mathcal{R}$	oersted	$\mathcal{R}(\text{oersteds}) = \frac{\mathcal{F}(\text{gilberts})}{\phi(\text{maxwells})}$				
Resistance..........	r, R	ohm	$R(\text{ohms}) = \frac{E(\text{volts})}{I(\text{amperes})}$	10^9	$\frac{1}{v^2}$	$\frac{1}{9 \cdot 10^{11}}$	1 ohm = 10^9 abohms $= \frac{1}{9 \cdot 10^{11}}$ statohm

electric charge and the unit magnetic pole of the electrostatic and electromagnetic systems, respectively, may be stated concisely, but it is not a simple matter to accurately determine experimentally the values specified. Moreover, most of the units in both systems are of inconvenient size for use in everyday electrical measurements. As a consequence a third system, the *international,* was developed and the electric units generally used are based on standards defined by international electrical congresses and in the United States legalized by Act of Congress in 1894. The *international* or *practical* system has the *international ohm, international ampere, centimeter* and *second* as fundamental units. The unit magnetic pole and the unit electric charge are relegated to subordinate positions, corresponding to their unimportance in practical work.

(*a*) The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 g. in mass, of a constant cross-sectional area and of a length of 106.3 cm. The ohm equals 10^9 units of resistance in the electromagnetic system.

(*b*) The *international ampere* is the unvarying electric current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0.0011180 gram per sec. The ampere equals 10^{-1} unit of current in the electromagnetic system.

(*c*) The *international volt* is the electrical pressure which, when steadily applied to a conductor the resistance of which is 1 international ohm, will produce a current of 1 international ampere. The volt equals 10^8 units of e.m.f. in the electromagnetic system. A more convenient subsidiary standard for the volt is found in the *Weston cadmium sulphate cell.* Under conditions that can readily be met the e.m.f. of the Weston cell is 1.0183 volts.

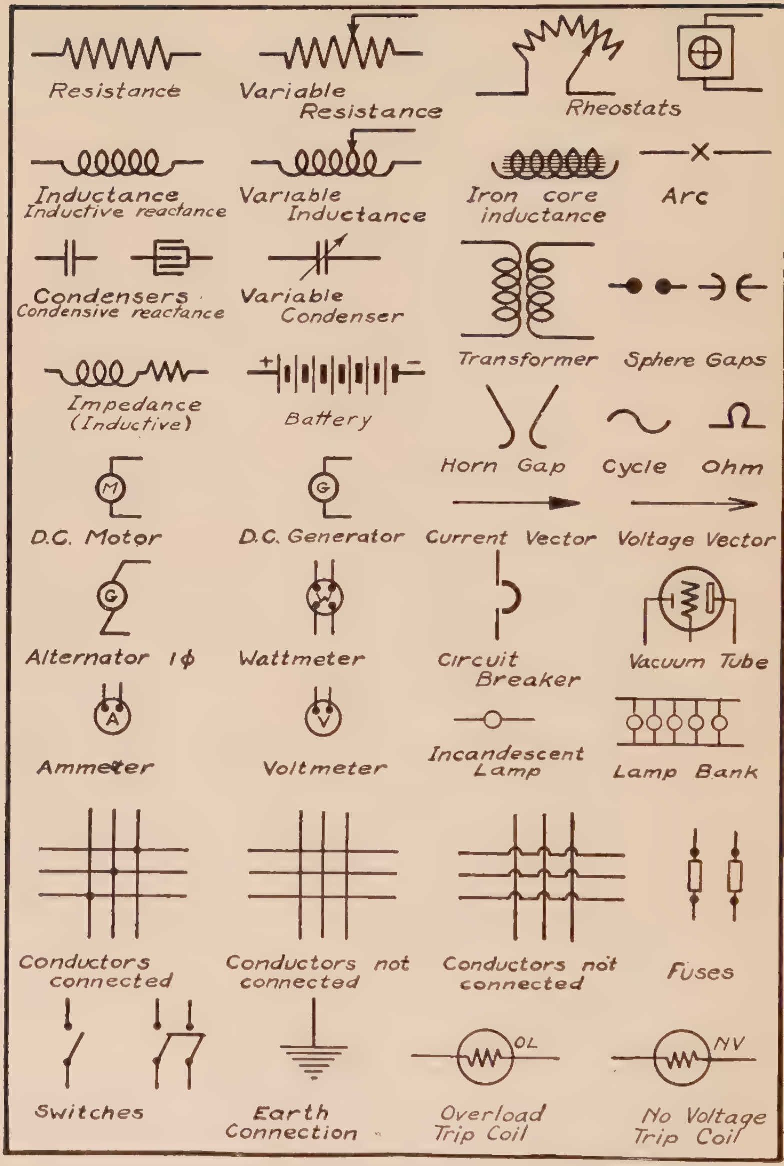
(*d*) The *international watt* is the energy expended per sec. by an unvarying electric current of 1 international ampere under an electric pressure of 1 international volt. The watt equals 10^7 units of power in the electromagnetic system.

Multiplying Factors.—In Table IV are given the quantitative relations of the units in the *electrostatic* or *stat* system, the *electromagnetic* or *ab* system, and the *international* or *practical* system. The multiplying factors are:

(*a*) 10^x, with x a positive or negative integer.

(*b*) 4π, the surface of a sphere of unit radius.

TABLE V.—STANDARD ELECTRICAL SYMBOLS



(*c*) $v = 3 \cdot 10^{10}$ cm. per sec., the velocity of propagation in cm.-sec. of an electromagnetic-dielectric field in free space; the same as the velocity of light.

(*d*) Ratio of time units used: seconds, minutes, hours, etc.

Other Systems.—Several other systems of electric units have been proposed and, in a few cases, used to a limited extent. The chief defect in the international system of units is the introduction of the 4π factor, due to the illogical assumption of one line of force per $\overline{\text{cm}}^2$. on the surface of a sphere of unit radius, instead of letting a single line represent the flux from a unit pole or a unit charge. In the Heaviside system of units this defect is eliminated at its source, while in some of the other systems the correction is made by including the 4π factor in the definitions for permeability and permittivity. However, the international or practical system, based upon the electromagnetic or absolute system, provides satisfactory practical units for the measurement of electric and magnetic quantities and is in general use.

Symbols.—In order to follow more readily discussions of electrical phenomena, machinery and appliances, an extensive system of symbolic representation has been developed; not merely letters, as for the units involved in Tables III and IV, but other figures in the form of compact drawings or stereotyped sketches, as illustrated in Table V, are also in general use.

The subscript notation used in the text may readily be kept in mind by referring to the corresponding circuit and vector diagrams. The subscripts in the lower right-hand corner give the position in the circuit while the subscripts in the lower left-hand corner indicate the nature of the circuit. Thus, ${}_Lx_2$ indicates inductive reactance in circuit 2; and ${}_cx_1$ indicates condensive reactance in circuit 1. Similarly, ${}_rE_3$ indicates the voltage consumed by the resistance r in circuit 3; and ${}_cI_2$ the condensance current flowing in circuit 2. The dots above $\dot{E}$ and $\dot{I}$ indicate a complex or vector nature of the voltage and current, while the same letters without the dot stand for the absolute or numerical values.

CHAPTER IV

INSTRUMENTS

In order to obtain data on the electric quantities in alternating-current circuits, instruments of many types and forms are in commercial use. While these instruments can be classified in various ways the basic grouping is generally made in relation to the specific factors to be measured. The more important quantities are *current*, *voltage*, *power*, *energy*, *frequency* and *power factor*. The corresponding instruments are *ammeters*, *voltmeters*, *wattmeters*, *watt-hour* or *kilowatt-hour meters*, *frequency meters*, *power-factor meters* and *oscillographs*. For gaining clear insight into the nature of alternating-current phenomena, the oscillographs form the most important group of all the alternating-current instruments. This is evident, as oscillographs indicate and record the instantaneous values of the current, voltage, power and frequency factors in the electric circuit, while ordinary ammeters, voltmeters and wattmeters merely indicate the corresponding effective values.

The ammeter, voltmeter and wattmeter groups may be subdivided into types based on the physical properties underlying their operation, namely:

1. Electromagnetic.
2. Electrodynamic.
3. Electrostatic.
4. Thermal.

Electrical instruments are also classified as portable, semiportable and stationary; the latter designed largely for switchboard service.

I. AMMETERS

Since ammeters measure the current, the instrument must be connected in series with the generator and load. Hence the internal resistance of ammeters must be small. For circuits carrying heavy currents, larger than can be passed through the instrument, connection is made through a current transformer of known ratio. The current transformer serves the same purpose

for alternating currents as shunts in the measuring of direct currents. Millivoltmeters in combination with shunts are also used for measuring current in alternating-current circuits.

(*a*) **Electromagnetic Type.**—The operation of this type of ammeter depends on the reaction between two magnetic fields. The current to be measured flows through a coil of a few turns, which forms the fixed or stationary element of the instrument. The Weston ammeter, Model 155, illustrated in Fig. 1.4. is a good example of the electromagnetic type.

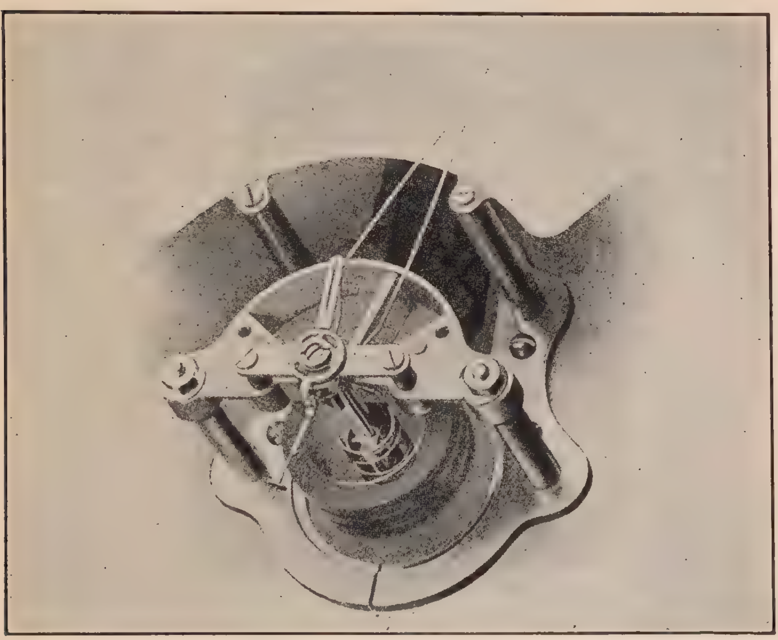


FIG. 1.4.—Elements of electromagnetic ammeter. (*Weston Electrical Instrument Company.*)

In this instrument the active element consists of two pieces or strips of soft iron placed inside the coil carrying the current to be measured. One of the iron strips is fixed in position with respect to the coil. The other iron strip is attached to the movable, pivoted element which also carries the pointer and a damping vane as shown in Fig. 1.4.

The field produced by the current flowing through the coil polarizes the two soft-iron strips. The polarization is always in the same direction in the two strips, relative to each other.

Every other half cycle of the alternating current the upper edges of the iron strips are north poles, as shown in Fig. 2.4, and the lower edges south poles. For the alternate half cycles the current reverses in direction and consequently both strips become

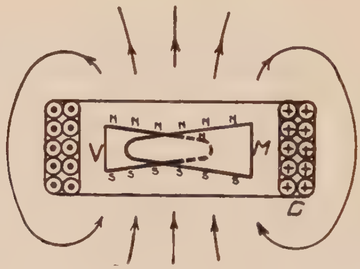


Fig. 2.4.—Polarized strips in electromagnetic type.

polarized in the reverse direction. In ammeters (Fig. 1.4) the coil C, Fig. 2.4, has few turns while in voltmeters (Fig. 9.4) it has many turns.

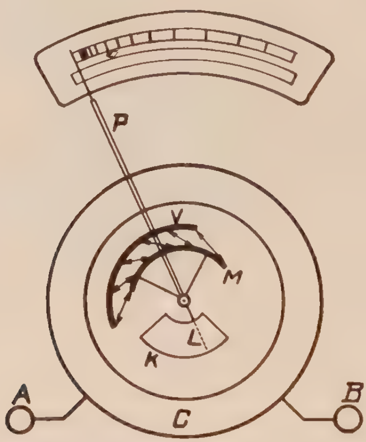


Fig. 3.4.—Torque from reaction of two polarized iron strips.

Due to the shapes and relative positions of the two iron strips, the magnetic reaction between them produces a turning moment or torque on the pivoted element, as indicated by the arrows in Fig. 3.4. This torque is balanced by a spiral spring adjusted so

as to hold the pointer at the zero position on the scale when no current flows through the instrument. The scale divisions are not uniform; the wider divisions being in the middle section of the scale.

By means of a metal damping vane, moving in a closed compartment and attached to the moving element, the instrument becomes dead-beat; that is, the pointer changes position with changes of current by one-way movements and not by a series of oscillations. This type of ammeter is only slightly affected by changes in the frequency of the current measured. If the instrument is calibrated with 60-cycle current, it can be used for all frequencies that normally occur in power circuits. For use in radio circuits the calibration must be made at radio frequencies.

The *induction type* of electromagnetic ammeter can be studied to best advantage after gaining insight into the basic principles of the induction motor, Chapter XIII.

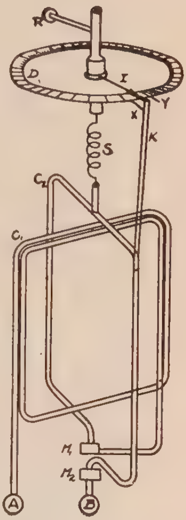


FIG. 4.4.—Diagram of electrodynamometer type of ammeter.

(*b*) **Electrodynamic or Electrodynamometer Type.**—In essence this instrument consists of two coils, connected in series. One of the coils is fixed in position with respect to the frame of the instrument. The other coil is held suspended by a spiral spring outside of the fixed coil as shown in Fig. 4.4. The spiral spring, S, supporting the movable coil C_2 is attached to a torsion head which may be turned by means of a lever, or milled knob, at the top of the instrument. A pointer attached to the torsion head moves over a graduated scale and thus indicates the angular degrees through which the torsion head is turned in order to bring the movable coil back to the zero position when the current measured flows through the instrument. This angle measures the force between the two coils tending to deflect the movable element from the zero position, since the force causing a twist in the spiral or helical spring is directly proportional to the angle of the twist. In order to avoid torque in the electrical connections the lower end of the coils dip into cups M_1 and M_2 filled with mercury. Since the movable and stationary coils are connected in series the repelling force between the coils is pro-

portional to the square of the current. For direct currents the relation of the current to the angular deflection is expressed by equation (2.4).

$$I^2 = K^2\Theta \tag{1.4}$$
$$I = K\sqrt{\Theta} \tag{2.4}$$

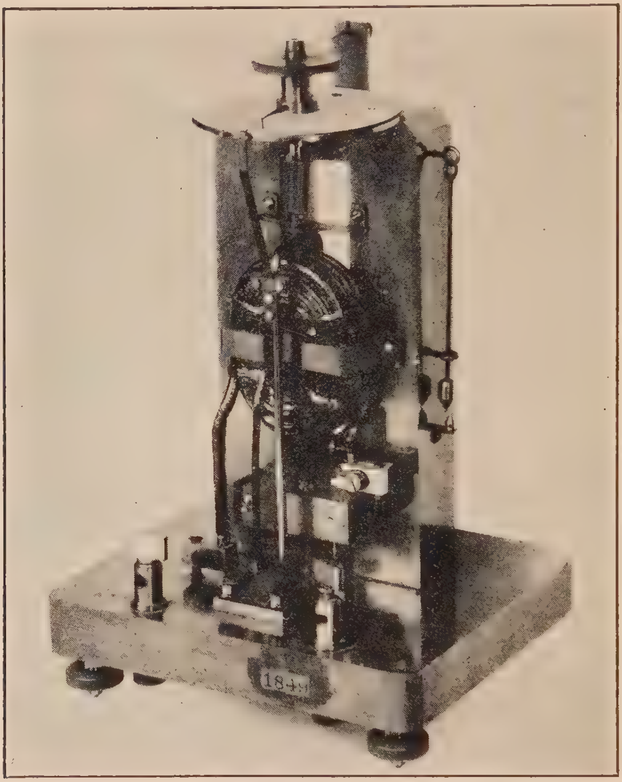


FIG. 5.4.—Electrodynamometer. Ammeter.

For alternating currents the value of the current would be expressed by equation (3.4) for any given instantaneous value of the current. Hence the square root of the average square gives

$$i^2 = K^2\Theta \tag{3.4}$$

the *root mean square*, or the *effective* value of the alternating current, as shown in equation (4.4).

$$I \text{ (effective)} = K\sqrt{\Theta} \tag{4.4}$$

The calibration of the electrodynamometer type of ammeter can therefore be made with direct currents for use in measuring both direct and alternating currents.

(c) **Thermal Type.**—The heat produced by the current passing through the instrument is the basis of operation in thermal-type ammeters. Two distinct effects produced by the heat are used, on which are based two types of thermal instruments: (1) expansion of a wire by heat, used in hot-wire ammeters; (2) thermocouple action, generating voltage by heating the junction of two metals.

(1) *Hot-wire Ammeters.*—In Fig. 6.4 is shown the circuit diagram of a hot-wire ammeter. The front appearance of a hot-wire ammeter, manufactured by the General Electric Company, is shown in Fig. 7.4.

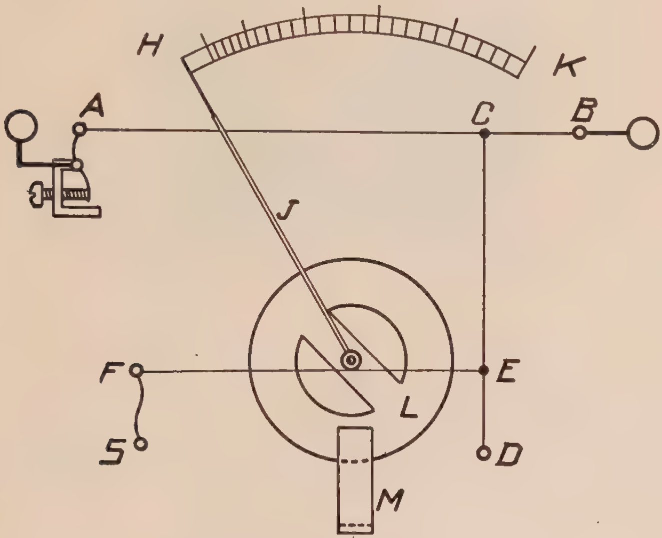


Fig. 6.4.—Circuit diagram of a hot-wire ammeter.

When the current flows through the wire AB the heat generated causes expansion or increases the length of the wire. The tension on the fine, bronze wire CD, produced by the spring SF through the silk cord EF, pulls the point C downwards. This causes the points E and F to move towards the left of the diagram. This movement of the cord EF turns the pointer in the clockwise direction as the silk cord EF is wound around the pivoted axis to which the pointer is attached. When properly calibrated, the pointer indicates on the scale HK the magnitude of the current flowing in AB. Since the heat generated is proportional to the square of the current the ammeter, if calibrated by using direct

currents, indicates the effective values for alternating current of any wave shape. For this inherent characteristic the hot-wire instruments are often used for determining the direct-current equivalent or effective value of alternating currents having distorted wave shapes. It is also evident that the hot-wire ammeter is not affected by the frequency or changes in frequency

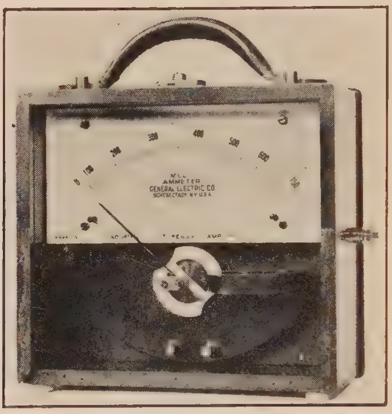


FIG. 7.4.—Hot-wire milliammeter. (*General Electric Company.*)

of the alternating current. For this reason the hot-wire instruments are of special importance in radio and for the measurement of high-frequency alternating currents.

The instrument is somewhat sluggish, and frequent calibrations and adjustments are necessary to keep the pointer at the zero point on the scale when no current is flowing.

In order to make the instrument dead-beat an aluminum disk L, fastened to the pointer shaft, moves between the poles of a permanent magnet M. Eddy currents in the disk, produced by the cutting of lines of force, cause a reaction between the magnet and the disk that tends to retard its motion.

(2) *Thermo-ammeters*—Heat applied at the junction of two metals, as antimony and bismuth, generates voltage which will cause a current to flow if the circuit is closed. The principle on which instruments of this type operate can readily be understood by reference to Fig. 8.4.

A coil of fine wire C, connected in series with a bismuth-antimony thermocouple, is pivoted and held in position by two spiral springs between the poles of a permanent magnet. The current to be

measured flows through a heater, or wire of high resistance which is placed in close proximity to the thermocouple. The heat produced by the current generates voltage in the thermocouple, causing a current to flow in the coil *C*. This current reacts with the field of the permanent magnet, producing a torque that

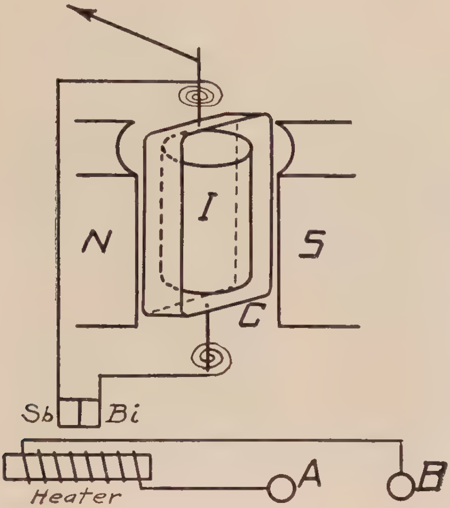


Fig. 8.4.—Diagram of thermo-ammeter.

twists the spiral springs. The angular deflection of the coil is measured by reading on a scale the change in position of a pointer attached to the moving element.

II. VOLTMETERS

Alternating-current voltmeters operate on the same basic principles as the corresponding types of ammeters. The difference is principally in design; the voltmeter having a high resistance, requires very small current, merely sufficient to operate the very light movable element so as to indicate on the scale the difference in potential of the given points on the circuit.

(*a*) **Electromagnetic Type.**—The electromagnetic type of voltmeter, illustrated in Fig. 9.4, showing the active elements of the Weston voltmeter, Model 155, operates on the same principle as explained for the same type of electromagnetic ammeter. The difference is largely in the number of turns and size of wire used in the coils of the two instruments. The reaction between two

strips of soft iron polarized in the same direction, relative to each other, by the current flowing in the coil, form the active element which produces the torque which is balanced by the spiral spring. The principle of operation is the same for the voltmeter as for the electromagnetic type of ammeter.

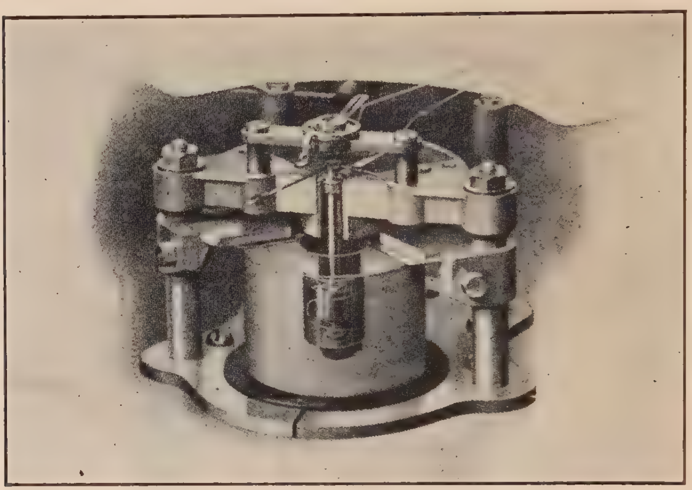


Fig. 9.4.—Inside view of electromagnetic type of voltmeter. (*Weston Electrical Instrument Company.*)

(*b*) **Electrodynamometer Type.**—The voltmeter of the electrodynamometer type operates on identically the same principles as the electrodynamic ammeter shown in Fig. 5.4. However, the currents flowing in the voltmeter are very small, in the better class of instruments less than one-twentieth of 1 amp.; the number of turns in both the stationary and movable coils are comparatively large. Hence, although both instruments operate on the same principle, the voltmeter can be designed and constructed so as to be far superior to the ammeter. Voltmeters of this type are used as laboratory standards giving indications, on properly graduated scales, within one-tenth of 1 per cent of the actual value.

The circuit diagram is shown in Fig. 11.4. The current in the voltmeter passes first through a stationary coil producing a magnetic field, then through the movable coil and, finally, through a non-inductive resistance R. Since the same current flows in both coils the resulting fields reverse simultaneously and, as a

consequence, the torque on the moving element will be continuously in the same direction.

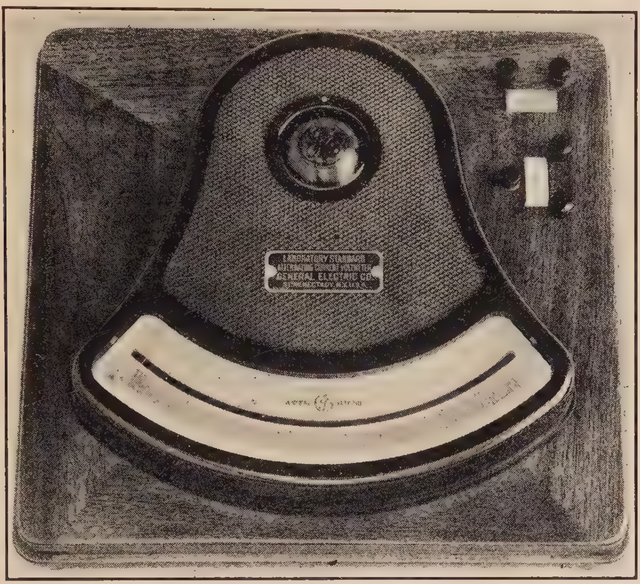


FIG. 10.4.—Laboratory standard voltmeter of the electrodynamometer type. (*General Electric Company.*)

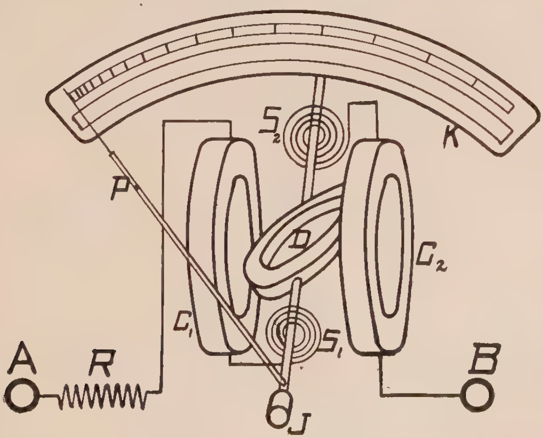


FIG. 11.4.—Circuit diagram of electrodynamometer voltmeter.

This turning movement is opposed or balanced by two spiral springs, one at each end of the moving element. A damping vane, moving in a closed air chamber, eliminates undesirable oscillations.

The torque varies directly as the product of the current i in the movable coil and the strength of the field.

$$\text{Torque} \propto i\phi \qquad (5.4)$$

Since the stationary or field coil is in series with the movable element, the field flux ϕ varies directly as the current.

$$\phi \propto i \qquad (6.4)$$

Hence:

$$\text{Torque} \propto i^2 \qquad (7.4)$$

For any given frequency the impedance of the voltmeter is constant and therefore the current i is directly proportional to the voltage e, impressed on the terminals of the instrument.

Therefore:

$$\text{Torque} \propto e^2 \qquad (8.4)$$

The average torque for a complete cycle is therefore proportional to the average square of the current or to the average square of the terminal voltage. By calibrating the scale of the instrument so as to give the square root of the average torque the electrodynamometer type of voltmeter indicates directly the *root mean square* of the instantaneous values; that is, the *effective* value of the voltage impressed on its terminals.

The voltmeter circuit has of necessity some inductance, even if small, because the torque required in its operation depends on the interaction of the magnetic fields produced by the current flowing in the stationary and movable coils. The current passing through the instrument is therefore equal to the voltage divided by the impedance.

$$i = \frac{e}{z} = \frac{e}{\sqrt{r^2 + x^2}} = \frac{e}{\sqrt{r^2 + (2\pi fL)^2}} \qquad (9.4)$$

e = voltage to be measured; impressed on the terminals of the meter.
z = impedance of voltmeter circuit.
r = internal resistance of the voltmeter.
x = reactance of voltmeter circuit.
L = inductance of voltmeter circuit.
f = frequency in cycles per second.

It is evident from equation (9.4) that the indications of the voltmeter will be affected by the frequency of the voltage measured. If the instrument be calibrated for one frequency, it will not indicate correctly an impressed voltage having any other

frequency. Fortunately, the resistances can be made very large, relative to the reactance, so that for frequencies as ordinarily found in power circuits the errors in the voltage indications introduced by the inductance become negligibly small even over a fairly wide range of frequencies. That is, a voltmeter calibrated by using a current having a frequency of 60 cycles per sec. can be used for measuring 25-cycle currents, or even direct currents, without introducing an appreciable error. However, for use in radio circuits the calibration of the voltmeter must be made by using current within the range of radio frequencies.

(*c*) **Electrostatic Type.**—The basic principle of the electrostatic voltmeter is the force of attraction between positive and negative electric charges. Instruments based on this principle are ordinarily called *electroscopes* or *electrometers*, of which the gold-leaf electroscope is a simple form. In essence the electrostatic voltmeter consists of two plates, one fixed in position and the other movable on a pivot, placed in close proximity and insulated from each other. The relative positions and shapes of the two plates are so designed that a turning moment is produced when a difference of potential exists between them. This torque is balanced by an adjustable weight attached to the lower edge of the movable plate as shown in Fig. 12.4. After the position of the weight has been determined the instrument is provided with a scale which is calibrated so as to indicate effective volts. For voltages ranging from 400 to 100,000 volts only one moving vane is required. For lower voltages the capacity of the instrument must be relatively large in order to produce sufficient torque. To gain this end, several vanes with corresponding stationary quadrants are arranged in front of each other; the movable vanes being attached to the same axis and pointer.

To secure satisfactory insulation in the instrument for measuring high voltages is extremely difficult. In the voltmeter shown in Fig. 12.4 the insulating properties of air and ebonite are used. In another design the active element is immersed in oil to secure the necessary insulation.

The chief advantages of the electrostatic voltmeter are: (1) no current is consumed; (2) they may be used for measuring very high voltages; (3) they are not affected by temperature changes, stray magnetic fields, changes in power factor and frequency. The weak torque inherent in this type is a marked disadvantage as the indicated readings will not be even of the same order of

accuracy as for the electromagnetic or electrodynamometer types. The electrostatic voltmeter is therefore used only under conditions that can not be met by either the electromagnetic or the electrodynamic types of voltmeters; as for example, to measure the potential difference of two points in a circuit without letting any current pass through the measuring instrument.

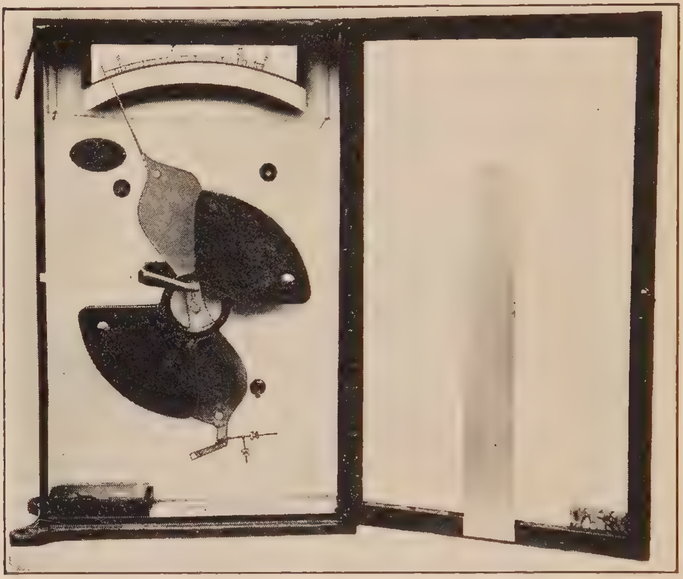


Fig. 12.4.—Electrostatic voltmeter.

III. WATTMETERS

Wattmeters are instruments designed and used for measuring power in electric circuits. Many types of wattmeters have been constructed; in fact, all of the basic principles underlying the operation of ammeters and voltmeters, namely, electromagnetic, electrostatic, electrodynamic and thermal, have been used. However, only two, the electrodynamic and the electromagnetic or induction types, are of commercial importance.

(*a*) **Electrodynamometer Type.**—Wattmeters of this type operate on the same principle as previously explained for the electrodynamic voltmeter. There is, however, one important difference. In wattmeters the two coils are not connected in series. The

stationary coil is connected in series with the load. The magnetic field is produced by the load current flowing through the stationary coil. The movable coil, in series with a constant high resistance, is connected across the circuit measured. The small current flowing through the movable coil is therefore proportional to the voltage of the circuit. The circuit connections are shown in Fig. 13.4. The reaction of the magnetic fields of the sta-

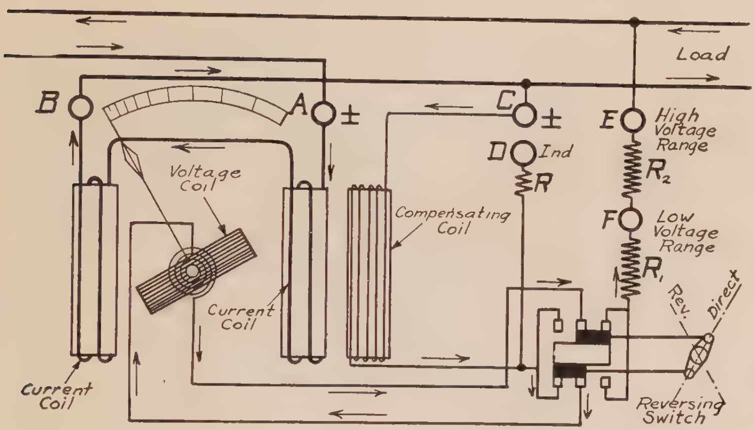


FIG. 13.4.—Circuit diagram for one type of compensated wattmeter.

tionary and movable coils therefore produces a torque which at any instant is proportional to the product of the current and the voltage; that is, to the power.

$$\text{Torque} \propto ie \tag{10.4}$$

The electrodynamometer type of wattmeter, of which the General Electric Company's type P_3 and Weston Electrical Instrument Company's Model 310 are good examples, may be used on either direct or alternating currents. Wattmeters of the electrodynamometer type are usually calibrated by using direct current, and indicate the power in effective watts when used on alternating-current circuits.

The external circuit connections to the wattmeter must be made properly to avoid damage to the instrument as incorrect connections produce excessive potentials between the stationary and movable coils. For this purpose one terminal of each coil is marked ± to indicate that both of these terminals must be

connected to the same side of the circuit to be measured. When the connections are made as shown in Fig. 13.4, there will be very little difference of potential between the two coils. However, if the coil connections for the voltage coil circuit should be reversed, that is with the high resistance R_1R_2 between the two coils, nearly full line potential would exist between the movable and stationary coils in the meter. This would make an arc-over probable, which, if it occurs, would ruin the meter. Even if the

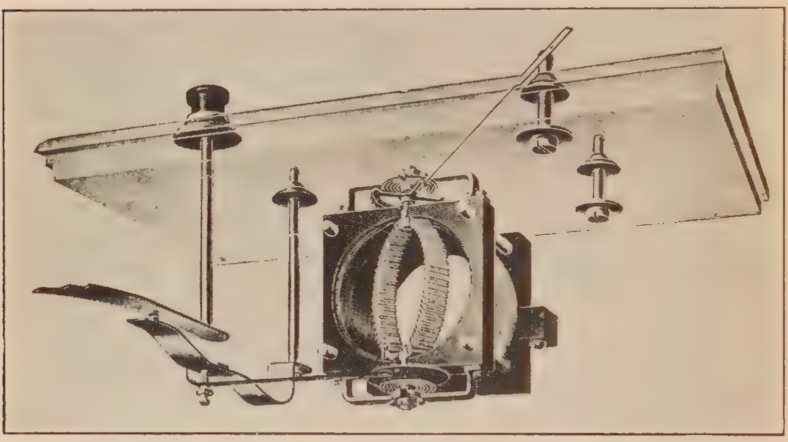


FIG. 14.4.—Elements of wattmeter. (*Weston Electrical Instrument Company.*)

insulation between the two coils should be sufficient to prevent an arc forming, the electrostatic forces would affect the torque, thereby introducing errors in the readings.

It should be noted that with the internal connections as shown in Fig. 13.4 the current in the stationary coil is the sum of the load current and the current flowing in the movable or voltage coil. The torque and hence the deflection are therefore too large by an amount equal to the power consumed by the potential coil circuit.

To correct for this error a *compensating coil*, consisting of a few turns, is placed in the meter and connected so as to produce a magnetic field in opposition to the field produced by the current in the main stationary coil. The sum of the fields produced by the current coil and the compensating coil, in a properly designed instrument, is therefore directly proportional to the load current.

It is evident that if the connections of the compensating coil with respect to the stationary coil should be reversed the error

due to the current in the voltage coil would be doubled. It is therefore important, whether the voltage of the circuit to be measured is high or low, that the terminals marked ±, one for the current coil and the other for the potential coil, be connected to the same side of the circuit in which power is to be measured.

When calibrating a wattmeter, using direct currents, the current for the voltage coil is obtained from a source independent of that for the stationary coil. For this purpose the independent terminal *D* is used in place of *C*, in order to by-pass the compensating coil. The resistance of the voltage coil circuit is not altered by this change in connection as a resistance *R*, equivalent to the resistance of the compensating coil, is in the *D* terminal circuit.

When used on alternating-current circuits the torque in the wattmeter occasionally tends to deflect the pointer in the reverse direction; that is, to the left of the zero reading. It is evident that in order to reverse the deflection, the direction of the current in one of the two coils must be reversed. To this end a convenient reversing switch, an integral part of the instrument, is placed in the potential coil circuit as shown in Fig. 13.4.

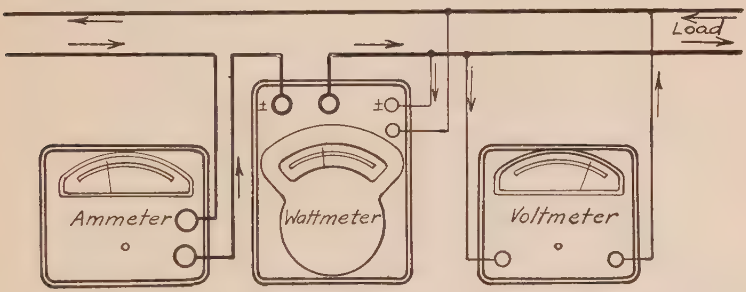


FIG. 15.4.—Circuit diagram of connections for ammeter, voltmeter and wattmeter.

Since both the current and voltage coils have inductance, a change in frequency will alter the reactance and therefore the impedance of the circuits. However, the relative value of the resistance to the reactance in the voltage coil circuit is so large that instruments calibrated by direct currents can be used on alternating-current circuits without corrections, for frequencies up to 133 cycles per sec. High-grade wattmeters will give indications that are correct within one-fourth of 1 per cent.

In Fig. 15.4 are shown the circuit connections for simultaneous use of ammeter, voltmeter and wattmeter. The instruments

are connected directly to the load circuit and hence the voltage between mains as well as the magnitude of the load current must not be greater than the rated capacity of the instruments used.

For making measurements on load circuits of higher potentials than the voltage rating of the voltmeter and wattmeter a potential transformer is inserted in the movable or voltage coil circuit as shown in Fig. 16.4. Likewise for circuits, in which the currents are larger than the rated current capacity of the ammeter and wattmeter, a current transformer is inserted into the stationary or current coil circuit as shown in Fig. 16.4. The ratio of the voltages in the potential transformer and the corresponding ratios of the currents in the current transformer must be known or accurately determined in order to apply the proper multiplying factors to the readings indicated by the instruments. For accurate measurements, corrections must be made for changes in phase angles between the current and voltage that are introduced into the circuit by the current and potential instrument transformers.

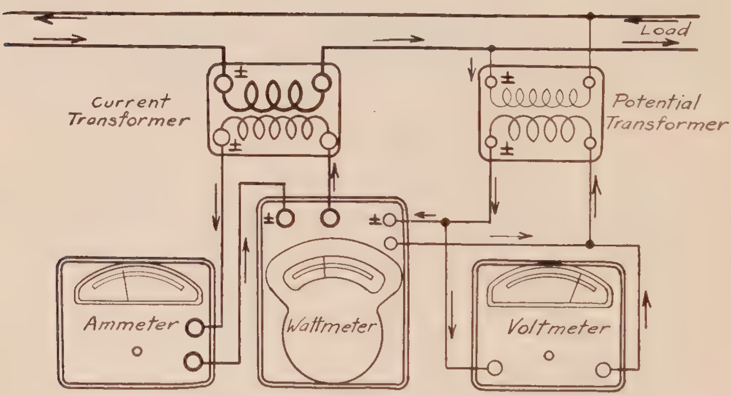


Fig. 16.4.—Circuit diagram of connections for ammeter, voltmeter and wattmeter when using current and voltage instrument transformers.

(*b*) **Electromagnetic or Induction Type.**—The induction type of wattmeter is in essence an induction motor and can be studied to best advantage after mastering the basic principles of induction motors (see page 275).

IV. WATT-HOUR OR KILOWATT-HOUR METERS

The watt-hour or kilowatt-hour meters integrate the product of the power, expressed in terms of watts or kilowatts at the

given point in the circuit, and the instantaneous elements of time over any desired period expressed in terms of hours. That is, the watt-hour meter measures the *energy* transmitted or delivered by the electric circuit during the time specified. The energy unit generally used is the kilowatt-hour (kw.h.).

All watt-hour meters are essentially small motors that rotate at a speed proportional to the power or energy flow in the circuit. A registering mechanism attached to the motor armature records the energy consumption. For alternating-current circuits two basically different types of watt-hour meters have been developed. The electrodynamometer or commutator type is similar to the watt-hour meter generally used for measuring electric energy in direct-current circuits. The instrument consists of a small, series motor having as load an aluminum disk rotating so as to cut the flux of one or more permanent magnets. This type is, however, seldom used, as watt-hour meters based on the induction principle are less expensive to build and have greatly superior operating characteristics.

The induction type of watt-hour meter operates on the principle of the rotating magnetic field and the active element is some form of induction motor (see page 275).

V. FREQUENCY METERS

Instruments indicating or recording the frequency of alternating currents, that is, the number of complete cycles made per second by the voltage or current in electric circuits, are called frequency meters. Many types and forms of frequency meters have been developed and several have proved to be of practical importance and are in commercial use. The more important types may be grouped on the basis of the principles on which they operate, namely, *vibrating reed, resonance, polarized reed, movable coil* and *magnetic induction.*

(*a*) **Vibrating-reed, Resonance and Polarized-reed Types.**—All of these instruments are based on the natural period of oscillation possessed by a reed or strip of metal, one end of which is rigidly fastened to a fixed support while the other is free to oscillate. The vibrating reed like the pendulum has a natural period of oscillation. The periodicity depends on its dimensions, on the weight, elasticity and, in some cases, on the magnetic properties of the material used. For a reed, of given material and cross-section, the periodicity depends on its length; the

shorter the reed the higher the frequency. An early type of frequency meter consisted of a single reed of variable effective length, as illustrated in Fig. 17.4. The length of the active part *R*, of the vibrating reed, is varied by moving the supporting contact *C*. An electromagnet *W*, energized by the current whose frequency is to be measured, is located near the free end of the reed. The alternating current produces a magnetic field which pulsates at double the frequency of the magnetizing current; that is, the upper magnetic pole will be alternately north and south as the current alternates in direction of flow.

The steel, of which the vibrating reed is made, is attracted towards the electromagnet by the magnetic field regardless of the

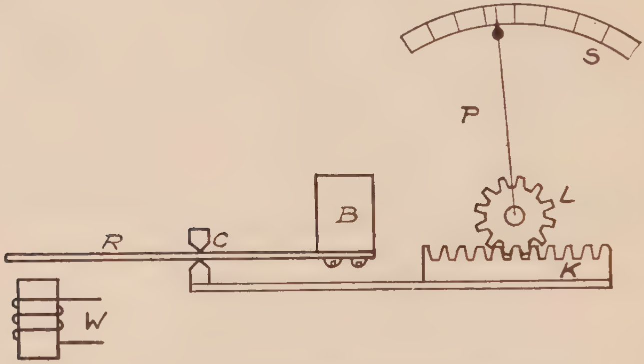


Fig. 17.4.—Single vibrating reed of variable length.

direction of the lines of force. The alternating current in the magnet will therefore produce a succession of pulls and releases on the free end of the reed, with the bending impulses at a frequency just twice that of the number of complete cycles per second made by the alternating current.

If the natural period of oscillation of the reed is the same, or very nearly the same, as that of the frequency of the impulses impressed upon it, the amplitude of the oscillations will increase to a maximum; that is, the successive energy impulses add and become stored in the oscillating body. If, however, the impressed impulses have either a higher or a lower frequency than the natural periodicity of the reed, the effect will not accumulate since the effect produced by one impulse will be neutralized by another.

In the operation of the single-reed instrument, shown diagrammatically in Fig. 17.4, the length of the vibrating reed is adjusted until a maximum oscillation is obtained. The ratchet mechanism transmits the movement of the supporting clamp C to the pointer which indicates on the calibrated scale S the frequency of the vibrations; or half that value, if the scale is made to indicate the frequency of the alternating current of the electromagnet.

In order to avoid the necessity of adjusting the length of the single vibrating reed, frequency meters of this type generally have a series or a set of reeds, tuned in steps over the desired range in frequency, which automatically indicate the frequency of the alternating current. An interior view of a resonance type of frequency meter is shown in Fig. 18.4.

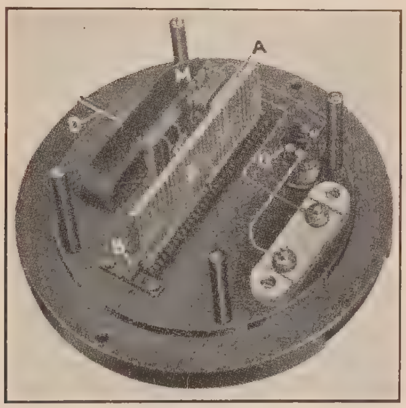


FIG. 18.4.—Interior view of Frahm frequency meter. A = armature of electromagnet M; B = bridge piece on which are mounted the reeds R and the armature A; D = amplitude screw for adjusting the air gap between A and M; G = protective resistance in series with M; R = tuned reeds, each one of which is adjusted by resonance to a mechanical vibration of definite frequency, produced in B through A by the alternating current in M.

The vibration of any given reed begins at about 2 per cent below the exact frequency to which the reed is tuned, reaches a maximum at the exact value, and extends with decreasing magnitude to about 2 per cent above the exact frequency of the reed. The range of frequencies suitable for this type of meter is from 15 to 200 cycles per sec.

A switchboard type of frequency meter having only one row of reeds is shown in Fig. 19.4. When in circuit the reeds of frequency nearest to that of the current in the electromagnet will

oscillate and present an appearance like that shown in Fig. 20.4. The reading of the instrument from the indications of the vibrating reeds is illustrated in Fig. 20.4.

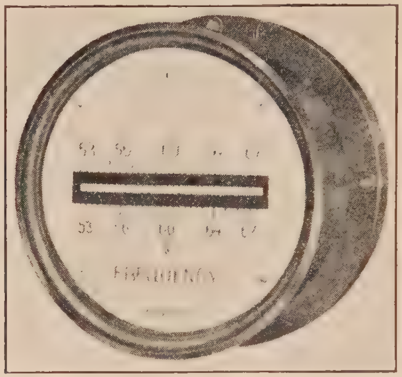


FIG. 19.4.—Switchboard type, Frahm frequency meter.

The range of the instrument can be extended by using two rows of reeds as shown in Fig. 21.4 and again doubled by *polarizing* the reeds. For the lower range (Fig. 21.4) the vibration of the

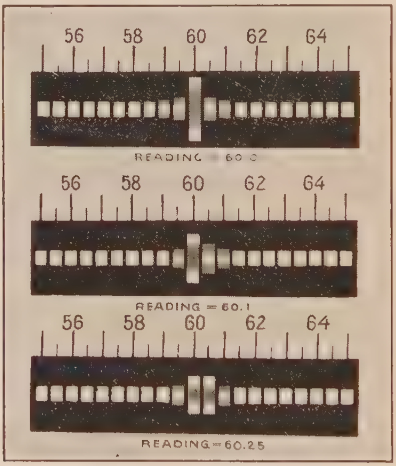


FIG. 20.4.—How to read Frahm frequency meter.

reeds is obtained by means of an electromagnet having a soft-iron core. As previously explained, two complete vibrations of the reed occur for each cycle of the current. For the higher range a second electromagnet, polarized by having a permanent magnet

for its core, is used. In the polarized electromagnet the successive half cycles of the alternating current strengthen and weaken the flux of the permanent magnet, thus causing but one vibration of the reed for each cycle of the current. Consequently, the reed will oscillate at the same frequency as the current in the polarized electromagnet. By means of terminal binding posts either of the two magnets can be used: the soft-iron core for the lower scale, and the polarized electromagnet for the upper scale.

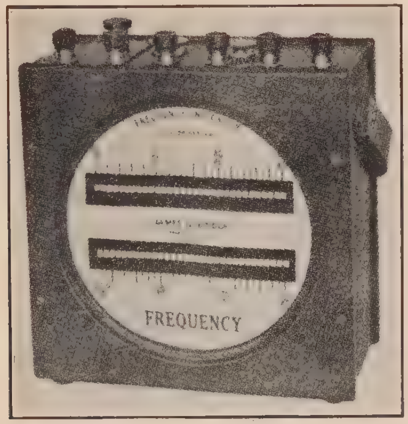


FIG. 21.4.—Portable-type Frahm vibrating-reed frequency meter.

Several forms of the resonance type of frequency meters are manufactured, but in essence the basic principle of operation is the same for all; namely, the periodicity characteristics of vibrating reeds.

(*b*) **Movable Iron Type.**—In the movable iron type of frequency meter, manufactured by the Weston Electrical Instrument Company, the armature or iron core attached to a pivot, is directed in position by the magnetic field produced by electric currents in two coils, *AA* and *BB* in Fig. 22.4. The coils are fixed in position at right angles to each other. The circuit diagram and the relative position of the coils and movable iron core are shown in Fig. 22.4.

The movable iron core to which the pointer is attached is located inside the coils *AA* and *BB*. The factors affecting the operation of the instrument are somewhat involved, but the basic feature is that the equivalent circuit between mains of coil *AA* has more inductance and less resistance than the correspond-

ing circuit for coil *BB*. Therefore, a change in frequency will affect the impedance and hence the magnitude of the current more in coil *AA* than in coil *BB*.

The values of the resistances and the inductance, shown diagrammatically in Fig. 22.4, are selected so that the currents in

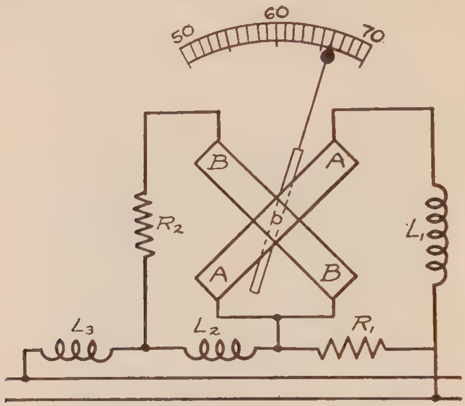


FIG. 22.4.—Circuit diagram of frequency meter, movable iron type. (*Weston Electrical Instrument Company.*)

the two coils are of the same magnitude for some given frequency, as 60 cycles or 25 cycles per sec.

Since the two coils are at right angles the magnetic fluxes produced by the coils will be in quadrature as shown by the vectors ϕ_A and ϕ_B in Fig. 23.4*a*. The vector ϕ_R gives the direction

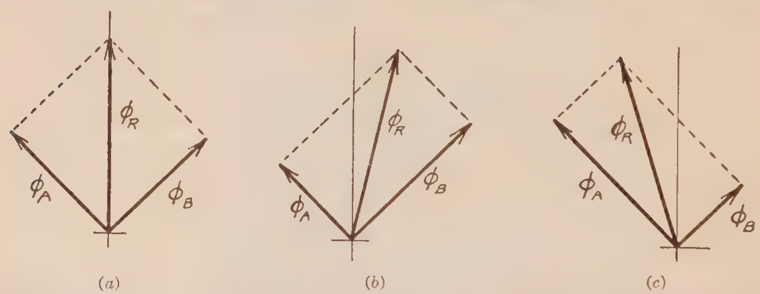


FIG. 23.4.—Vector diagrams for circuit in Fig. 22.4 for three frequencies.

and magnitude of the resultant field flux which determines the position of the movable iron core. Under the given conditions the pointer indicates the mid-scale reading.

If the frequency of the current in the coils *AA* and *BB* is increased, less current will flow through *AA* than in *BB* and the corresponding field fluxes are represented by the vectors ϕ_A and

ϕ_B in Fig. 23.4b The resultant flux ϕ_R will therefore be changed in direction and the movable iron core will turn so as to be in line with the resultant flux. The pointer will turn and indicate the higher frequency on the calibrated scale of the instrument.

If the frequency is decreased so as to be less than the mid-scale reading, the current in coil AA will be greater than in BB. The corresponding magnitude and direction of the resultant flux ϕ_R are shown in Fig. 23.4c. The movable iron core and hence the pointer will therefore be in line with the flux ϕ_R in Fig. 23.4c and the frequency of the current indicated on the calibrated scale of the instrument.

VI. POWER-FACTOR METERS

The term power factor as used for electric circuits is explained and defined in Chap. VIII. A discussion of the power-factor meter is found in the latter part of the chapter (see page 134).

VII. OSCILLOGRAPHS

The importance of oscillographs in the study of alternating-current phenomena can hardly be overestimated. The oscillograph gives a continous record by indicating and recording the instantaneous values of current, voltage, power, energy, frequency and power factor in electric circuits. For the study of transient electric phenomena oscillographs are indispensable and they are also of outstanding importance for gaining insight into, and quantitative data of, alternating-current circuits. Only the galvanometer type of oscillographs will be discussed, as the various forms of cathode-ray oscillographs are used mainly for investigations on electric transients.

The first recording oscillograph was developed by Duddell in 1894 and, although the designs of present-day instruments are far superior, the same principles underlie the operations of all oscillographs of the galvanometer type.

The Duddell oscillograph consisted essentially of a modified moving coil galvanometer combined with a rotating or vibrating mirror, a moving photographic film or a falling photographic plate. The galvanometer section, often referred to as the oscillograph, is shown diagrammatically in Fig. 24.4.

In a narrow gap between the poles N and S of a powerful magnet are stretched two parallel conductors s and s', formed by

bending a very thin strip of phosphor bronze back on itself over an ivory pulley, *P*. A spiral spring attached to this pulley serves to keep the tension on the strips uniform and a guide piece, *G*, limits the length of the vibrating portion to the part actually in the magnetic field. A small mirror, *M*, is attached to both strips as shown in Fig. 24.4. The effect produced by passing a current through such a vibrating element is to cause one of the strips to advance while the other recedes, thus turning the mirror about a vertical axis.

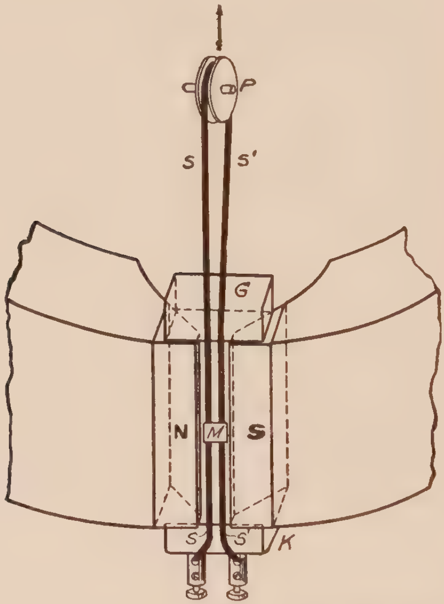


FIG. 24.4.—Vibrator element in Duddell oscillograph.

The whole of the *vibrator*, as this part of the instrument is called, is immersed in oil, the object of the oil immersion being to damp the movement of the strips, thereby making the instrument dead-beat.

A beam of light coming from an arc lamp and reflected by the mirror is received on a photographic film or photographic plate. If the strength of the magnetic field is constant the magnitude of the linear displacement of the spot of light focused on the film or plate is directly proportional to the current flowing in the vibrator. With alternating currents the spot of light oscillates to-and-fro as the current varies, which, with the film stationary,

would thus trace a straight line. To obtain an image of the wave form, the photographic film is moved in a direction at right angles to the direction of motion of the spot of light. For visual observation a second mirror is interposed in the path of the beam of light and this mirror is caused to vibrate or rotate so as to impart to the beam of light a uniform motion about an axis at right angles to the to-and-fro oscillation of the beam in the initial plane of vibration. The spot of light will, in the latter case, trace on a stationary plate or screen the vibrator current-time curve. If the vibrations are periodic, as in alternating currents, the mirrors can be synchronized and the spot of light will trace the wave form over and over again.

By having two vibrators in the same field, arranged so as to throw the beam of light on the same photographic film, the current-time and the voltage-time curves of a circuit may be recorded simultaneously. A circuit diagram of a two-element oscillograph for recording simultaneously the instantaneous values of both the current and the voltage in a low-tension circuit is shown in Fig. 25.4.

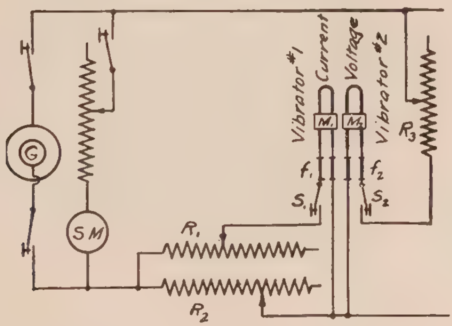


FIG. 25.4.—A circuit diagram of two-element oscillograph recording current-time and voltage-time curves.

In vibrator 1 the magnitude of the current and hence the amplitude of the vibration may be regulated by adjustment of the rheostats R_1 and R_2. The load is adjusted by R_2 while R_1 is an adjustable shunt by which the ratio of the current flowing in the vibrator to the total load current is determined. The beam of light from vibrator 1 will record the current-time curve, on the moving photographic film, giving on a continuous curve the instantaneous values of the current.

Similarly, for vibrator 2 the magnitude of the oscillations can be regulated by adjusting the resistance R_3. For a constant value of R_3 the current flowing in the vibrator 2 will, at all instants, be proportional to the voltage between the points spanned on the circuit. Hence the beam of light reflected from the mirror on vibrator 2 will trace on the moving photographic film the voltage-time curve, giving the instantaneous values of the voltages. Thus, both the current-time and voltage-time curves are recorded simultaneously on the same photographic film.

Oscillographs are calibrated by means of direct currents so that the oscillations of the alternating current and voltage waves may be measured respectively in amperes and volts.

A diagram of the optical train, horizontal projection, of a three-element oscillograph is shown in Fig. 26.4. The diagram for the corresponding vertical projection is shown in Fig. 27.4.

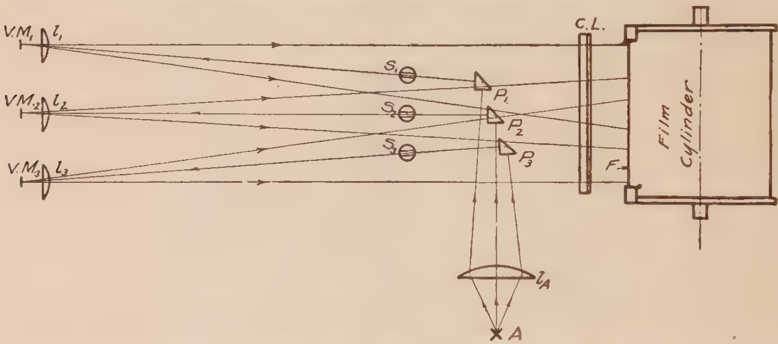


FIG. 26.4.—Diagram of optical train, three-element oscillograph. Horizontal projection.

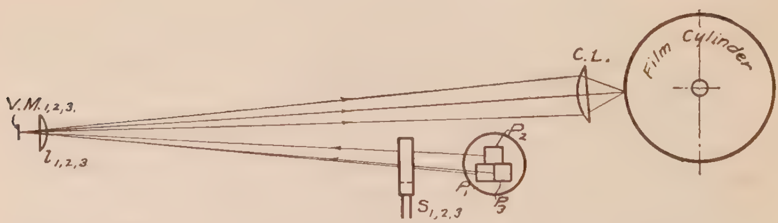


FIG. 27.4.—Diagram of optical train, vertical projection.

An arc lamp is at A and the arrows indicate the paths of the beams of light. P_1, P_2, P_3 are right-angled prism mirrors; S_1, S_2, S_3, adjustable slits; l_A, l_1, l_2, l_3, condensing lenses; VM_1, VM_2, VM_3, the vibrating mirrors; CL, a cylindrical lens focusing the light rays on the photographic film on the revolving cylinder.

A considerable number of oscillographs of excellent design are now on the market. The designs differ in size, number of elements, dimensions of parts, etc., but all operate on the same basic principle.

The Osiso, Fig. 28.4, is a single-element oscillograph of extremely compact design, the whole outfit weighing less than 14 lb. The instrument proper is $6\frac{1}{4}$ in. wide, 9 in. high and $10\frac{1}{2}$ in. long. It has a specially sensitive galvanometer or vibrating element, a 4-volt incandescent lamp with a straight filament as the source

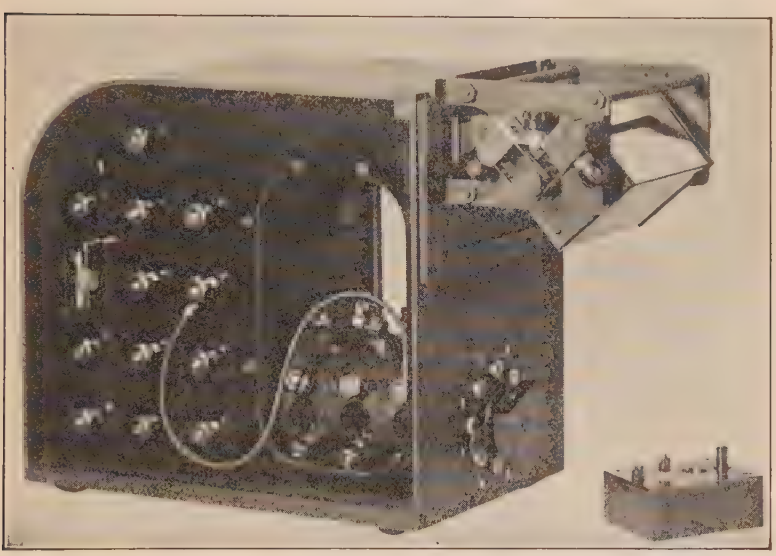


Fig. 28.4.—The Osiso with motor-driven commutator and viewing mirror attached. The permanent-magnet galvanometer is shown at the lower right corner. (*Westinghouse Electric and Manufacturing Company.*)

of light and an effective optical system. A simple mechanism is provided by which the single vibrator can be used successively in six different circuits, by inserting proper resistances and shifting the crossline so as to mark for identification each of the six curves recorded on the film. The three-pole, six-position switch may be connected to six different circuits, in succession, ranging in potential from 0.1 to over 220 volts. Thus, for example, by using 0.05-ohm to 5-amp. shunts in standard transformer circuits, the alternating-current waves for any six circuits may be recorded on one film.

Besides the Osiso, three-element, six-element (Fig. 29.4) and nine-element oscillographs of excellent design are manufactured by the Westinghouse Electric and Manufacturing Company.

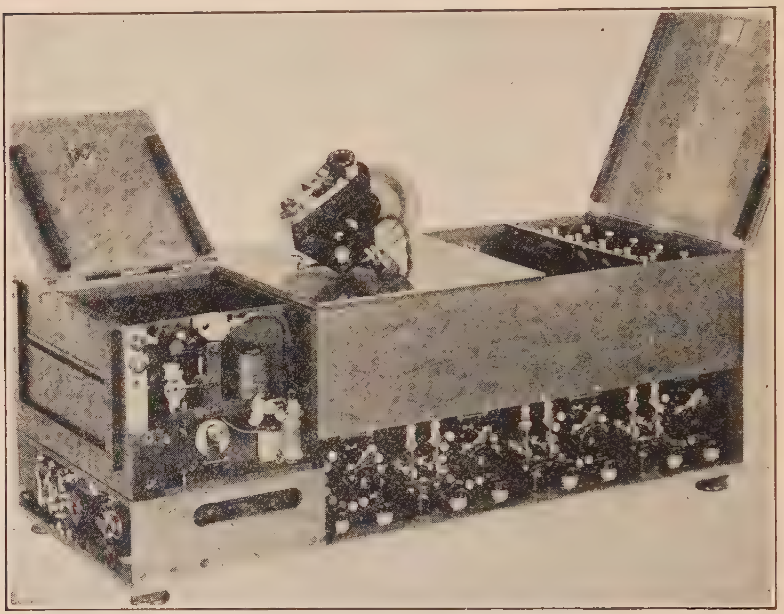


FIG. 29.4.—Six-element oscillograph, showing covers raised over optical box and galvanometer. The driving head is shown resting on top. (*Westinghouse Electric and Manufacturing Company.*)

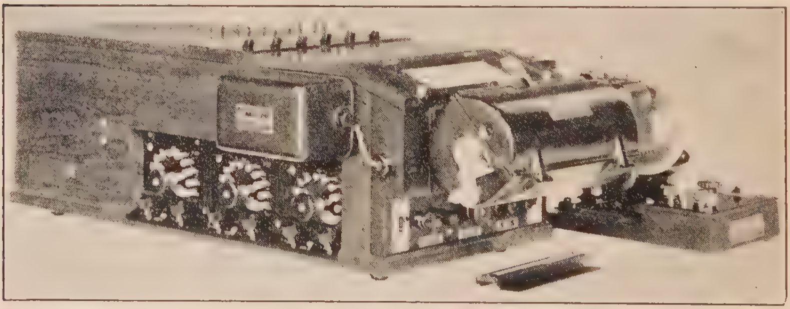
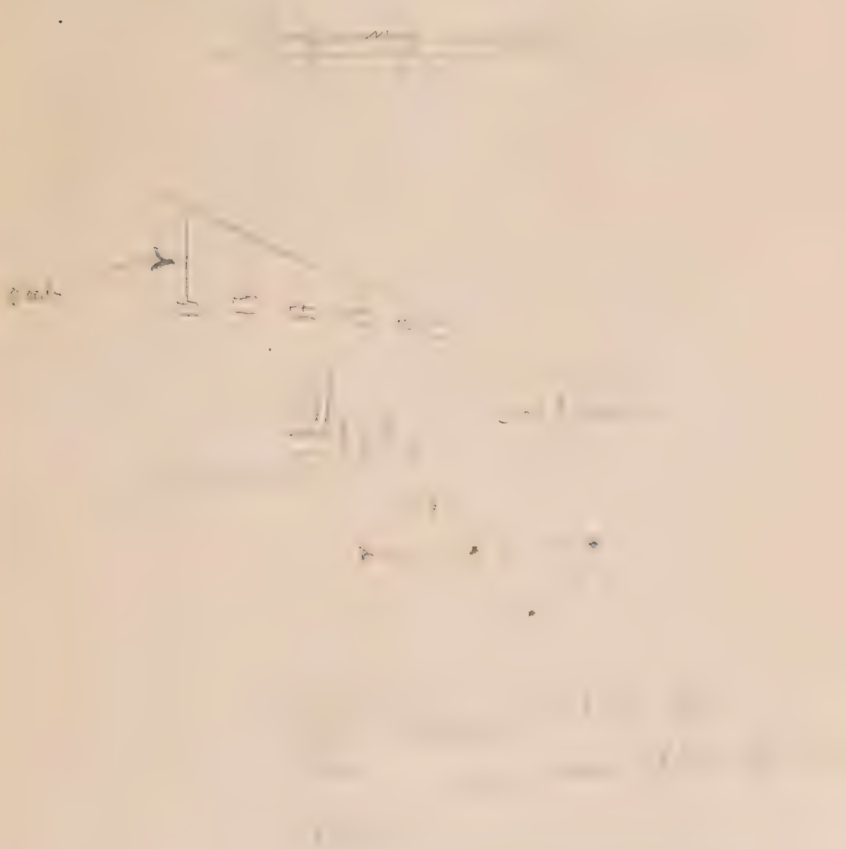


FIG. 30.4.—Six-element, general-purpose oscillograph. Type PM-10. (*General Electric Company.*)

The elements are alike in design and operation but the more vibrator elements the more oscillographic records can be recorded simultaneously on the same film. In the nine-element oscillo-

graph the current-time, voltage-time or watts-time records can be obtained simultaneously on the same film for nine different points in the electric system.

The Type PM-10, general-purpose oscillograph, shown in Fig. 30.4 and manufactured by the General Electric Company, is admirably designed for operation under all test conditions. It has six vibrator elements for current or voltage measurements and, in addition, a vibrator for timing purposes. Besides the six current and voltage vibrator elements, there are also available watt galvanometers for indicating and recording the power in both single-phase and three-phase circuits.



CHAPTER V

FORMS OF REPRESENTATION

The importance of the simple sine wave in the study of alternating-current phenomena has made it desirable to use several forms of representation in order that the interrelations between the several factors in a given problem may be more easily understood and the resultant effect readily calculated. The methods in common use may be grouped as follows:

- I. Graphic
  1. Rectangular coördinates.
  2. Polar coördinates.
  3. Vector diagrams. (*a*) Crank-phase diagrams.
     (*b*) Polar-phase diagrams.
  4. Topographic (see Chap. XXVII).
- II. Analytic
  1. Trigonometric functions.
  2. Complex algebra.
  3. Exponential functions (see Chap. XXVII).

Rectangular and Polar Coördinates.—In alternating currents of the simplest and most desirable form, the relation of the instantaneous values of the voltage or the current with respect to time is expressed by the sine or cosine equation:

$$e = {}^{M}E \sin (\omega t) \tag{1.5}$$

$$i = {}^{M}I \sin (\omega t - \theta) \tag{2.5}$$

e, i = instantaneous values.

${}^{M}E, {}^{M}I$ = maximum values.

$\omega = 2\pi f$.

f = frequency or number of complete cycles per sec.

t = time in sec.

θ = the time angle lag or lead of the current with respect to the voltage.

The relations expressed by equations (1.5) and (2.5) may be represented graphically by curves drawn in Cartesian or *rectangular coördinates*, as illustrated in Figs. 1.5 and 3.5, or by radius vectors and circles in *polar coördinates* as in Figs. 2.5 and 4.5.

In Figs. 1.5 and 3.5 the abscissæ represent time, positive to the right of the origin as indicated by the arrow. The instantaneous values of the current and voltage are plotted as ordinates. In Fig. 1.5 the origin, that is, the instant from which time was measured, is at the zero point of the voltage wave followed by increasing values. The diagram shows that the current reaches zero value $\theta°$ later than the voltage, or the current lags the voltage by $\theta°$.

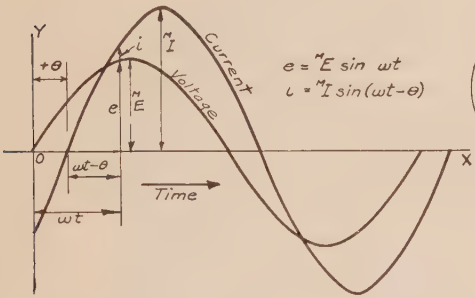


Fig. 1.5.—Rectangular diagram, current lagging $\theta°$.

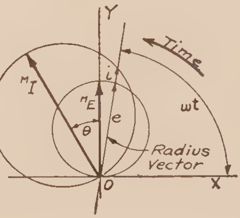


Fig. 2.5.—Polar diagram, same data as for Fig. 1.5.

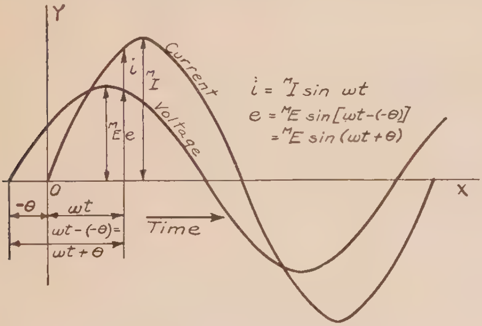


Fig. 3.5.—Rectangular diagram, current lagging $\theta°$.

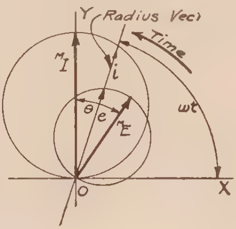


Fig. 4.5.—Polar diagram, corresponds to Fig. 3.5.

The diagram in Fig. 3.5 represents the same voltage and current waves as in Fig. 1.5, the only difference being in the choice of origin, so that in this case time was counted from the instant the current wave was at zero value on the positive slope of the curve.

In *polar coördinates* time is measured by the angle between the rotating *radius vector* and the reference axis. Rotation in the counterclockwise direction is positive; and the magnitude of the instantaneous values are given by the length of the radius

vector. For sine waves the locus of the polar diagram is a circle. The diameter of the circle represents the maximum value and its position, with respect to the Y-axis, the starting phase angle or epoch. Thus, in Fig. 2.5, ${}^{M}E$ is the diameter of the voltage locus, a circle. Similarly, ${}^{M}I$ is the diameter of the current locus and is drawn at an angle $\theta°$ to the left of the Y-axis. Time is generally counted from the X-axis and considered positive in the counterclockwise direction. The diameters ${}^{M}E$ and ${}^{M}I$ of the loci do not move but are stationary with respect to OX or OY. The radius vector rotates and the length of this line, intercepted by the voltage and current loci, represents the instantaneous values, e and i.

In Fig. 4.5 is shown the polar diagram corresponding to the current and voltage waves in Fig. 3.5. Note that diameter ${}^{M}E$ of the voltage locus lies $\theta°$ to the right of ${}^{M}I$ and the Y-axis. That is, the radius vector in rotating in the counterclockwise direction will reach the maximum value of the voltage, ${}^{M}E$, $\theta°$ before it reaches the maximum value of the current, ${}^{M}I$.

It is evident that the same equations for instantaneous values of current and voltage apply for both the rectangular and polar diagrams.

For Figs. 1.5 and 2.5:

$$e = {}^{M}E \sin (\omega t) \tag{3.5}$$
$$i = {}^{M}I \sin (\omega t - \theta) \tag{4.5}$$

For Figs. 3.5 and 4.5:

$$i = {}^{M}I \sin \omega t \tag{5.5}$$
$$e = {}^{M}E \sin [\omega t - (-\theta)] \tag{6.5}$$
$$= {}^{M}E \sin (\omega t + \theta) \tag{7.5}$$

Vector Diagrams.—Quantities that can be fully determined by giving the value of one factor only are termed *scalar*. Examples: mass, length, time, common numbers. Such quantities have magnitude only and can be represented graphically for comparison by straight lines of proportionate lengths. Scalar quantities may be grouped or divided by the usual arithmetical operations.

The sum of certain other quantities cannot be determined from the known magnitude of the components. For example: If a person walks 3 miles east and 2 miles south, his distance from the starting point is not determined by stating that he walked 5 miles. If three forces act on a body the resultant

would equal the sum of the three only under the condition that all were applied at the same point and acting in the same direction. If a force of 6 lb. acts horizontally due north, a second force of 2 lb. due west, while a third of 9 lb. is applied in a vertical direction, the resultant force acting on the body is not 6 + 2 + 9, but 11 lb. If two simple harmonic waves of the same frequency are combined, the resultant maximum value will be equal to the sum of the component maxima only if the two reach their maximum values at the same time. For every other phase relation the resultant maximum will be less. A physical quantity that has both magnitude and either time-phase or space-phase position and direction is called a *vector quantity.* A line which by its position and direction represents the time phase or space phase, and by its length the magnitude of the physical quantity, is termed a *vector.* In all vector diagrams it is customary to measure time as positive when the vector rotates in the counter-clockwise direction.

Polar-phase Diagram.—In polar coördinates the length of the radius vector represents the instantaneous values of the function.

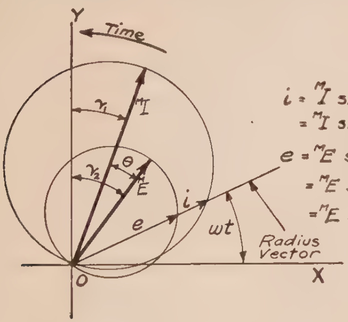


FIG. 5.5.—Polar-phase diagram, current lagging $\theta°$.

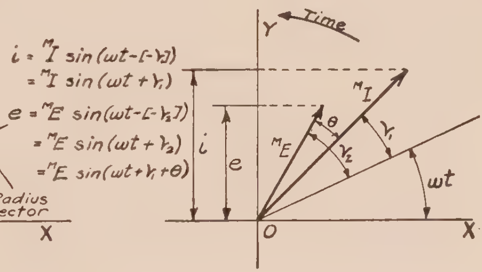


FIG. 6.5.—Crank-phase diagram, current lagging $\theta°$.

For sine waves the loci are circles, and in order to keep the diagram as simple as possible it is customary to draw only the diameter of the locus, that is, the maximum value of the current or voltage. Since the current and voltage have the same frequency the condition of chief importance is their *relative* time position. In most problems in alternating currents the only factors under consideration are the relative time-phase positions and the maximum or effective values of the currents and voltages under discussion. The phase-vector diagram is thus independent of ωt and can be drawn in any position. Thus in Fig. 5.5 the mag-

nitude and relative phase position of the maximum values of a current and voltage are shown in the first quadrant.

Crank Diagrams.—In dealing with simple harmonic functions it is often convenient to keep the rotating vector a constant in magnitude and let the instantaneous values be represented by the projections of the vector upon any fixed line through the origin selected as reference axis. This representation is called a *crank-* or *clock-phase* or *vector* diagram. Thus in Fig. 7.5 let the line B, of constant length, rotate in a counterclockwise direction. The time is measured by the angle ωt while the instantaneous values of the sine function are represented by the projection of the line upon either the X- or the Y-axis. Comparing the quantities in the crank diagram with an equation for a current wave of sine form, $i = {}^MI \sin(\omega t)$, we have as follows: The vector OB represents the maximum value ${}^MI$; the angle XOB represents the time angle ωt; and the projection upon the Y-axis represents i.

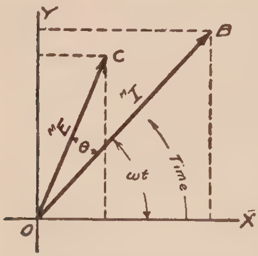


FIG. 7.5.—Crank-phase diagram, current lagging $\theta°$.

The resultant of two sine waves having the same frequency but differing in phase is also a simple harmonic function. The plotting of these waves in rectangular coördinates is tedious and the values of the resultant can be more easily found by using the crank- or polar-vector diagrams.

In Figs. 8.5, 9.5 and 10.5 are represented sine waves as given by equations (8.5) and (9.5); and the resultant sine wave as given by equation (10.5).

$$e_1 = {}^ME \sin[\omega t - (-\gamma_1)] = {}^ME \sin(\omega t + \gamma_1) \qquad (8.5)$$
$$e_2 = {}^ME \sin(\omega t - \gamma_2) \qquad (9.5)$$
$$e_3 = {}^ME \sin[\omega t - (-\gamma_3)] = {}^ME \sin(\omega t + \gamma_3) \qquad (10.5)$$

The quantitative numerical values used in Figs. 8.5, 9.5 and 10.5 are:

$${}^ME_1 = 86 \text{ volts};\ {}^ME_2 = 57 \text{ volts};\ {}^ME_3 = 108 \text{ volts}.$$
$$\gamma_1 = -54°;\ \gamma_2 = 16°;\ \gamma_3 = -23°.$$

For both polar and crank diagrams, time is measured by the angle from the X-axis and counted as positive in the counter-

clockwise direction. It should be noted that vectors $^ME_1$ and $^ME_2$ reverse their position relative to $^ME_3$ in Figs. 8.5 and 10.5. This follows directly from what the diagrams represent. In the polar diagram, Fig. 10.5, the length of the radius vector represents the instantaneous value and hence the position of the vectors $^ME_1$, $^ME_2$ and $^ME_3$ representing the maximum values of the three functions are determined by the values of γ_1, γ_2 and γ_3. With the ωt line representing time rotating in the counterclockwise direction the function having the smallest value for γ will be first to reach its maximum, and likewise the function having the largest value for γ will be last in order to reach its maximum

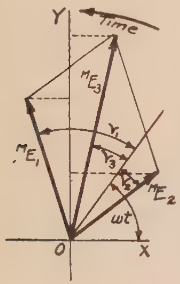


FIG. 8.5.—Crank-phase diagram.

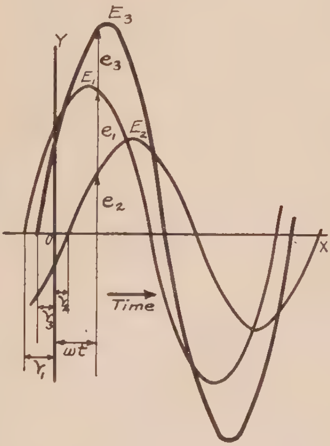


FIG. 9.5.—Rectangular diagram.

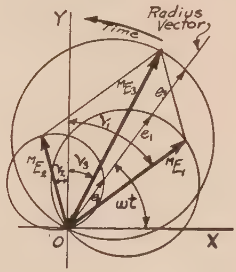


FIG. 10.5.—Polar-phase diagram.

value. Hence the vectors $^ME_2$, $^ME_3$, $^ME_1$ lie in the order given above the X-axis. In the crank diagram, Fig. 8.5, the instantaneous values are represented by the projections of the vectors upon the Y-axis. Hence the maximum values rotate as vectors with the time, and therefore the vector $^ME_1$ with the largest γ leads and $^ME_2$ comes last in order, and the relative positions of $^ME_1$ and $^ME_2$ are reversed when compared with the polar diagram. This reversal in position of vectors in the polar and crank diagrams requires a reversal in signs when the functions are expressed by complex algebraic symbols as explained in the next paragraph. This change in sign has led to confusion To remove this difficulty the International Electrical Congress in 1912 adopted as

standard the complex notation corresponding to the crank-phase diagram.

It should be noted that in solving many alternating-current problems a modified form of the crank-vector diagram is used, in which the length of the vectors represents the effective instead of the maximum voltage and current values. In general, the instantaneous values of the voltage and current do not enter into problems relating to power or electric energy delivered,

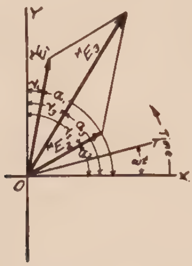


FIG. 11.5.—Polarphase diagram.

FIG. 12.5.

but the effective voltage and current values, as obtained directly from meter readings, and the θ angle of current lag or lead are of prime importance.

Symbol of Rotation.—Mathematical symbols are useful for expressing concise statements in compact form. Such symbols as $+$, $-$, $\int$, log, $\sqrt{}$, etc., state completely more or less complicated operations. In dealing with scalar quantities the minus sign means simply subtraction; that is, part of the positive quantity has been removed. In vector quantities the minus sign indicates opposite direction from the vector with the positive sign. Since the application of the minus sign reverses the direction of the vector, it is evident that the symbol $-$ may be considered as an operator indicating rotation through 180°. Two applications of this operator would result in a rotation of twice 180° or 360°. In algebra $(-) \times (-) = (+)$. Successive applications make the direction of the vector alternately positive and negative. While working with harmonic functions, it is convenient to use an operator indicating a rotation of 90°. In the notation generally used when discussing alternating-current phenomena the symbol "j" indicates a rotation of 90° in the counter-clockwise direction. Thus, if in Fig. 12.5 F represents a vector quantity, a force, for instance, then $-F$ would represent a force along the X-axis but in the opposite direction from F.

Imp.

Likewise jF would indicate an equal force turned 90° so as to lie in the positive direction along the Y-axis. It follows that $-jF$ must denote a like force in the negative direction along the Y-axis. If the operator j indicates a rotation of 90° then two applications must result in a rotation of 180°. Stated in the form of an equation:

$$j \times j = j^2 = (180°) = -1 \tag{11.5}$$

Hence

$$j = \sqrt{-1} \tag{12.5}$$
$$j^4 = 1 \tag{13.5}$$

In ordinary algebra, quantities having the factor $\sqrt{-1}$ are often called *imaginary*. The term is misleading by giving the impression of lack of reality of these quantities. The term *quadrature component* is more appropriate, as the relations between quantities having the factors $\sqrt{-1}$ and the real values may be shown graphically by placing the two quantities in quadrature. The factor $\sqrt{-1}$ or j may therefore be considered as an operator indicating a counterclockwise rotation of 90° and is used to indicate a quadrature component or a quantity in time or space quadrature to the reference axis.

Complex Quantities.—Expressions containing both the *real* and the *imaginary* or *quadrature* components are termed *complex*, and the system of consistent operations for these quantities the *algebra of complex quantities*. Only the few simple operations used in alternating currents will be discussed.

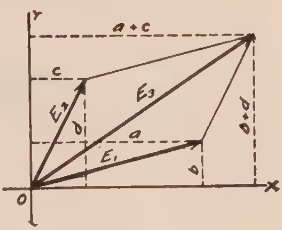


FIG. 13.5.

The laws of ordinary addition and subtraction apply with the one proviso that *real* and *quadrature components* must be taken separately, or graphically. The X and Y components taken separately may be added and subtracted in the same way as if there were only X or only Y quantities. In Fig. 13.5 let the horizontal and vertical components of E_1 and E_2 be a, c and b, d respectively. The corresponding complex equations would be (14.5) and (15.5).

$$\dot{E}_1 = a + jb \tag{14.5}$$
$$\dot{E}_2 = c + jd \tag{15.5}$$

The sum of the X or *real* components is $a + c$; and the sum of the Y or *quadrature* components is $b + d$. Hence the result-

ant vector E_3 is a complex quantity having $a + c$ for the X component and $b + d$ along the Y-axis.

$$\dot{E}_3 = a + c + j(b + d) \tag{16.5}$$

It is readily seen that this method may be extended for three or more vector quantities and that the resultant can be found both graphically and algebraically. This is illustrated in Fig. 14.5 and equations (17.5), (18.5), (19.5) and (20.5).

$$\dot{E}_1 = 15 + j8 \tag{17.5}$$
$$\dot{E}_2 = 12 - j20 \tag{18.5}$$
$$\dot{E}_3 = -7 - j12 \tag{19.5}$$

The resultant:

$$\dot{E}_4 = 20 - j24 \tag{20.5}$$

The rules for ordinary multiplication and division can be applied to complex quantities with the additional factor j or $\sqrt{-1}$ indicating a rotation of 90°.

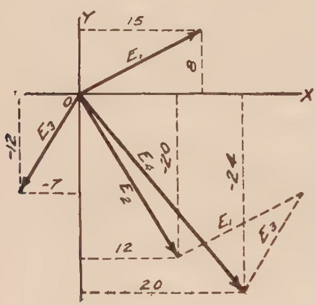


FIG. 14.5.

Multiplying the complex equations (21.5) and (22.5) gives the resultant complex equation (23.5). The corresponding graphical representation is shown in Fig. 15.5.

$$A = 5 + j4 \tag{21.5}$$
$$B = 2 + j3 \tag{22.5}$$
$$A \times B = (5 + j4)(2 + j3)$$
$$= 10 + j8 + j15 + j^2 12$$
$$= -2 + j23 \tag{23.5}$$

In division the operation is stated in the form of a fraction. The denominator is rationalized by multiplying both numerator and denominator with the conjugate quantity as shown in equation (24.5).

$$\frac{A}{B} = \frac{a + jb}{c - jd} = \frac{(a + jb)(c + jd)}{(c - jd)(c + jd)} = \frac{(a + jb)(c + jd)}{c^2 + d^2}$$
$$= \frac{ac - bd}{c^2 + d^2} + \frac{j(cb + ad)}{c^2 + d^2} \tag{24.5}$$

For multiplication of vectors in polar coördinates the process is: (a) find the direction by adding the angles and, (b) obtain the magnitude by multiplying the numerical or absolute values of the component vectors.

For Fig. 16.5:

$$\dot{A} = 2\underline{/\theta_A} = 2\underline{/40°} \tag{25.5}$$

$$\dot{B} = 3.5\underline{/\theta_B} = 3.5\underline{/72°} \tag{26.5}$$

$$\dot{C} = \dot{A}\dot{B} = 2 \cdot 3.5\underline{/\theta_A + \theta_B} = 7\underline{/112°} \tag{27.5}$$

For division of vectors subtract the angles and find the quotient of the numerical or absolute values of the given vectors. Thus,

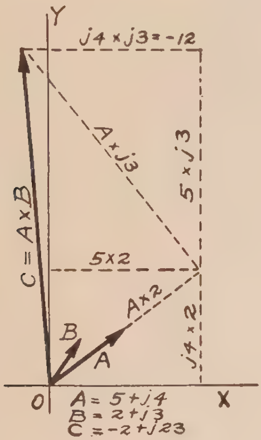


FIG. 15.5.

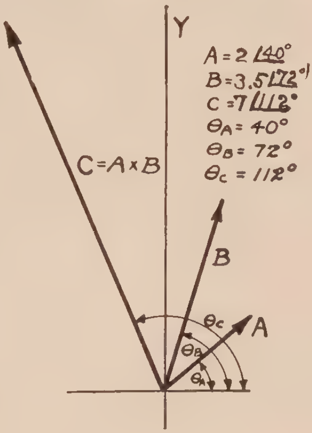


FIG. 16.5.

if it is desired to divide vector C by vector A in Fig. 16.5 the procedure is shown in equation (28.5.)

$$\frac{\dot{C}}{\dot{A}} = \frac{7}{2}\underline{/\theta_C - \theta_A} = 3.5\underline{/72°} = \dot{B} \tag{28.5}$$

The crank diagram is advantageous for adding and subtracting vector quantities, while the multiplication and division of vectors can more readily be performed by using polar coördinates.

PROBLEMS

1.5. Given: $e = 80 \sin (\omega t \mp 15°)$ volts.
$i = 60 \sin (\omega t + 40°)$ amp.

(*a*) Draw curves for e and i in rectangular coördinates.

(*b*) Draw the corresponding polar-phase diagram.

(*c*) Draw the corresponding crank diagram. Note on all the diagrams the angles ωt, 15° and 40°.

2.5. Two alternators of the same frequency are connected in series.

$$e_1 = 120 \sin (\omega t)$$
$$e_2 = 100 \sin (\omega t - 45°)$$

(*a*) Draw the polar-phase diagram of the three voltages.
(*b*) Draw the crank-phase diagram.
(*c*) Find graphically the value of ${}^{M}E_3$.
(*d*) Find the value of E_3.
(*e*) Write the equation for e_3, similar to those given for e_1 and e_2.

3.5. Given: $\dot{E}_1 = 20 + j15.$
$\dot{E}_2 = -12 - j8.$
$\dot{E}_3 = -17 + j10.$

(*a*) Find $\dot{E}_4$ the resultant of $\dot{E}_1$, $\dot{E}_2$ and $\dot{E}_3$.
(*b*) Draw the corresponding vector diagram (crank) for $\dot{E}_1$, $\dot{E}_2$ $\dot{E}_3$ and $\dot{E}_4$.

4.5. Given: a circuit with $z = 5 + j3$ ohms.

$$\dot{I} = 8 - j7 \text{ amp.}$$
$$\dot{E} = z\dot{I} \text{ volts.}$$

(*a*) Find the values of $\dot{E}$ and E.
(*b*) Draw the vector diagram for $\dot{E}$ and $\dot{I}$.

5.5. Given: a circuit with $z = 4 + j3$.

$$\dot{E} = 50 - j20.$$
$$\dot{I} = \frac{\dot{E}}{z}.$$

(*a*) Find the values of $\dot{I}$, I and ${}^{M}I$.
(*b*) Draw the crank-vector diagram for $\dot{I}$, ${}^{M}\dot{I}$, $\dot{E}$ and ${}^{M}\dot{E}$.

6.5. Given a circuit in which:

$$e = 140 \sin (\omega t + 10°) \text{ volts.}$$
$$i = 42 \sin (\omega t - 12°) \text{ amp.}$$

(*a*) Find the effective values E and I.
(*b*) Draw curves for e and i in rectangular coördinates.
(*c*) Draw the corresponding polar-phase diagram.
(*d*) Draw the corresponding crank-vector diagram.

In (*b*), (*c*) and (*d*), show ${}^{M}E$, ${}^{M}I$ and the angles ωt, 10° and 12° as given in the equations.

CHAPTER VI

SERIES CIRCUITS, PARALLEL CIRCUITS

In direct currents the relation between the current and voltage in an electric circuit is given by the equation $E = RI$. In alternating currents magnetic and dielectric induction enter as essential factors and must be taken into consideration with the resistance in determining both the quantitative and phase relations of the currents and voltages. Most commercial systems consist of networks with the three factors, resistance, inductance and condensance (capacity, capacitance), distributed and intermingled in a more or less complex manner. It may be looked upon as a combination of a number of simple series and parallel circuits. In a simple-series circuit the current is the same throughout while the voltage may be separated into parts. The voltage consumed by the resistance, inductance and condensance (capacity, capacitance) may be found separately and the total obtained from the resultant of the several components.

Resistance.—The effect of the resistance factor is the same for alternating as for direct currents.

Assuming a sine wave:

$$i = {}^{M}I \sin (\omega t) \tag{1.6}$$

For instantaneous values:

$$ {}_{r}e = ri = r\,{}^{M}I \sin (\omega t) \tag{2.6}$$

For maximum values:

$$ {}^{M}_{r}E = r\,{}^{M}I \tag{3.6}$$

For effective values:

$$ {}_{r}E = rI. \tag{4.6}$$

The voltage is similar to, and in time phase with, the current.

Let the current vector be placed along the X-axis. Since the voltage is in time phase with the current the voltage vector will also lie along the X-axis, Fig. 1.6.

As already stated, the length of the vector represents the maximum value of the voltage or current. Since the ratio

between the maximum and effective value for sine waves is $\sqrt{2}$, a constant, it is evident that the length of the vector may also be drawn so as to represent effective values of voltage or current. It should be noted that when effective values are used the diagram represents in magnitude constant quantities and also the phase relations of the maximum values.

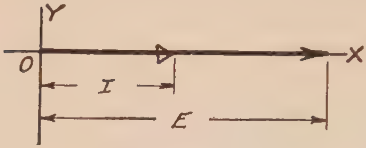


FIG. 1.6.

Inductive Reactance.—The magnetic field surrounding a conductor is proportional to the current.[1]

$$\phi = Li \tag{5.6}$$

The voltage consumed by the inductance in the circuit is proportional to the rate of change in the lines of force.

$${}_Le = \frac{d\phi}{dt} \tag{6.6}$$

From equations (5.6) and (6.6):

$${}_Le = L\frac{di}{dt} \tag{7.6}$$

$di = I^m \cos(\omega t)\,\omega\, dt$

Assuming the current a sine wave from equation (1.6), for instantaneous values:

$${}_Le = L\frac{d({}^MI \sin(\omega t))}{dt} = \omega L\,{}^MI \cos(\omega t) = {}_Lx\,{}^MI \cos(\omega t) \tag{8.6}$$

For maximum values:

$${}^M_LE = j\omega L\,{}^MI = j\,{}_Lx\,{}^MI \tag{9.6}$$

For effective values:

$${}_LE = j\omega LI = j\,{}_LxI \tag{10.6}$$

The voltage consumed by the inductance is in time quadrature with the current; that is, the voltage will reach its positive maximum value 90° before the current reaches the corresponding maximum.

For the maxima and effective values the phase relation is indicated by the symbol j. Placing the current vector along the X-axis, the voltage vector will be at right angles and lie upward along the Y-axis, Fig. 2.6. By comparing the vector diagrams, Figs. 1.6 and 2.6, it is seen that for the same current the voltage consumed by the inductance is in time quadrature

[1] See footnote on p. 4.

with the voltage taken by the resistance. The quantity ωL or $2\pi f L$ bears the same relation to the voltage consumed by the inductance as r to the resistance voltage. It is called the *inductive reactance* of the circuit and is represented by the symbol ${}_L x$. It should be noticed that ${}_L x$ is directly proportional to the frequency, while the resistance is independent of the frequency.

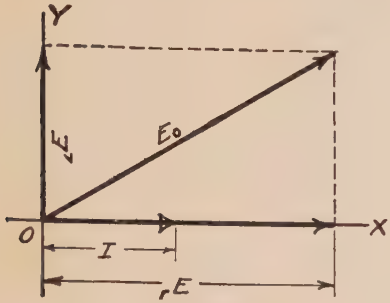


FIG. 2.6.

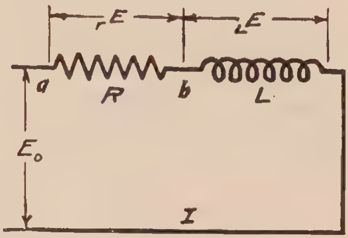


FIG. 3.6.

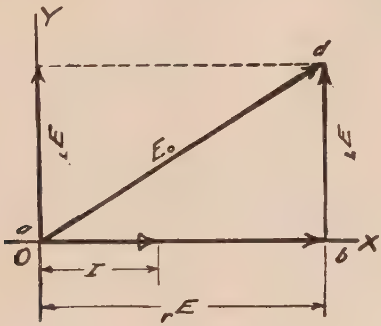


FIG. 4.6.

In a circuit having resistance and inductive reactance in series, as indicated in Fig. 3.6, the voltage consumed by the inductive reactance is in time quadrature with the resistance voltage. Hence the total voltage is the vector sum of the two component vectors.

For instantaneous values:

$$e_0 = r\,{}^M I \sin(\omega t) + {}_L x\,{}^M I \cos(\omega t) \tag{11.6}$$

For maximum values:

$$\begin{aligned} {}^M\dot{E}_0 &= r\,{}^M I + j\,{}_L x\,{}^M I = {}^M_r E + j\,{}^M_L E \\ &= {}^M\dot{E}_{ab} + {}^M\dot{E}_{bd} \end{aligned} \tag{12.6}$$

For effective values:

$$\dot{E}_0 = rI + j\,{}_Lx I = {}_rE + j\,{}_LE = \dot{E}_{ab} + \dot{E}_{bd} \qquad (13.6)$$

In Fig. 4.6 is shown the vector diagram of the voltages and current in a circuit having a resistance of 8 ohms in series with an inductive reactance of 5 ohms. The current is 12 amp.

Since the vectors for ${}_re$ and ${}_Le$ are in phase quadrature their sum equals the square root of the sum of the squares.

$$E_0 = \sqrt{{}_rE^2 + {}_LE^2} = \sqrt{r^2I^2 + {}_Lx^2I^2} = I\sqrt{r^2 + {}_Lx^2} \qquad (14.6)$$

Hence the three voltmeters connected as indicated in Fig. 3.6 would read ${}_rE = 96$, ${}_LE = 60$ and $E_0 = 112.7$ volts.

Condensive Reactance.—The dielectric flux between two conductors is proportional to their difference of potential.[1]

$$\psi = Ce \qquad (15.6)$$

Any increase or decrease in the dielectric flux requires or produces a corresponding flow of current in the condenser circuit.

$$i = \frac{d\psi}{dt} \qquad (16.6)$$

$$i = C\frac{d_ce}{dt} \qquad (17.6)$$

Hence:

$$_ce = \int \frac{idt}{C} = \frac{{}^MI}{C}\int \sin(\omega t)dt \qquad (18.6)$$

For instantaneous values:

$$_ce = -\frac{{}^MI}{\omega C}\cos(\omega t) = -{}_cx\,{}^MI\cos(\omega t) \qquad (19.6)$$

For maximum values:

$${}^M_cE = -j\frac{{}^MI}{\omega C} = -j\,{}_cx\,{}^MI \qquad (20.6)$$

For effective values:

$$_cE = -j\frac{I}{\omega C} = -j\,{}_cxI \qquad (21.6)$$

The factor $\frac{1}{\omega C}$ is called the *condensive reactance* and is represented by the symbol ${}_cx$.

A circuit having resistance and condensance in series is represented in Fig. 5.6. The corresponding vector diagram is shown in Fig. 6.6. The voltage consumed by the condensance is repre-

[1] See footnote on p. 5.

sented by a vector along the Y-axis and 90° behind the resistance voltage. The total voltage is the resultant of the two component voltages.

The circuit represented in Figs. 5.6 and 6.6 has a 6-ohm resistance, 12-ohm reactance and a current of 5 amp.

For instantaneous values:

$$e_0 = r\,{}^{M}I \sin(\omega t) - {}_{c}x\,{}^{M}I \cos(\omega t) \tag{22.6}$$

For maximum values:

$${}^{M}E_0 = r\,{}^{M}I - j\,{}_{c}x\,{}^{M}I = {}^{M}_{r}E - j\,{}^{M}_{c}E = {}^{M}\dot{E}_{ab} + {}^{M}\dot{E}_{bd} \tag{23.6}$$

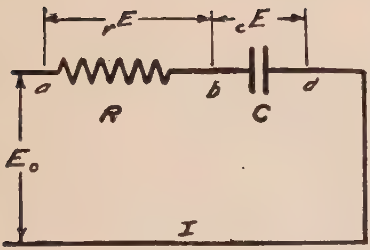


FIG. 5.6.

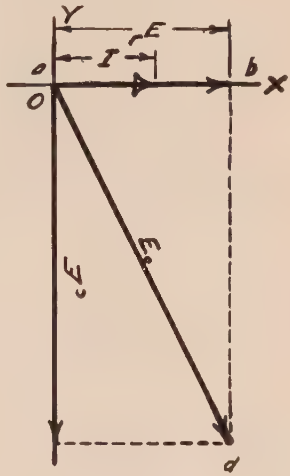


FIG. 6.6.

For effective values:

$$E = rI - j\,{}_{c}xI = {}_{r}E - j\,{}_{c}E = \dot{E}_{ab} + \dot{E}_{bd} \tag{24.6}$$

The absolute or numerical value of the total voltage is the square root of the sum of the squares of the voltages consumed by the resistance and reactance.

$$E_0 = \sqrt{r^2I^2 + {}_{c}x^2I^2} = I\sqrt{r^2 + {}_{c}x^2} = \sqrt{{}_{r}E^2 + {}_{c}E^2} \tag{25.6}$$

Impedance.—In Fig. 7.6 is represented a circuit having resistance, inductance and condensance in series.

Let

$I = 7.5$ amp.; $r = 5.4$ ohms; ${}_{L}x = 4.0$ ohms; and ${}_{c}x = 8.2$ ohms.

The equation of the circuit is:

For instantaneous values:

$$e_0 = r^MI \sin(\omega t) + {}_Lx^MI \cos(\omega t) - {}_cx^MI \cos(\omega t) \tag{26.6}$$

$$e_0 = 5.4 \cdot \sqrt{2} \cdot 7.5 \sin(\omega t) + 4.0 \cdot \sqrt{2} \cdot 7.5 \cos(\omega t)$$
$$- 8.2 \cdot \sqrt{2} \cdot 7.5 \cos(\omega t) \text{ volts.}$$

For maximum values:

$${}^M\dot{E}_0 = r^MI + j\,{}_Lx^MI - j\,{}_cx^MI \tag{27.6}$$
$$= 5.4 \cdot \sqrt{2} \cdot 7.5 + j4.0 \cdot \sqrt{2} \cdot 7.5 - j8.2 \cdot \sqrt{2} \cdot 7.5 \text{ volts.}$$

For effective values:

$$\dot{E}_0 = rI + j\,{}_LxI - j\,{}_cxI = rI + jI({}_Lx - {}_cx) \tag{28.6}$$
$$= 5.4 \cdot 7.5 + j7.5(4.0 - 8.2) \text{ volts}$$
$$= \dot{E}_{ab} + \dot{E}_{bd} + \dot{E}_{de}.$$

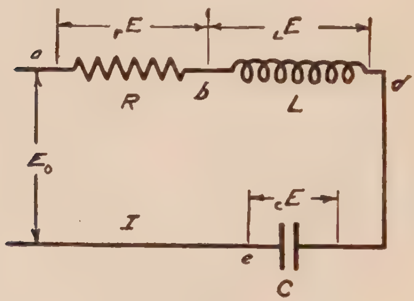


FIG. 7.6.

Let

$$x = {}_Lx - {}_cx = -4.2 \tag{29.6}$$
$$\dot{E}_0 = \dot{I}(r + jx) \tag{30.6}$$
$$= 7.5(5.4 - j4.2) \text{ volts.}$$

Let

$$z = r + jx \text{ or, } Z = \sqrt{r^2 + x^2} \tag{31.6}$$
$$= 5.4 - j4.2 = \sqrt{5.4^2 + 4.2^2} = 6.85 \text{ ohms.}$$
$$\dot{E}_0 = z\dot{I} \text{ or } E_0 = ZI \tag{32.6}$$
$$= 6.85 \cdot 7.5 = 51.4 \text{ volts.}$$

This factor Z or z in equation (32.6) is called the *impedance* of the circuit. By replacing the resistance as used in direct currents with the impedance, *Ohm's law is reestablished for alternating currents, or stated in its complete form for all electric circuits.* In Fig. 8.6 is shown the vector diagram corresponding to the circuit in Fig. 7.6. The resultant voltage vector along the

Y-axis is the difference of the voltages consumed by $_Lx$ and $_Cx$. The total voltage is the vector sum of the resistance voltage on the X-axis and the difference of the two reactance voltages. Instead of drawing the voltage vectors through the origin, as in Fig. 8.6,

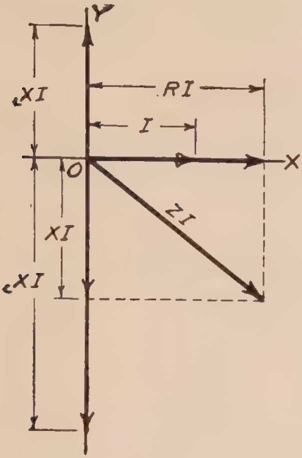


Fig. 8.6.

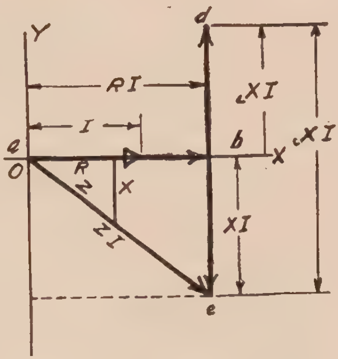


Fig. 9.6.

it is sometimes less labor to draw a triangle of the voltages as shown in Fig. 9.6.

Since I is a common factor, a right triangle may also be formed from x, r and z as shown in Fig. 9.6. This is called the *impedance triangle*. The complex expression $z = r + jx$, from equation (31.6), indicates the same relation. Resistance, reactance and impedance are, however, not vector quantities like current and voltage, but merely act as operators.

Conductance, Susceptance, Admittance.—In a series circuit the same current passes through all the parts, while in parallel circuits the voltage is the common factor. In drawing the vector diagrams for the series circuits the current is generally taken as the reference vector since it is the common factor. Similarly, it is usually desirable to take the voltage as reference vector in parallel circuits. Just as in dealing with series circuits in the preceding para-

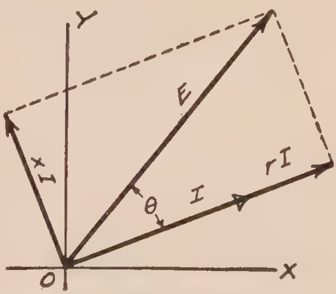


Fig. 10.6.

graphs the total voltage was composed of several components, so in parallel circuits the total current may be considered as consisting of vector components from the several parallel circuits. In equation (33.6) and Fig. 10.6 the voltage is separated into one part rI in phase with the current, and another jxI leading the current by 90°. Solving the equation for current we have equation (34.6).

$$E = rI + jxI \tag{33.6}$$

$$I = \frac{E}{r + jx} \tag{34.6}$$

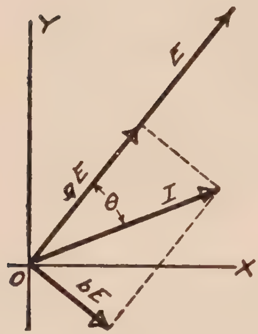


Fig. 11.6.

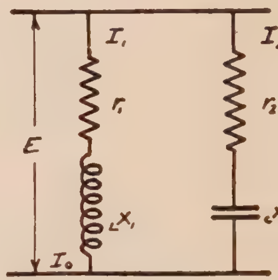


Fig. 12.6.

Rationalizing the fraction by multiplying both numerator and denominator by $r - jx$ gives

$$I = E\frac{(r - jx)}{r^2 + x^2} = \frac{rE}{r^2 + x^2} - \frac{jxE}{r^2 + x^2} \tag{35.6}$$

For convenience let:

$$\frac{r}{r^2 + x^2} = g; \text{ and } \frac{x}{r^2 + x^2} = b \tag{36.6}$$

Then:

$$I = gE - jbE \tag{37.6}$$

Equation (37.6) states that the current consists of two components, one in phase and the other in quadrature with the voltage. The corresponding vector diagram is shown in Fig. 11.6. The minus sign before the quadrature component indicates that the voltage leads the current. That this is the same relation as indicated by equation (33.6) is evident; for, if the current lags behind the voltage, necessarily the voltage leads the current.

The factor $\dfrac{r}{r^2 + x^2}$ is called the *conductance* of the circuit and is represented by the symbol g.

The factor $\dfrac{x}{r^2 + x^2}$ is called the *susceptance* of the circuit and is represented by the symbol b.

Let

$$y = g - jb; \text{ or } Y = \sqrt{g^2 + b^2} \tag{38.6}$$

Then y or Y is the *admittance* of the circuit.

From equations (37.6) and (38.6):

$$\dot{I} = y\dot{E}, \text{ or } I = YE \tag{39.6}$$

Combining this with equation (32.6):

$$Y = \frac{1}{Z}, \text{ or } Z = \frac{1}{Y} \tag{40.6}$$

The practical unit for admittance, conductance and susceptance is the reciprocal ohm, called the *mho.*

For an illustration take two circuits as represented in Fig. 12.6.

In the first circuit: $r_1 = 12$ ohms and ${}_Lx_1 = 10$ ohms.

In the second circuit: $r_2 = 8$ ohms and ${}_cx_2 = 15$ ohms.

Let the voltage for both circuits be 120 volts.

$$g_1 = \frac{r_1}{r_1^2 + {}_Lx_1^2} = \frac{12}{12^2 + 10^2} = 0.0492 \text{ mho.}$$

$$b_1 = \frac{{}_Lx_1}{r_1^2 + {}_Lx_1^2} = \frac{10}{12^2 + 10^2} = 0.0409 \text{ mho.}$$

$$I_1 = g_1E - jb_1E = 0.0492E - j0.0409E \text{ amp.} \tag{41.6}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_cx_2^2} = \frac{8}{8^2 + 15^2} = 0.0276 \text{ mho.}$$

$$b_2 = \frac{{}_cx_2}{r_2^2 + {}_cx_2^2} = \frac{15}{8^2 + 15^2} = 0.0519 \text{ mho.}$$

$$\dot{I}_2 = g_2\dot{E} + jb_2\dot{E} = 0.0276\dot{E} + j0.0519\dot{E} \text{ amp.} \tag{42.6}$$

The total current is the vector sum of the component currents as shown graphically in Fig. 13.6 and analytically by equation (43.6).

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = 0.0492\dot{E} - j0.0409\dot{E} + 0.0276\dot{E} + j0.0519\dot{E}$$

$$= (0.0768 + j0.011)\dot{E} = (g_0 + jb_0)\dot{E} \tag{43.6}$$

$$I = E\sqrt{(g_0^2 + b_0^2)} = 0.0843E = 10.12 \text{ amp.} \tag{44.6}$$

Ohm's Law.—From the discussion in this and the previous chapter it is evident that Ohm's law may be expressed by either equation (45.6) or equation (46.6).

$$\dot{E} = z\dot{I} \tag{45.6}$$

$$\dot{I} = y\dot{E} \tag{46.6}$$

Therefore *the joint impedance of a number of series-connected impedances, if expressed in complex quantities, is equal to the sum of the individual impedances; the joint admittance of a number of parallel-connected admittances, if expressed in complex quantities, is equal to the sum of the individual admittances. In diagrammatic representation, combination of vectors by the parallelogram law takes the place of addition of the complex quantities.*

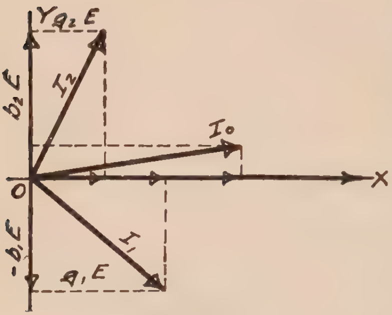


FIG. 13.6.

Kirchhoff's Laws.—For direct currents Kirchhoff's laws state:

(*a*) *The sum of all the currents flowing to a point is zero.*

$$\sum I = 0.$$

(*b*) *The sum of all the electromotive forces in a circuit equals zero.*

$$\sum E = 0.$$

In alternating currents the phase relations as well as the magnitude of the component parts must be included. Any voltage wave can be resolved into a component in phase with the current and a second component in quadrature with the current. In combining several voltages these components must be taken separately, as neither can have any effect on the magnitude of the other. Likewise the currents can be resolved into one component in phase with the voltage and a second component in quadrature with the voltage. For the same reason neither of these components can have any effect on the magnitude of the other, and therefore must be taken separately when several currents are combined.

Kirchhoff's laws, therefore, reestablished for alternating currents are:

(*c*) The sum of the real components and the sum of the quadrature components of currents flowing to a point are separately equal to zero.

$$\sum \dot{I} = 0, \text{ or } \sum {}_a\dot{I} = 0 \text{ and } \sum {}_b\dot{I} = 0. \qquad (47.6)$$

(*d*) The sum of all the real components and the sum of all the quadrature components of the e.m.f.'s. in a circuit are separately equal to zero.

$$\sum \dot{E} = 0, \text{ or } \sum {}_r\dot{E} = 0 \text{ and } \sum {}_x\dot{E} = 0 \qquad (48.6)$$

PROBLEMS

1.6. Given a circuit as shown in Fig. 3.6. $I = 7$ amp.; $r = 5$ ohms; $L = 0.02$ henry; $f = 60$ cycles

$$z = r + j(2\pi fL) = r + j\,{}_Lx$$

(*a*) Find ${}_rE$, ${}_LE$, E_0.
(*b*) Draw vector diagram for $\dot{I}$, ${}_r\dot{E}$, ${}_L\dot{E}$ and $\dot{E}_0$.
(*c*) Draw the impedance triangle.

2.6. Same circuit and data as for problem 1.6 except the frequency is 25 cycles.

3.6. Circuit diagram as in Fig. 5.6. The impressed voltage, $E_0 = 220$ volts; $r = 8$ ohms; $C = 320$ mf.; $f = 60$ cycles.

$$z = r - j\left(\frac{1}{2\pi fC}\right) = r - j\,{}_cx$$

(*a*) Find I, ${}_rE$ and ${}_cE$.
(*b*) Draw vector diagram showing $\dot{I}$, ${}_r\dot{E}$, ${}_c\dot{E}$ and $\dot{E}_0$.
(*c*) Draw the impedance triangle.

4.6. Same circuit and data as for problem 3.6 except the frequency is 25 cycles.

5.6. Circuit diagram as in Fig. 7.6. The impressed voltage, $E_0 = 240$ volts; $r = 6$ ohms; $L = 0.05$ henry; $C = 273$ mf.; $f = 60$.

$$z = r + j\left(2\pi fL - \frac{1}{2\pi fC}\right) = r + j({}_Lx - {}_cx)$$

(*a*) Find I, ${}_rE$, ${}_LE$, ${}_cE$ and ${}_xE$.
(*b*) Draw a vector diagram showing $\dot{I}$, ${}_r\dot{E}$, ${}_L\dot{E}$, ${}_c\dot{E}$, ${}_x\dot{E}$, $\dot{E}_0$.
(*c*) Draw the impedance triangle.

6.6. Same circuit and data as in problem 5.6 except the frequency is 25 cycles.

7.6. Readings taken in the laboratory with an alternating-current voltmeter (*a*) in series with a telephone condenser and (*b*) directly across a 60-cycle line, are respectively 47.0 and 114.0 volts. The resistance of the voltmeter is 3,500 ohms and its reactance is negligible. What are the impedance and condensance of the condenser, assuming that there is no energy loss in it?

8.6. Given a circuit having a constant reactance of 0.5 ohm in series with a resistance that can be varied from 0 to 3 ohms. Plot curves for imped-

ance, admittance, conductance and susceptance as ordinates and with resistance as abscissæ.

9.6. Given a circuit having a constant resistance of 0.5 ohm in series with a reactance that can be varied from +2 ohms (inductive) to −2 ohms (condensive). Plot curves for impedance, admittance, conductance and susceptance as ordinates and with reactance as abscissæ.

10.6. A hot-wire galvanometer having a resistance of 4.3 ohms is shunted with a 0.5-mf. condenser. What percentage of the total current will pass through the meter at frequencies of 3,000, 30,000, 300,000 and 3,000,000 cycles?

CHAPTER VII

SOLUTION OF CIRCUITS

Having, in the preceding chapters, shown that Ohm's and Kirchhoff's laws, when expressed in their complete form, apply to alternating currents, we are in position to calculate alternating-current-circuit networks containing resistance, inductive reactance and condensive reactance in as concise a manner as for direct currents. The symbols and expressions for the circuit constants and the equations for Ohm's and Kirchhoff's laws as applied to alternating currents are grouped for ready reference.

$$\text{Resistance} = r = \frac{g}{g^2 + b^2} = \frac{g}{g^2 + ({}_cb - {}_Lb^2)} \tag{1.7}$$

$$\text{Reactance} = x = {}_Lx - {}_cx = \frac{b}{g^2 + b^2} = \frac{{}_cb - {}_Lb}{g^2 + ({}_cb - {}_Lb)^2} \tag{2.7}$$

$$\text{Impedance} = z = r \pm jx = r + j({}_Lx - {}_cx) = \frac{1}{y} \tag{3.7}$$

$$\text{Impedance} = Z = \sqrt{r^2 + x^2} = \sqrt{r^2 + ({}_Lx - {}_cx)^2} = \frac{1}{Y} \tag{4.7}$$

$$\text{Conductance} = g = \frac{r}{r^2 + x^2} = \frac{r}{r^2 + ({}_Lx - {}_cx)^2} \tag{5.7}$$

$$\text{Susceptance} = b = \frac{x}{r^2 + x^2} = \frac{{}_Lx - {}_cx}{r^2 + ({}_Lx - {}_cx)^2} \tag{6.7}$$

$$\text{Admittance} = y = g \mp jb = g - j({}_Lb - {}_cb) = \frac{1}{z} \tag{7.7}$$

$$\text{Admittance} = Y = \sqrt{g^2 + b^2} = \sqrt{g^2 + ({}_cb - {}_Lb)^2} = \frac{1}{Z} \tag{8.7}$$

Ohm's law in notation indicating both magnitude and phase relations:

$$\dot{E} = z\dot{I} = (r + jx)\dot{I} = [r + j({}_Lx - {}_cx)]\dot{I} \tag{9.7}$$

$$\dot{I} = y\dot{E} = (g - jb)\dot{E} = [g - j({}_Lb - {}_cb)]\dot{E} \tag{10.7}$$

For absolute values without directly indicating phase relations:

$$E = ZI = I\sqrt{r^2 + x^2} = I\sqrt{r^2 + ({}_Lx - {}_cx)^2} \tag{11.7}$$

$$I = YE = E\sqrt{g^2 + b^2} = E\sqrt{g^2 + ({}_cb - {}_Lb)^2} \tag{12.7}$$

Kirchhoff's law in notation indicating both magnitude and phase relations:

$$\sum \dot{E} = 0 \text{ or } \sum {}_rE = 0 \text{ and } \sum j_xE = 0 \qquad (13.7)$$

$$\sum \dot{I} = 0 \text{ or } \sum {}_gI = 0 \text{ and } \sum j_bI = 0 \qquad (14.7)$$

From the above equations it is evident that the field of constant direct currents is merely the special case of alternating currents when $f = 0$. The energy stored in both the magnetic and the dielectric fields under this condition remains constant, and hence only the electric circuit, in a restricted sense, need be considered in direct-current problems.

It is obviously impossible to discuss all of the infinite variety of combinations which can be imagined or which may exist in a network of circuits; therefore, only some of the simpler and more common types will be considered.

The notation used in the text may readily be kept in mind by referring to the corresponding circuit and vector diagrams. The subscripts in the lower right-hand corner give the position in the circuit, while the subscripts in the lower left-hand corner indicate the nature of the circuit. Thus, ${}_Lx_2$ indicates inductive reactance in circuit No. 2; and ${}_cx_1$ indicates condensive reactance in circuit No. 1. Similarly, ${}_rE_3$ indicates the voltage consumed by the resistance r in circuit No. 3; and ${}_cI_2$ the condensance current flowing in circuit No. 2. The dots above $\dot{E}$ and $\dot{I}$ indicate a complex or vector nature of the voltage and current, while the same letters without the dot stand for the absolute or numerical values.

I. SERIES CIRCUITS

a. **Resistance and Inductance.**

Problem 1.7.—Circuit diagram, Fig. 1.7. Vector diagram, Fig. 2.7.

Given: $r = 4.8$ ohms; $L = 0.02$ henry; $I = 10$ amp., $f = 60$ cycles.

Find: ${}_rE$, ${}_LE$, $\dot{E}_0$, E_0, θ_0, $\cos\theta_0$.

$${}_rE = rI = 48 \text{ volts} = E_{ab}.$$

$${}_LE = {}_LxI = 2\pi fLI = 75.4 \text{ volts} = E_{bd}.$$

$$\dot{E}_0 = \dot{I}(r + j\,{}_Lx) = {}_r\dot{E} + j\,{}_L\dot{E} = 48 + j75.4 \text{ volts} = \dot{E}_{ad}.$$

$$E_0 = \sqrt{{}_rE^2 + {}_LE^2} = \sqrt{48^2 + 75.4^2} = 89.4 \text{ volts} = E_{ad}.$$

$$\theta_0 = \tan^{-1}\frac{{}_Lx}{r} = 57^\circ\,31'; \cos\theta_0 = 0.537.$$

Problem 2.7.—Given the same circuit as in Fig. 1.7 with r, L and I the same as in problem 1.7 but $f = 25$ cycles.

Find: ${}_rE$, ${}_LE$, $\dot{E}_0$, E_0, θ_0 and $\cos\theta_0$.

Draw the corresponding vector diagram.

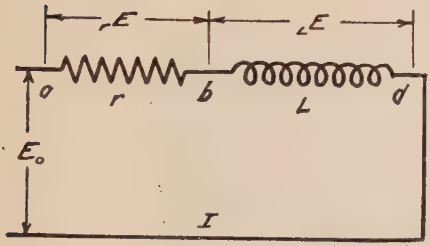


FIG. 1.7.

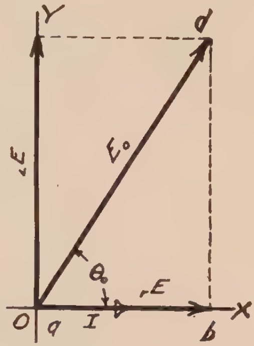


FIG. 2.7

Problem 3.7.—Circuit diagram, Fig. 3.7. Vector diagram, Fig. 4.7.

Given: $r_1 = 8.2$ and $r_2 = 2.7$ ohms; $L_1 = 0.01$ and $L_2 = 0.03$ henry; $f = 25$ cycles; $I = 10$ amp.

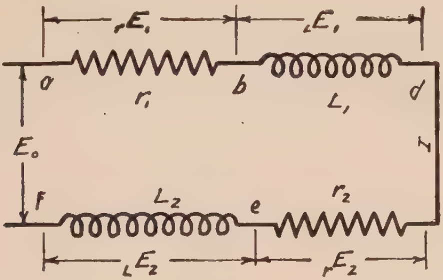


FIG. 3.7

Find: ${}_rE_1$, ${}_LE_1$, ${}_rE_2$, ${}_LE_2$, ${}_rE$, ${}_LE$, $\dot{E}_0$, E_0, θ_1, θ_2, θ_0, $\cos\theta_0$.

$${}_rE_1 = r_1 I = 82.0 \text{ volts} = E_{ab}.$$
$${}_LE_1 = {}_Lx_1 I = 15.7 \text{ volts} = E_{bd}.$$
$$E_1 = I\sqrt{r_1^2 + {}_Lx_1^2} = 83.5 \text{ volts} = E_{ad}.$$
$${}_rE_2 = r_2 I = 27.0 \text{ volts} = E_{de}.$$
$${}_LE_2 = {}_Lx_2 I = 47.1 \text{ volts} = E_{ef}.$$
$$E_2 = I\sqrt{r_2^2 + {}_Lx_2^2} = 54.2 \text{ volts} = E_{df}.$$
$${}_rE = {}_rE_1 + {}_rE_2 = 109.0 \text{ volts}.$$
$${}_LE = {}_LE_1 + {}_LE_2 = 62.8 \text{ volts}.$$
$$\dot{E}_0 = {}_rE + j\,{}_LE = 109.0 + j62.8 \text{ volts} = \dot{E}_{af}.$$
$$E_0 = \sqrt{{}_rE^2 + {}_LE^2} = 125.7 \text{ volts} = E_{af}.$$

$$\theta_1 = \tan^{-1}\frac{{}_Lx_1}{r_1} = 10°\ 50'. \quad \theta_2 = \tan^{-1}\frac{{}_Lx_2}{r_2} = 60°\ 10'.$$

$$\theta_0 = \tan^{-1}\frac{{}_Lx_1 + {}_Lx_2}{r_1 + r_2} = 30°;\ \cos\theta_0 = 0.866.$$

Problem 4.7.—Same circuit and constants as in problem 3.7 except the frequency is 60 cycles per sec.

Draw vector diagram and find ${}_rE_1$, ${}_LE_1$, ${}_rE_2$, ${}_LE_2$, E_1, E_2, ${}_rE$, ${}_LE$, $\dot{E}_0$, E_0, θ_1, θ_2, θ_0 and $\cos\theta_0$.

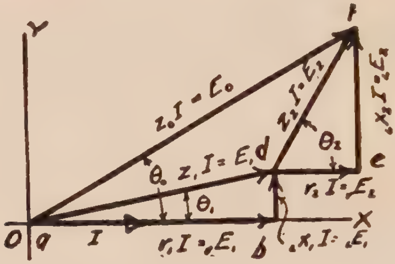


FIG. 4.7.

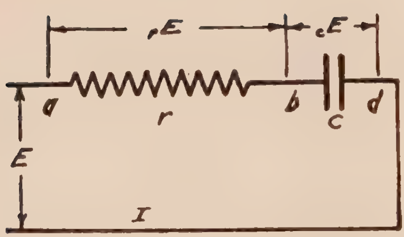


FIG. 5.7.

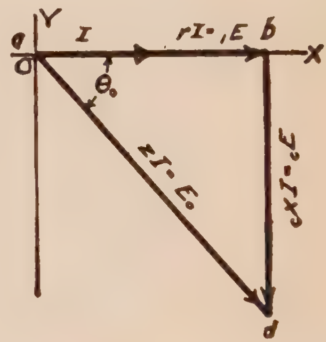


FIG. 6.7.

b. **Resistance and Condensance.**

Problem 5.7.—Circuit diagram, Fig. 5.7. Vector diagram, Fig. 6.7.

Given: $r = 6.3$ ohms; $C = 440$ mf.; $I = 10$ amp.; $f = 50$ cycles.

Find: ${}_rE$, ${}_cE$, $\dot{E}_0$, E_0, θ_0 and $\cos\theta_0$.

$${}_rE = rI = 63.0 \text{ volts} = E_{ab}.$$

$${}_cE = {}_cxI = 72.4 \text{ volts} = E_{bd}.$$

$$\dot{E}_0 = \dot{I}(r - j\,{}_cx) = {}_r\dot{E} - j\,{}_c\dot{E} = 63.0 - j72.4 \text{ volts} = \dot{E}_{ad}.$$

$$E_0 = I\sqrt{r^2 + {}_cx^2} = \sqrt{{}_rE^2 + {}_cE^2} = 95.9 \text{ volts} = E_{ad}.$$

$$\theta = \tan^{-1}\frac{7.24}{6.3} = 49°;\ \cos\theta = 0.656.$$

Problem 6.7.—Given the same circuit and constants as in problem 5.7 except $f = 60$ cycles.

Find: ${}_rE$, ${}_cE$, $\dot{E}_0$, E_0, θ_0 and $\cos\theta_0$.

Draw the vector diagram.

Problem 7.7.—Circuit diagram, Fig. 7.7. Vector diagram, Fig. 8.7.

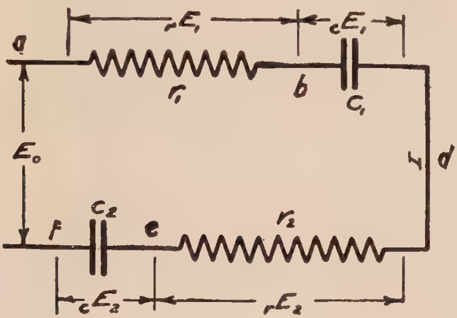


FIG. 7.7.

$r_1 = 8.0$ and $r_2 = 3.5$ ohms; $C_1 = 800$ and $C_2 = 250$ mf.; $I = 10$ amp.; $f = 60$ cycles.

Find: ${}_rE_1$, ${}_cE_1$, ${}_rE_2$, ${}_cE_2$, ${}_rE$, ${}_cE$, E_1, E_2, $\dot{E}_0$, E_0, θ_1, θ_2, and θ_0.

$${}_rE_1 = r_1 I = 80.0 \text{ volts} = E_{ab}.$$
$${}_rE_2 = r_2 I = 35.0 \text{ volts} = E_{de}.$$
$${}_cE_1 = {}_cx_1 I = \frac{I}{2\pi f C_1} = 33.2 \text{ volts} = E_{bd}.$$
$${}_cE_2 = {}_cx_2 I = 106.1 \text{ volts} = E_{ef}.$$
$${}_rE = {}_rE_1 + {}_rE_2 = 115.0 \text{ volts}.$$
$${}_cE = {}_cE_1 + {}_cE_2 = 139.3 \text{ volts}.$$
$$\dot{E}_1 = I(r_1 - j\,{}_cx_1) = {}_rE_1 - j\,{}_cE_1 = 80 - j33.2 \text{ volts} = E_{ad}.$$
$$\dot{E}_2 = I(r_2 - j\,{}_cx_2) = {}_rE_2 - j\,{}_cE_2 = 35 - j106.1 \text{ volts} = E_{df}.$$
$$\dot{E}_0 = \dot{I}[(r_1 + r_2) - j({}_cx_1 + {}_cx_2)] = 115 - j139.3 \text{ volts} = E_{af}.$$
$$E_0 = \sqrt{{}_rE^2 + {}_cE^2} = 180.6 \text{ volts} = E_{af}.$$
$$\theta_1 = \tan^{-1}\frac{{}_cx_1}{r_1} = 22°\ 33'.$$
$$\theta_2 = \tan^{-1}\frac{{}_cx_2}{r_2} = 71°\ 45'.$$
$$\theta_0 = \tan^{-1}\frac{{}_cx_1 + {}_cx_2}{r_1 + r_2} = 50°\ 26'.$$

Problem 8.7.—Given the same circuit and constants as in problem 7.7 except $f = 25$ cycles.

Find: ${}_rE_1$, ${}_rE_2$, ${}_LE_1$, ${}_cE_2$, ${}_rE$, ${}_cE$, $\dot{E}_1$, $\dot{E}_2$, $\dot{E}_0$, E_0, θ_1, θ_2 and θ_0.
Draw the corresponding vector diagram.

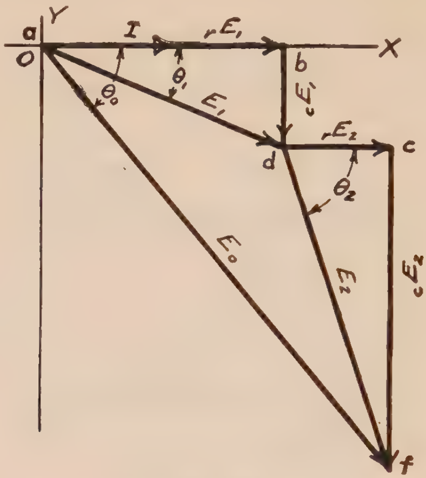


FIG. 8.7.

c. **Resistance, Inductance and Condensance.**

Problem 9.7.—Circuit diagram, Fig. 9.7. Vector diagram, Fig. 10.7.

Given: $r = 4.2$ ohms; $L = 0.03$ henry; $C = 450$ mf.; $I = 10$ amp.; $f = 50$ cycles.

Find: ${}_rE$, ${}_LE$, ${}_cE$, $\dot{E}_0$, E_0 and θ_0.

$${}_rE = rI = 42.0 \text{ volts.}$$
$${}_LE = 2\pi fLI = {}_Lx I = 94.2 \text{ volts.}$$
$${}_cE = \frac{I}{2\pi fC} = {}_cxI = 70.7 \text{ volts.}$$
$$\dot{E}_0 = \dot{I}[r + j({}_Lx - {}_cx)] = {}_r\dot{E} + j({}_L\dot{E} - {}_c\dot{E}) = 42.0 + j23.5 \text{ volts.}$$
$$E_0 = I\sqrt{r^2 + ({}_Lx - {}_cx)^2} = 48.1 \text{ volts.}$$
$$\theta_0 = \tan^{-1}\frac{x}{r} = \tan^{-1}\frac{{}_Lx - {}_cx}{r} = 29° \; 15'.$$

Problem 10.7.—Given the same circuit and constants as in problem 9.7 except the frequency is 25 cycles per sec.

Find: ${}_rE$, ${}_LE$, ${}_cE$, $\dot{E}_0$, E_0 and θ_0. Draw vector diagram.

Problem 11.7.—Given the same circuit and constants as in problem 9.7 except $f = 40$ cycles.

Find: ${}_rE$, ${}_LE$, ${}_cE$, $\dot{E}_0$, E_0 and θ_0. Draw vector diagram.

In a series circuit all the voltage absorbed by resistance is in phase with the current. Hence the total resistance voltage is the product of the arithmetical sum of all the resistances by

the current. This will be true whether the resistance is distributed over the circuit or concentrated at one or more points.

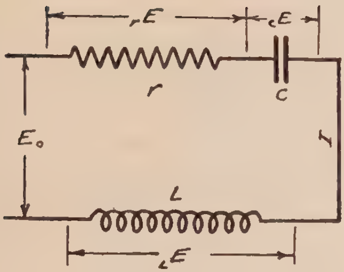


FIG. 9.7.

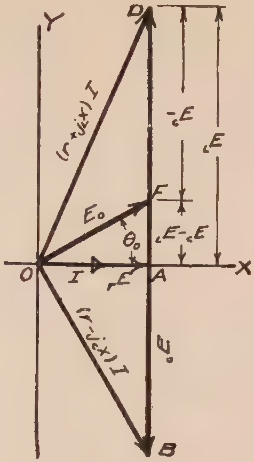


FIG. 10.7.

Since the condensive reactance is of opposite sign to the inductive reactance, the sum of all the reactances in the circuit is equal

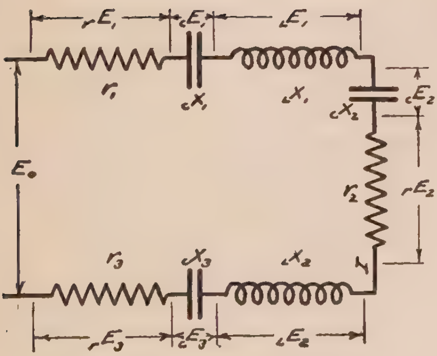


FIG. 11.7.

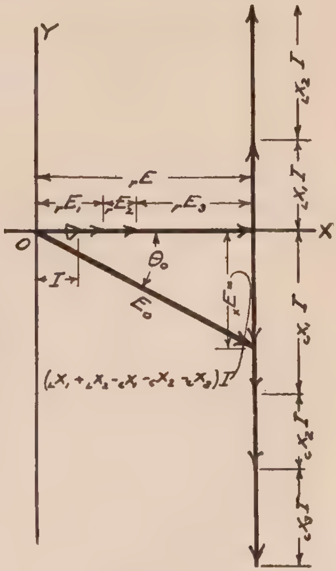


FIG. 12.7.

to the algebraic sum of the separate inductive and condensive reactances. Thus in a series circuit as shown in Fig. 11.7 the total

resistance voltage is found by multiplying the sum of all the resistances by the current. Likewise the total voltage taken in quadrature with the current is found by taking the product of the current into the algebraic sum of all the reactances.

Problem 12.7.—Circuit diagram, Fig. 11.7. Vector diagram, Fig. 12.7.

Given: $r_1 = 5.0$; ${}_Lx_1 = 6.6$; ${}_cx_1 = 12.0$; $r_2 = 2.5$; ${}_cx_2 = 5.5$; ${}_Lx_2 = 10.0$; ${}_cx_3 = 7.4$; $r_3 = 8.8$—all in ohms; $I = 10$ amp.; $f = 60$ cycles.

Find: $\dot{E}_0$, E_0 and θ_0.

$$\dot{E}_0 = \dot{I}\{(r_1 + r_2 + r_3) + j[({}_Lx_1 + {}_Lx_2) - ({}_cx_1 + {}_cx_2 + {}_cx_3)]\} = 10[16.3 + j(16.6 - 24.9)] = 163 - j83 \text{ volts.}$$

Hence the voltage in phase with the current is 163 volts and the quadrature component is 83 volts, with the current leading the voltage.

$$E_0 = \sqrt{163^2 + 83^2} = 182.7 \text{ volts.}$$

$$\theta = \tan^{-1}\frac{83}{163} = 26° 56'.$$

Problem 13.7.—Given the same circuit and the same resistance, inductance and condensance as in problem 12.7, $f = 50$ cycles.

Find: $\dot{E}_0$, E_0 and θ_0. Draw vector diagram.

Problem 14.7.—Given the same circuit and the same inductance, condensance and resistance as in problem 12.7, $f = 25$ cycles.

Find: $\dot{E}_0$, E_0 and θ_0. Draw vector diagram.

It is seen that this method of solving circuits may readily be applied to all possible combinations of resistance, inductive reactance and condensive reactance in series. The order of the various parts is of no consequence in determining the relations of the current and the total voltage. Since the current is the same throughout, it can to best advantage be selected as the reference vector. Then all the resistance will absorb voltage in phase with the current. Hence the total voltage absorbed by the several resistances in the circuit must be equal to the product of the sum of all the resistances into the current. The vector for the resistance voltage is in phase with the current vector.

Likewise the total reactance is the algebraic sum of all the reactances in the circuit. Inductive reactance is considered as positive and condensive reactance as negative. The total voltage absorbed by all the reactances is equal to the product of the algebraic sum of all the reactances in the circuit by the current. The reactance voltage is in time quadrature with the

current and hence the corresponding vector will be in quadrature with the current vector.

The total voltage consumed by both the resistance and the reactance is the square root of the sum of the squares of the resistance and the reactance voltages. These relations are expressed by equations (16.7) and (17.7). Let the sum of all the resistances in the circuit be $r_1 + r_2 + r_3 +$ etc. $= R$. Let the sum of all the inductive reactances in the circuit be ${}_Lx_1 + {}_Lx_2 + {}_Lx_3 +$ etc. $= {}_LX$; and the sum of all the condensive reactances be ${}_cx_1 + {}_cx_2 + {}_cx_3 +$ etc. $= {}_cX$.

$$\dot{E}_0 = [R + j({}_LX - {}_cX)]\dot{I}_0 \qquad (16.7)$$

$$E_0 = I_0\sqrt{R^2 + ({}_LX - {}_cX)^2} = ZI_0 \qquad (17.7)$$

By referring to the vector diagram certain simple relations between the total voltage and the voltages of the parts of the circuit may be noted.

a. If non-inductive resistances are connected in series, the sum of the several voltages is equal to the total voltage. Likewise if condensers are connected in series with short pieces of wire so as to make the resistance negligible, the sum of the several voltages is equal to the total voltage. Commerical inductive reactances have more or less resistance, but if the resistance is relatively small the voltage will be almost entirely absorbed by the inductive reactance.

b. If the *time constant*, that is, the ratio between the inductance and the resistance, is the same in two coils connected in series, then the total voltage is equal to the sum of the two voltages across the coils. By drawing a vector diagram it becomes evident that the impedance drops for the two coils lie in the same straight line.

c. When resistance, inductive reactance and condensive reactance are connected in series the sum of the separate voltages is always larger than the total voltage. Since the inductive and condensive reactances give opposing effects the total voltage may be less than the voltage consumed by either the inductive or the condensive reactances.

d. **Resonance.**—In any series circuit having a given inductance and condensance the relative value of the inductive and condensive reactance depends on the frequency of the current, ${}_Lx = 2\pi fL; {}_cx = \dfrac{1}{2\pi fC}$.

Since ${}_Lx$ is directly and ${}_cx$ inversely proportional to f, a frequency may be found at which ${}_Lx = {}_cx$ or $2\pi fL = \frac{1}{2\pi fC}$, and hence for resonance, $f = \frac{1}{2\pi\sqrt{LC}}$. (15.7)

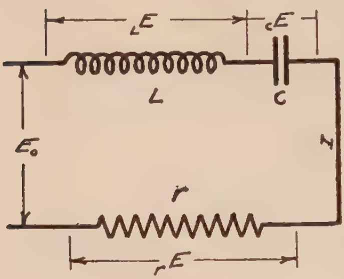


Fig. 13.7.

For the frequency at which ${}_Lx = {}_cx$ the circuit is in resonance and the resistance alone opposes the flow of current. Hence under resonance conditions $E = rI$, since $x = {}_Lx - {}_cx = 0$.

It should be kept clearly in mind that a circuit must contain both inductance and condensance in order to give resonance.

Problem 15.7.—Circuit diagram, Fig. 13.7.
Let

$r = 0.5$ ohm; $L = 0.05$ henry; $C = 400$ mf.; $E_0 = 100$ volts.

(*a*) Find the frequency for resonance.

(*b*) Under resonance conditions, find I, ${}_LE$, ${}_cE$, ${}_rE$, and draw vector diagram.

(*c*) Plot curves using frequencies from 0 to 100 as abscissæ and the corresponding values of ${}_Lx$, ${}_cx$, x, z, I, ${}_LE$, ${}_cE$ and ${}_rE$ as ordinates.

II. PARALLEL CIRCUITS

The constants of parallel circuits are often given in terms of resistance, inductance and condensance, or resistance, inductive and condensive reactance and impedance of each of the parallel circuits. In order to find the total current, the relative phase positions as well as the magnitudes must be taken into consideration. By converting the constants given in ohms to the corresponding values in mhos, that is, into conductance and susceptance, the components of the current in phase with the voltage and the part in quadrature may be separated. The total current can then be found in the same manner as the total voltage is found in series circuits.

a. Resistance and Inductance.

Problem 16.7.—Circuit diagram, Fig. 14.7. Vector diagram, Fig. 15.7.

Given: $g_1 = 0.22$; $b_1 = 0.00$; $g_2 = 0.00$; and ${}_Lb_2 = 0.12$ mho; $E_L = 100$ volts.

Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_0$, I_0, θ_0.

$$\dot{I}_1 = g_1\dot{E} - jb_1\dot{E} = 0.22\dot{E} = 22.0 \text{ amp.}$$
$$\dot{I}_2 = g_2\dot{E} - j\,{}_Lb_2\dot{E} = -j0.12\dot{E} = -j12.0 \text{ amp.}$$
$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = 22.0 - j12.0 \text{ amp.}$$
$$I_0 = \sqrt{I_1^2 + I_2^2} = 25.0 \text{ amp.}$$
$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = -28°\ 35'.$$

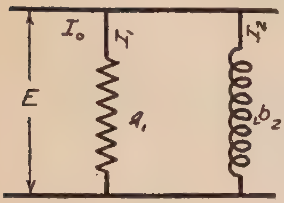


Fig. 14.7.

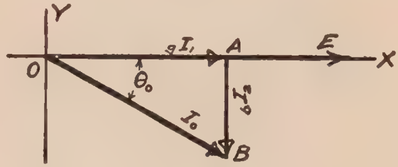


Fig. 15.7.

Problem 17.7.—Circuit diagram, Fig. 16.7. Vector diagram, Fig. 17.7.

$$r_1 = 6.4 \text{ ohms } {}_Lx_1 = 4.8 \text{ ohms.}$$
$$r_2 = 2.4 \text{ ohms, } {}_Lx_2 = 8.0 \text{ ohms.}$$
$$E = 100 \text{ volts, } f = 60 \text{ cycles.}$$

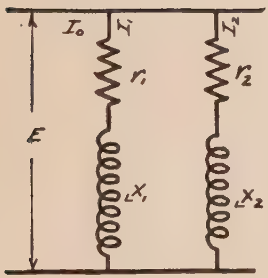


Fig. 16.7.

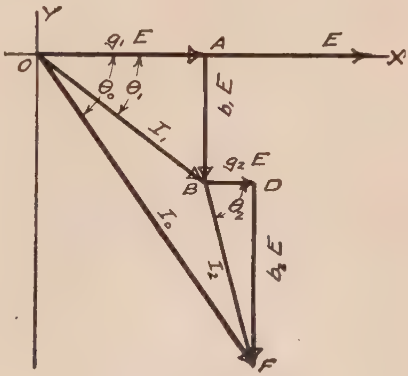


Fig. 17.7.

Find: g_1, g_2, g_0, ${}_Lb_1$, ${}_Lb_2$, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_a\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_0.

$$g_1 = \frac{r_1}{r_1^2 + {}_Lx_1^2} = 0.10 \text{ mho.}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_Lx_2^2} = 0.0344 \text{ mho.}$$

$$g_0 = g_1 + g_2 = 0.1344 \text{ mho.}$$
$$_Lb_1 = \frac{_Lx_1}{r_1^2 + _Lx_1^2} = 0.0750 \text{ mho.}$$
$$_Lb_2 = \frac{_Lx_2}{r_2^2 + _Lx_2^2} = 0.1148 \text{ mho.}$$
$$b_0 = _Lb_1 + _Lb_2 = 0.1898 \text{ mho.}$$
$$y_0 = g_0 - jb_0 = 0.1344 - j0.1898 \text{ mho.}$$
$$\dot{I}_1 = (g_1 - j\,_Lb_1)\dot{E} = (0.10 - j0.075)\dot{E} \text{ amp.}$$
$$\dot{I}_2 = (g_2 - j\,_Lb_2)\dot{E} = (0.0344 - j0.1148)\dot{E} \text{ amp.}$$
$$_g\dot{I} = g_0\dot{E} = 13.44 \text{ amp.}$$
$$_b\dot{I} = b_0\dot{E} = 18.98 \text{ amp.}$$
$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = (g_0 - jb_0)\dot{E} = (0.1344 - j0.1898)\dot{E} \text{ amp.}$$
$$I_0 = \sqrt{_gI^2 + _bI^2} = E\sqrt{g_0^2 + b_0^2} = 23.25 \text{ amp.}$$
$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = -54° \, 40'.$$

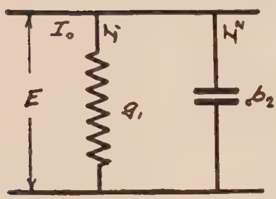


FIG. 18.7.

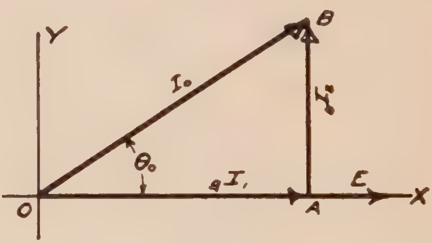


FIG. 19.7.

Problem 18.7.—Given the same circuit as in problem 17.7 and let $E = 100$ volts and $f = 25$ cycles.

Find: g_0, b_0, y_0, $_g\dot{I}$, $_b\dot{I}$, $\dot{I}_0$, I_0 and θ_0. Draw the vector diagram.

b. Resistance and Condensance.

Problem 19.7.—Circuit diagram, Fig. 18.7. Vector diagram, Fig. 19.7.

Given: $g_1 = 0.32$; $_cb_1 = 0.00$; $g_2 = 0.00$; $_cb_2 = 0.21$ mho; $E = 100$ volts.

Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_0$, I_0 and θ_0.

$$\dot{I}_1 = (g_1 + j\,_cb_1)\dot{E} = 0.32\dot{E} \text{ amp.}$$
$$\dot{I}_2 = (g_2 + j\,_cb_2)\dot{E} = j0.21\dot{E} \text{ amp.}$$
$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = (0.32 + j0.21)\dot{E} \text{ amp.}$$
$$I_0 = E\sqrt{0.32^2 + 0.21^2} = 38.25 \text{ amp.}$$
$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 33° \, 17'.$$

Problem 20.7.—Circuit diagram, Fig. 20.7. Vector diagram, Fig. 21.7.

Given: $r_1 = 9.0$; $r_2 = 12.0$; ${}_c x_1 = 15.0$; ${}_c x_2 = 10.0$ ohms; $E = 100$ volts; $f = 25$ cycles.

Find: g_1, g_2, g_0, ${}_c b_1$, ${}_c b_2$, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2 and θ_0.

$$g_1 = \frac{r_1}{r_1^2 + {}_c x_1^2} = 0.0294 \text{ mho.}$$

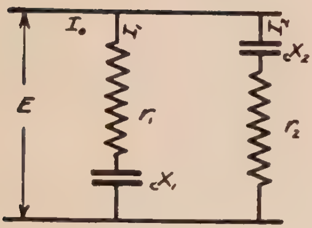


FIG. 20.7.

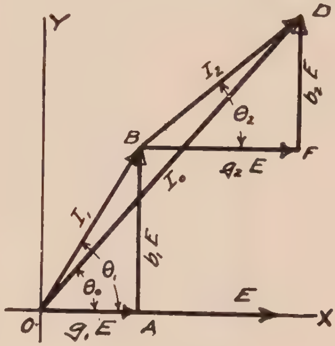


FIG. 21.7.

$$ {}_c b_1 = \frac{{}_c x_1}{r_1^2 + {}_c x_1^2} = 0.0490 \text{ mho.}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_c x_2^2} = 0.0492 \text{ mho.}$$

$$ {}_c b_2 = \frac{{}_c x_2}{r_2^2 + {}_c x_2^2} = 0.0410 \text{ mho.}$$

$$g_0 = g_1 + g_2 = 0.0786 \text{ mho.}$$

$$b_0 = {}_c b_1 + {}_c b_2 = 0.0900 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.0786 + j0.0900 \text{ mho.}$$

$$\dot{I}_1 = (g_1 + j\,{}_c b_1)\dot{E} = (0.0294 + j0.0490)\dot{E} \text{ amp.}$$

$$\dot{I}_2 = (g_2 + j\,{}_c b_2)\dot{E} = (0.0492 + j0.0410)\dot{E} \text{ amp.}$$

$$ {}_g\dot{I} = g_0\dot{E} = 0.0786\dot{E} \text{ amp.}$$

$$ {}_b\dot{I} = jb_0\dot{E} = j0.0900\dot{E} \text{ amp.}$$

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = [0.0786 + j0.0900]\dot{E} \text{ amp.}$$

$$I_0 = \sqrt{{}_gI^2 + {}_bI^2} = E\sqrt{g_0^2 + b_0^2} = 11.95 \text{ amp.}$$

$$\theta_1 = \tan^{-1}\frac{{}_c b_1}{g_1} = 59^\circ\ 1'.$$

$$\theta_2 = \tan^{-1}\frac{{}_c b_2}{g_2} = 39^\circ\ 52'.$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 48^\circ\ 52'.$$

Problem 21.7.—Given the same circuit as in problem 20.7 with $E = 100$ volts and $f = 60$ cycles.

Find: g_1, g_2, g_0, ${}_cb_1$, ${}_cb_2$, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2, θ_0. Draw the vector diagram.

c. **Resistance, Inductance and Condensance.**

Problem 22.7.—Circuit diagram, Fig. 22.7. Vector diagram, Fig. 23.7.

Given: $r_1 = 3.4$; $r_2 = 8.5$; ${}_cx_1 = 6.6$; ${}_Lx_2 = 4.1$ ohms; $E = 100$ volts; $f = 50$ cycles.

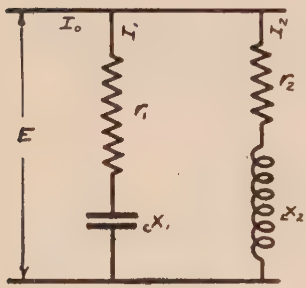


FIG. 22.7.

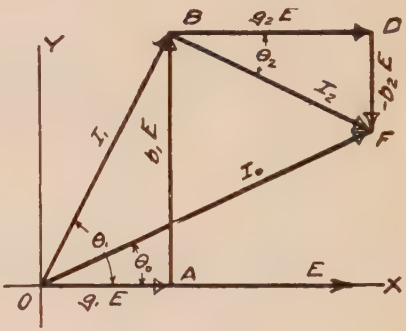


FIG. 23.7.

Find: g_0, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2 and θ_0.

$$g_0 = g_1 + g_2 = \frac{r_1}{r_1^2 + {}_cx_1^2} + \frac{r_2}{r_2^2 + {}_Lx_2^2} = 0.062 + 0.096 = 0.158 \text{ mho.}$$

$$b_0 = {}_cb_1 - {}_Lb_2 = \frac{{}_cx_1}{r_1^2 + {}_cx_1^2} - \frac{{}_Lx_2}{r_2^2 + {}_Lx_2^2}$$
$$= 0.120 - 0.046 = 0.074 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.158 + j0.074 \text{ mho.}$$
$$\dot{I}_1 = (g_1 + j{}_cb_1)\dot{E} = (0.062 + j0.120)\dot{E} \text{ amp.}$$
$$\dot{I}_2 = (g_2 - j{}_Lb_2)\dot{E} = (0.096 - j0.046)\dot{E} \text{ amp.}$$
$${}_g\dot{I} = g_0\dot{E} = 0.158\dot{E} = 15.8 \text{ amp.}$$
$${}_b\dot{I} = jb_0\dot{E} = j0.074\dot{E} = j7.4 \text{ amp.}$$
$$\dot{I}_0 = (g_0 + jb_0)\dot{E} = (0.158 + j0.074)\dot{E} \text{ amp.}$$
$$I_0 = \sqrt{{}_gI^2 + {}_bI^2} = \sqrt{g_0^2 + b_0^2}E = 17.5 \text{ amp.}$$
$$\theta_1 = \tan^{-1}\frac{{}_cb_1}{g_1} = 62° \, 40'; \; \theta_2 = \tan^{-1}\frac{{}_Lb_2}{g_2} = -25° \, 39';$$
$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 25° \, 8'.$$

Problem 23.7.—Given the same circuit as in problem 22.7, with $E = 100$ volts and $f = 40$ cycles.

Find: g_0, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_b\dot{I}$, ${}_g\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2 and θ_0. Draw the vector diagram.

Problem 24.7.—Circuit diagram, Fig. 22.7. Vector diagram, similar to Fig. 23.7.

Given: $r_1 = 0.1$; $r_2 = 0.3$; ${}_cx_1 = 8.4$; ${}_Lx_2 = 5.4$ ohms; $E = 100$ volts; $f = 60$ cycles.

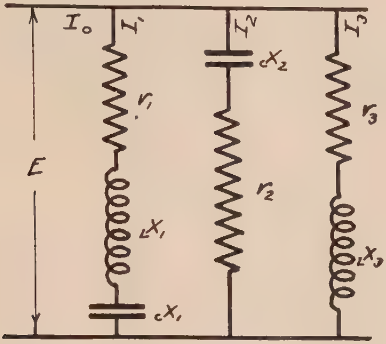


FIG. 24.7.

Find: g_0, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2, θ_0.

$$g_0 = g_1 + g_2 = \frac{r_1}{r_1^2 + {}_cx_1^2} + \frac{r_2}{r_2^2 + {}_Lx_2^2} = 0.0014 + 0.0102 = 0.0116 \text{ mho.}$$

$$b_0 = {}_cb_1 - {}_Lb_2 = \frac{{}_cx_1}{r_1^2 + {}_cx_1^2} - \frac{{}_Lx_2}{r_2^2 + {}_Lx_2^2} = 0.1190 - 0.1844 = -0.0654 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.0116 - j0.0654 \text{ mho.}$$

$$\dot{I}_1 = (g_1 + j\,{}_cb_1)\dot{E} = (0.0014 + j0.1190)\dot{E} \text{ amp.}$$

$$\dot{I}_2 = (g_2 - j\,{}_Lb_2)\dot{E} = (0.0102 - j0.1844)\dot{E} \text{ amp.}$$

$${}_g\dot{I} = g_0\dot{E} = 0.0116\dot{E} = 1.16 \text{ amp.}$$

$${}_b\dot{I} = jb_0\dot{E} = -j0.0654\dot{E} = j6.54 \text{ amp.}$$

$$\dot{I}_0 = (g_0 + jb_0)\dot{E} = (0.0116 - j0.654)\dot{E} \text{ amp.}$$

$$I_0 = \sqrt{{}_gI^2 + {}_bI^2} = \sqrt{g_0^2 + b_0^2}E = 6.64 \text{ amp.}$$

$$\theta_1 = \tan^{-1}\frac{{}_cb_1}{g_1} = 89°\ 19'; \quad \theta_2 = \tan^{-1}\frac{{}_Lb_2}{g_2} = -86°\ 35'.$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = -79°\ 45'.$$

Problem 25.7.—Given the same circuit as in problem 24.7 with $E = 100$ volts and $f = 25$ cycles.

Find: g_0, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2, θ_0. Draw vector diagram.

Problem 26.7.—Circuit diagram, Fig. 24.7. Vector diagram, Fig. 25.7.

Given: $r_1 = 1.5$; $r_2 = 5.6$; $r_3 = 7.0$; ${}_Lx_1 = 3.4$; ${}_cx_1 = 6.5$; ${}_cx_2 = 4.7$; ${}_Lx_3 = 8.2$ ohms; $E = 100$ volts; $f = 60$ cycles.

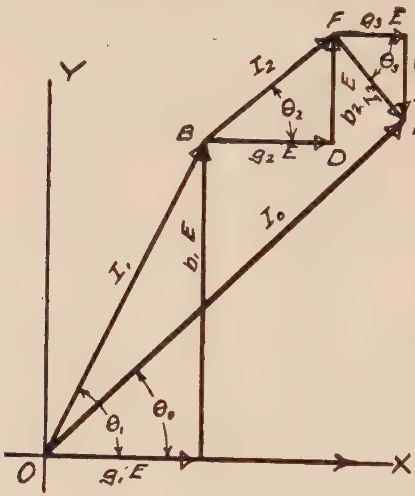


FIG. 25.7.

Find: g_0, b_0, y_0, $\dot{I}_0$, I_0, θ_0.

$$g_0 = g_1 + g_2 + g_3 = \frac{r_1}{r_1^2 + ({}_Lx_1 - {}_cx_1)^2} + \frac{r_2}{r_2^2 + {}_cx_2^2} + \frac{r_3}{r_3^2 + {}_Lx_3^2}$$

$$= 0.1264 + 0.1045 + 0.0601 = 0.2910 \text{ mho.}$$

$$b_0 = b_1 + b_2 + b_3 = \frac{-({}_Lx_1 - {}_cx_1)}{r_1^2 + ({}_Lx_1 - {}_cx_1)^2} + \frac{{}_cx_2}{r_2^2 + {}_cx_2^2} - \frac{{}_Lx_3}{r_3^2 + {}_Lx_3^2}$$

$$= 0.260 + 0.088 - 0.0705 = 0.2775 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.2910 + j0.2775 \text{ mho.}$$

$$\dot{I}_0 = y_0\dot{E} = (0.2910 + j0.2775)\dot{E} \text{ amp.}$$

$$I_0 = Y_0E = \sqrt{g_0^2 + b_0^2}E = 40.2 \text{ amp.}$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 43° \, 38'.$$

Problem 27.7.—In the same circuit as in problem 26.7, let the applied voltage be 100 volts and the frequency 25 cycles.

Find: g_0, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_0, θ_1, θ_2, θ_3. Draw the vector diagram.

d. **Resonance.**[1]—In series circuits, Fig. 13.7, under resonance conditions the voltage across either the inductance or the condensance may be greater than the total voltage impressed on the circuit. Similarly in parallel circuits the current in either circuit as I_1 or I_2 in Fig. 22.7, may be greater than their resultant I_0. To produce a marked difference, r_1 and r_2 must be small in comparison with ${}_Lx_1$ and ${}_cx_2$, and hence g_1 and g_2 must be small in comparison with ${}_Lb_1$ and ${}_cb_2$. The system is in resonance at the frequency for which the total admittance, $y_1 + y_2$, is a minimum. For values of r_1 and r_2 for which ${}_Lg_1$ and ${}_cg_2$ are negligibly small in comparison with ${}_Lb_1$ and ${}_cb_2$ resonance exists when ${}_Lb_1 + {}_cb_2 = 0$ or for

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{18.7}$$

the same frequency as for the corresponding series circuit.

Problem 28.7.—Find the frequency for resonance in a circuit similar to Fig. 22.7 with $r_1 = r_2 = 0.1$ ohm; ${}_Lx_2 = 17.2$ ohm; ${}_cx_1 = 12.7$ ohm; $f = 60$ cycles. Under resonance conditions find: g_0, b_0, y_0, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, I_0, θ_1, θ_2 and θ_0. Draw vector diagram.

Problem 29.7.—For the circuit used in problem 28.7, plot curves in rectangular coördinates with frequencies from 20 to 80 as abscissae and the corresponding values for I_0, I_1 and I_2 as ordinates.

e. **Graphic Method for Parallel Circuits.**

Problem 30.7.—Given the same circuit as in problem 20.7.

The symbolic method is often tedious and the solution may be gained more easily graphically. Thus in problem 20.7, let the voltage be represented by the line OX in Fig. 26.7. From O draw the line OA at an angle of θ_1°, above OX. $\theta_1 = \tan^{-1}\frac{{}_cx_1}{r_1} = 59° \; 1'$. Then OA is equal to r_1I_1. XA is equal to ${}_cx_1I_1$, and θ_1 is the angle of lead. The current in this branch, I_1, is equal to $\frac{OA}{r_1} = \frac{AX}{X_1} = 5.7 = OA'$, and laid off from O along OA. The current in the second branch, $\dot{I}_2$, is found in a similar manner. From O lay off the line OB at an angle θ_2°, above OX. $\theta_2 = \tan^{-1}\frac{{}_cx_2}{r_2} = 39° \; 52'$. The current I_2 is equal to $\frac{OB}{r_2} = 6.2 = OB'$ and laid off along OB.

[1] MAGNUSSON, C. E., "Electric Transients," Chap. VIII, McGraw-Hill Book Company, Inc.

The total current I_0 is the vector sum of I_1 and I_2 and is represented in the diagram by the line OD'. The voltage consumed by the equivalent resistance of the two circuits in parallel is therefore represented by the line OD and the voltage taken by the equivalent reactance of the two parallel circuits by the line XD.

The angle XOD is the angle of lead for the total current and $= \theta_0 = 48° 52'$.

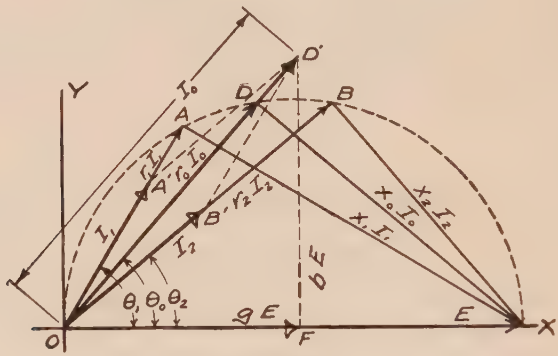


FIG. 26.7.

By dropping a perpendicular line from D' to OX, the current I_0 is separated into its components in parallel and in quadrature with the voltage. Thus OF is equal to ${}_gI_0$, and FD' to ${}_bI_0$.

Problem 31.7.—Solve problem 21.7 graphically.
Problem 32.7.—Solve problems 22.7, 23.7, 26.7 and 27.7 graphically.
Problem 33.7.—Find the frequency for resonance in problems 9.7, 12.7, 22.7 and 24.7.

III. SERIES AND PARALLEL CIRCUITS

By the symbolic method as explained for series and parallel circuits any network of circuits can be solved. Whenever two or more circuits are in parallel the constants are expressed in terms of conductance and susceptance. By adding all the conductances the total conductance is obtained; and the algebraic sum of all the susceptances gives the total susceptance. Thus equivalent conductance, susceptance and admittance of the parallel parts may be obtained. In order to combine this equivalent circuit with quantities in series, the constants must be trans-

formed into equivalent resistance and reactance. An example will make the method clear.

Problem 34.7.—Given the circuit diagram shown in Fig. 27.7, beginning at the right-hand end, that is, farthest away from the generator, there are two circuits in parallel, up to the dotted line AA', with the constants:

$$r_1 = 3.0; \ {}_Lx_1 = 4.0; \ r_2 = 6.2 \text{ and } {}_cx_2 = 3.3 \text{ ohms.}$$

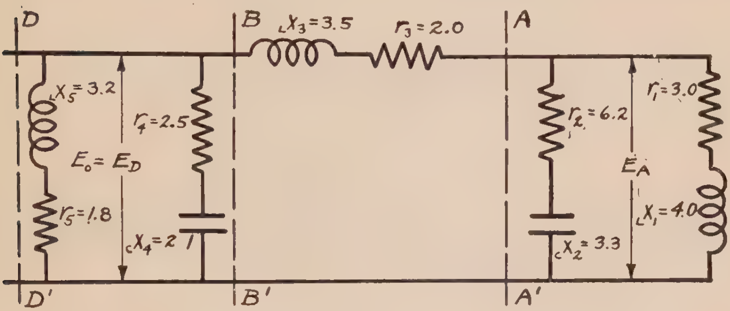


FIG. 27.7.

To combine parallel circuits the constants must be expressed in terms of conductance and susceptance.

$$g_1 = \frac{r_1}{r_1^2 + {}_Lx_1^2} = 0.120 \text{ mho.}$$

$$ {}_Lb_1 = \frac{{}_Lx_1}{r_1^2 + {}_Lx_1^2} = 0.160 \text{ mho.}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_cx_2^2} = 0.125 \text{ mho.}$$

$$ {}_cb_2 = \frac{{}_cx_2}{r_2^2 + {}_cx_2^2} = 0.067 \text{ mho.}$$

$$g_A = g_1 + g_2 = 0.245 \text{ mho.}$$

$$b_A = {}_cb_2 - {}_Lb_1 = 0.093 \text{ mho.}$$

$$y_A = g_A - jb_A = 0.245 - j0.093 \text{ mho.}$$

The equivalent resistance, AA':

$$r_A = \frac{g_A}{g_A^2 + b_A^2} = 3.58 \text{ ohms.}$$

The equivalent reactance:

$$x_A = \frac{b_A}{g_A^2 + b_A^2} = 1.35 \text{ ohms.}$$

The equivalent impedance:

$$z_A = \frac{1}{g_A - jb_A} = \frac{g_A}{g_A^2 + b_A^2} + j\frac{b_A}{g_A^2 + b_A^2} = r_A + jx_A$$
$$= 3.58 + j1.35 \text{ ohms.}$$

Having derived expressions for the equivalent resistance and reactance of the two circuits on the right of AA', this can now be combined with the part between AA' and BB'. Let the value of the constants for this part be:

$$r_3 = 2.0 \text{ and } {}_Lx_3 = 3.5 \text{ ohms.}$$

Therefore at the dotted line BB' the combined resistance and reactance for circuits 1, 2 and 3 are

$$r_B = r_A + r_3 = 5.58 \text{ ohms.}$$
$$x_B = x_A + {}_Lx_3 = 4.85 \text{ ohms.}$$

The next step will be the combination of the two parallel circuits 4 and 5 between BB' and DD' with the equivalent circuit just obtained from circuits 1, 2 and 3.

$${}_cx_4 = 2.1 \text{ ohms.}$$
$$r_4 = 2.5 \text{ ohms.}$$
$$r_5 = 1.8 \text{ ohms.}$$
$${}_Lx_5 = 3.2 \text{ ohms.}$$
$$z_B = r_B + jx_B = 5.58 + j4.85 \text{ ohms.}$$
$$z_4 = r_4 - j\,{}_cx_4 = 2.5 - j2.1 \text{ ohms.}$$
$$z_5 = r_5 + j\,{}_Lx_5 = 1.8 + j3.2 \text{ ohms.}$$

This will give the desired solution for both magnitude and phase relations of the voltage and current at DD'. In order to combine these parallel circuits, the resistance and reactance of each circuit must be changed into conductance and susceptance.

$$g_B = \frac{r_B}{r_B^2 + x_B^2} = 0.1018 \text{ mho.}$$

$$b_B = \frac{x_B}{r_B^2 + x_B^2} = 0.0884 \text{ mho.}$$

$$g_4 = \frac{r_4}{r_4^2 + {}_cx_4^2} = 0.2345 \text{ mho.}$$

$$b_4 = \frac{{}_cx_4}{r_4^2 + {}_cx_4^2} = 0.1970 \text{ mho.}$$

$$g_5 = \frac{r_5}{r_5^2 + {}_Lx_5^2} = 0.1335 \text{ mho.}$$

$$b_5 = \frac{{}_Lx_5}{r_5^2 + {}_Lx_5^2} = 0.2377 \text{ mho.}$$

The total conductance at DD' is the sum of the conductances in the three parallel circuits.

$$g_D = g_B + g_4 + g_5 = 0.1018 + 0.2345 + 0.1335 = 0.470 \text{ mho.}$$

The total susceptance is equal to the algebraic sum of the susceptances in the three parallel circuits.

$$b_D = b_B - b_4 + b_5 = 0.088 - 0.197 + 0.238 = 0.129 \text{ mho.}$$

Hence the total admittance of the circuit at DD':

$$y_D = g_D - jb_D = 0.470 - j0.129 \text{ mho.}$$

$$Y_D = \sqrt{g_D^2 + b_D^2} = 0.488 \text{ mho.}$$

$$\theta_D = \tan^{-1}\frac{b_D}{g_D} = 15^\circ\ 20'.$$

Having determined the equivalent conductance and susceptance for the parallel circuits and the resistance and reactance for the series circuits, the value of the currents in any part of the circuit may be found for any assumed voltage; or conversely, if the current is measured in any part of the circuit, voltage and current may be calculated for any part of the network.

The process outlined above and as illustrated by the solution of problem 34.7 is somewhat tedious. For many circuit networks solutions may be gained more expeditiously by using the symbolic expressions for impedances or admittances throughout, instead of converting to or from impedances in series circuits, from or to admittances for the parallel sections. That is, the solution of alternating-current networks can be made similar to that of the corresponding direct-current circuits by merely using impedance in place of resistance or admittance in place of conductance in the application of Ohm's and Kirchhoff's laws.

For direct currents the total resistance R of two parallel circuits having resistances of R_1 and R_2, respectively, is given by equation (19.7).

$$R = \frac{R_1R_2}{R_1 + R_2}. \tag{19.7}$$

Similarly, the relation of the total impedance z for two parallel circuits, as in Fig. 16.7, is expressed by equations (20.7) and (21.7):

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}. \tag{20.7}$$

$$z = \frac{z_1z_2}{z_1 + z_2}. \tag{21.7}$$

The same process can be applied to parallel circuits using admittance y for alternating-current circuits in place of conductance for direct currents. The laws for combining impedances and admittances (expressed in complex notation) in alternating-current networks are analogous in form to those for resistances and conductances in direct currents.

The process or method can be shown to best advantage by the solution of specific problems and is illustrated in problems 35.7 to 38.7, inclusive.

Problem 35.7.—Given the series-parallel circuit shown in Fig. 28.7 with quantitative values of the circuit constants as indicated in the diagram.

Let

$$\dot{E}_1 = 100 \text{ volts}; f = 60 \text{ cycles.}$$

(a) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{I}_4$, $\dot{I}_5$, $\dot{I}_0$, $\dot{E}_2$ and $\dot{E}_0$.

(b) Draw vector diagram.

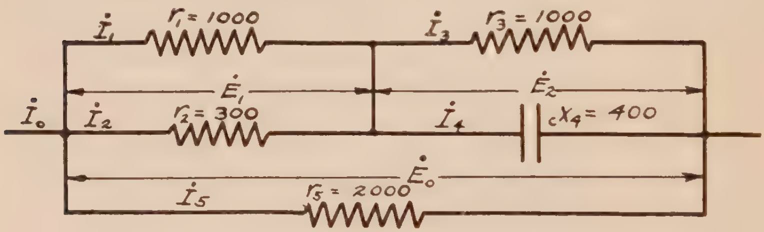


FIG. 28.7.—Circuit diagram for problem 35.7.

$$I_1 = {}^{100}\!/_{1,000} = 0.1 \text{ amp.}$$

$$I_2 = {}^{100}\!/_{300} = 0.333 \text{ amp.}$$

$$\frac{1}{z_{3,4}} = \frac{1}{z_3} + \frac{1}{z_4} = \frac{z_3 + z_4}{z_3 z_4} \therefore z_{3,4} = 137.8 - j345 \text{ ohms.}$$

$$\dot{I}_1 + \dot{I}_2 = \dot{I}_3 + \dot{I}_4.$$

$$\dot{E}_2 = z_{3,4}(\dot{I}_3 + \dot{I}_4) = (137.8 - j345)(0.433) = 59.75 - j149.5 \text{ volts.}$$

$$\dot{E}_0 = \dot{E}_1 + \dot{E}_2 = 159.75 - j149.5 \text{ volts.}$$

$$\dot{I}_3 = \frac{\dot{E}_2}{z_2} = \frac{59.75 - j149.5}{1,000} = 0.0597 - j0.1495 \text{ amp.}$$

$$\dot{I}_4 = \frac{\dot{E}_2}{z_4} = \frac{59.75 - j149.5}{-j400} = 0.3740 + j0.1495 \text{ amp.}$$

$$\dot{I}_5 = \frac{\dot{E}_0}{z_5} = \frac{159.75 - j149.5}{2,000} = 0.0797 - j0.0747 \text{ amp.}$$

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 + \dot{I}_5 = \dot{I}_3 + \dot{I}_4 + \dot{I}_5 = 0.5127 - j0.0747 \text{ amp.}$$

The corresponding vector diagram is shown in Fig. 29.7.

Problem 36.7.—Given the circuit and quantitative data shown in Fig. 30.7.

Let

$$\dot{E}_0 = 100 \text{ volts}; f = 60 \text{ cycles.}$$

(a) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{E}_1$ and $\dot{E}_2$.
(b) Draw vector diagram.

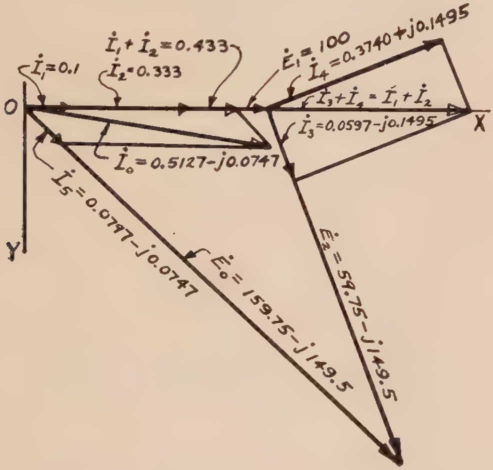


FIG. 29.7.—Vector diagram for problem 35.7.

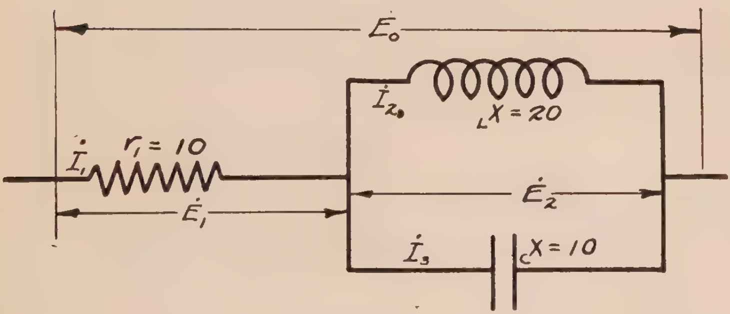


FIG. 30.7.—Circuit diagram for problem 36.7.

$$y_2 = \frac{1}{z_2} = \frac{1}{j20} = \frac{-j20}{400} = -j0.05 \text{ mhos.}$$

$$y_3 = \frac{1}{z_3} = \frac{1}{-j10} = \frac{j10}{100} = j0.1 \text{ mhos.}$$

$$y_{2,3} = y_2 + y_3 = j0.05 \text{ mhos.}$$

$$z_{2,3} = \frac{1}{y_{2,3}} = \frac{1}{j0.05} = \frac{-j0.05}{0.0025} = -j20 \text{ ohms.}$$

$$z_0 = z_1 + z_{2,3} = 10 - j20 \text{ ohms.}$$

$$\dot{I}_0 = \dot{I}_1 = \frac{\dot{E}_0}{z_0} = \frac{100}{10 - j20} = \frac{1{,}000 + j2{,}000}{500} = 2 + j4 \text{ amp.}$$

$$\dot{E}_1 = z_1\dot{I}_1 = 10(2 + j4) = 20 + j40 \text{ volts.}$$

$$\dot{E}_2 = \dot{E}_0 - \dot{E}_1 = 100 - (20 + j40) = 80 - j40 \text{ volts.}$$

$$\dot{I}_2 = y_2\dot{E}_2 = -j0.05(80 - j40) = -2 - j4 \text{ amp.}$$

$$\dot{I}_3 = y_3\dot{E}_2 = \frac{\dot{E}_2}{z_3} = j0.1(80 - j40) = 4 + j8 \text{ amp.}$$

As check $\dot{I}_2 + \dot{I}_3 = (-2 - j4) + (4 + j8) = 2 + j4 = \dot{I}_1 = \dot{I}_0.$

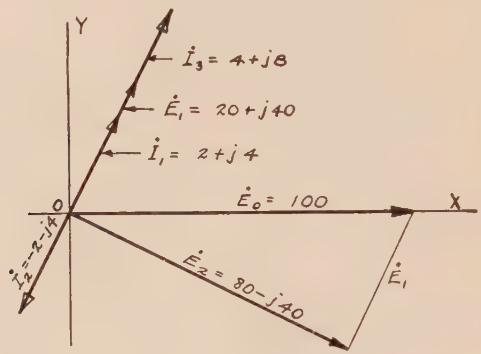


FIG. 31.7.—Vector diagram for problem 36.7.

Problem 37.7.—Given the circuit and corresponding data as shown in Fig. 32.7.

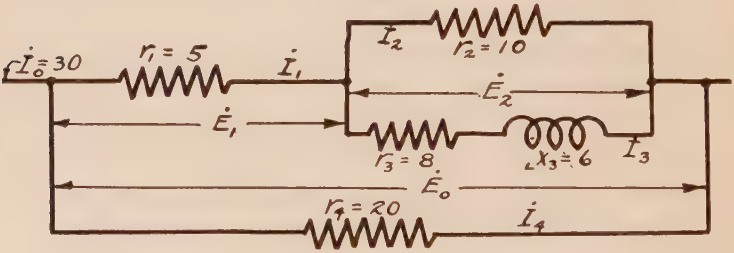


FIG. 32.7.—Circuit diagram for problem 37.7.

Let

$$\dot{I}_0 = 30 \text{ amp.};\ f = 60 \text{ cycles.}$$

(*a*) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{I}_4$, $\dot{E}_1$, $\dot{E}_2$, and $\dot{E}_0$.

In this case it is necessary to find the equivalent impedance or admittance of the circuit.

$$y_2 = \frac{1}{z_2} = \frac{1}{10} = 0.1 \text{ mho.}$$

$$y_3 = \frac{1}{z_3} = \frac{1}{8 + j6} = \frac{8 - j6}{100} = 0.08 - j0.06 \text{ mhos.}$$

$$y_{2,3} = 0.18 - j0.06 \text{ mhos.}$$

$$z_{2,3} = \frac{1}{0.18 - j0.06} = \frac{0.18 + j0.06}{0.036} = 5 + j1.67 \text{ ohms.}$$

$$z_{1,2,3} = z_1 + z_{2,3} = 5 + (5 + j1.67) = 10 + j1.67 \text{ ohms.}$$

$$y_{1,2,3} = \frac{1}{z_{1,2,3}} = \frac{1}{10 + j1.67}$$

$$= \frac{10 - j1.67}{101.5} = 0.0984 - j0.01643 \text{ mhos.}$$

$$y_4 = \frac{1}{z_4} = \frac{1}{20} = 0.05 \text{ mho.}$$

$$y_0 = y_{1,2,3} + y_4 = 0.1484 - j0.01643 \text{ mhos.}$$

$$\dot{E}_0 = \frac{\dot{I}_0}{y_0} = \frac{30}{0.1484 - j0.01643}$$

$$= \frac{4.45 + j0.493}{0.02237} = 198.9 + j22.05 \text{ volts.}$$

$$\dot{I}_4 = y_4\dot{E}_0 = 0.05(198.9 + j22.05) = 9.945 + j1.102 \text{ amp.}$$

$$\dot{I}_1 = \dot{I}_0 - \dot{I}_4 = 30 - (9.945 + j1.102) = 20.055 - j1.102 \text{ amp.}$$

$$\dot{E}_1 = z_1\dot{I}_1 = 5(20.055 - j1.102) = 100.275 - j5.510 \text{ volts.}$$

$$\dot{E}_2 = \dot{E}_0 - \dot{E}_1 = (198.9 + j22.05) - (100.27 - j5.51)$$

$$= 98.63 + j27.56 \text{ volts.}$$

$$\dot{I}_2 = \frac{\dot{E}_2}{z_2} = \frac{98.63 + j27.56}{10} = 9.863 + j2.756 \text{ amp.}$$

$$\dot{I}_3 = y_3\dot{E}_2 = (0.08 - j0.06)(98.63 + j27.56) = 9.55 - j3.72 \text{ amp.}$$

(*b*) Draw vector diagram by plotting the vector values of the currents and voltages as expressed in the above equations.

Problem 38.7.—Given the circuit shown in Fig. 33.7.

Let

$$\dot{E}_3 = 80 + j60;\ f = 60 \text{ cycles.}$$

(*a*) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{I}_4$, $\dot{I}_5$, $\dot{E}_1$, $\dot{E}_2$ and $\dot{E}_0$.

$$\dot{I}_4 = \frac{\dot{E}_3}{z_4} = \frac{80 + j60}{10} = 8 + j6 \text{ amp.}$$

$$\dot{I}_5 = \frac{\dot{E}_3}{z_5} = \frac{80 + j60}{-j5} = -12 + j16 \text{ amp.}$$

$$\dot{I}_3 = \dot{I}_0 = \dot{I}_4 + \dot{I}_5 = -4 + j22 \text{ amp.}$$

$$y_1 = \frac{1}{6 + j8} = \frac{6 - j8}{100} = 0.06 - j0.08 \text{ mhos.}$$

$$y_2 = \frac{1}{3 + j4} = \frac{3 - j4}{25} = 0.12 - j0.16 \text{ mhos.}$$

$$y_{1,2} = 0.18 - j0.24 \text{ mhos.}$$

$$\dot{I}_1 + \dot{I}_2 = \dot{I}_3 = y_{1,2}\dot{E}_1$$

$$\therefore \dot{E}_1 = \frac{-4 + j22}{0.18 - j0.24} = \frac{-6.00 + j3.00}{0.09} = -66.67 + j33.33 \text{ volts.}$$

$$\dot{I}_1 = y_1\dot{E}_1 = (0.06 - j0.08)(-66.67 + j33.33)$$
$$= -1.33 + j7.32 \text{ amp.}$$
$$\dot{I}_2 = y_2\dot{E}_1 = (0.12 - j0.16)(-66.67 + j33.33)$$
$$= -2.67 + j14.68 \text{ amp.}$$
$$\dot{E}_2 = \dot{I}_3 z_3 = (-4 + j22)(j10) = -220 - j40 \text{ volts.}$$
$$\dot{E}_0 = \dot{E}_1 + \dot{E}_2 + \dot{E}_3$$
$$= (-66.67 + j33.33) + (-220 - j40) + (80 + j60)$$
$$= -206.67 + j53.33 \text{ volts.}$$

(*b*) Draw vector diagram by plotting the vector values of the currents and voltages as expressed in the above equations.

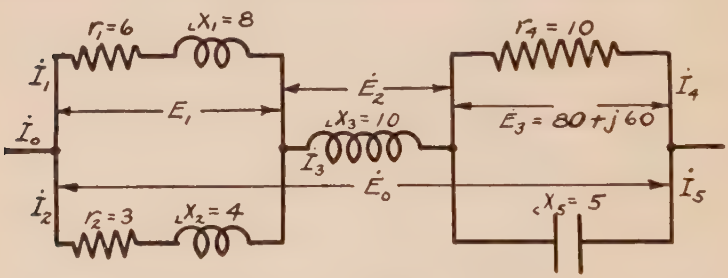


FIG. 33.7.—Circuit diagram for problem 38.7.

Problem 39.7.—Same circuit as in Problem 35.7.
Let

$E_1 = 220$ volts; $f = 25$ cycles.

(*a*) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{I}_4$, $\dot{I}_5$, $\dot{I}_0$, $\dot{E}_2$ and $\dot{E}_0$ by the same method as used in Problem 35.7.

(*b*) Draw the corresponding vector diagram.

Problem 40.7.—Same circuit as in Problem 36.7.
Let

$E_0 = 120$ volts; $f = 50$ cycles.

(*a*) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{E}_1$ and $\dot{E}_2$.

(*b*) Draw the corresponding vector diagram.

Problem 41.7.—Same circuit as in Problem 37.7.
Let

$\dot{I}_0 = 50$ amp.; $f = 30$ cycles.

(a) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{I}_4$, $\dot{E}_1$, $\dot{E}_2$ and $\dot{E}_0$.

(b) Draw the corresponding vector diagram.

Problem 42.7.—Same circuit as in Problem 38.7.
Let

$\dot{E}_3 = 60 - j80$; $f = 50$ cycles.

(a) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, $\dot{I}_4$, $\dot{I}_5$, $\dot{E}_1$, $\dot{E}_2$ and $\dot{E}_0$.

(b) Draw the corresponding vector diagram.

IV. VARIABLE RESISTANCE OR REACTANCE. CURRENT LOCI

a. **Constant Resistance and Variable Reactance.**—Let a constant voltage be impressed upon a circuit having a constant

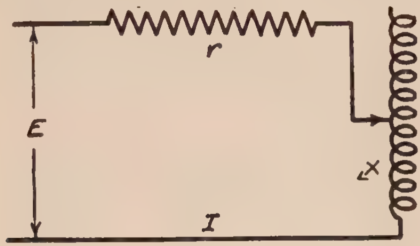


FIG. 34.7.

resistance and a reactance that can be varied as shown in the circuit diagram, Fig. 34.7. The corresponding vector diagram and current locus are shown in Fig. 35.7. Let the line *OA* along

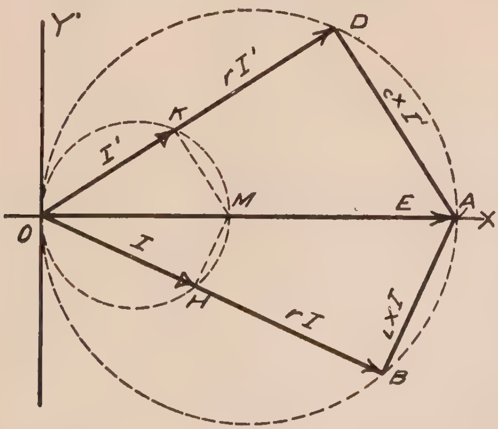


FIG. 35.7.

the *X*-axis represent the impressed voltage *E*; and the line *OM* the current when only the resistance *r* is in the circuit.

On OA and OM as diameters describe circles. With an inductive reactance ${}_Lx$ in series with the constant resistance, the voltages consumed by the resistance and reactance respectively are represented by the lines OB and BA, and the current by OH. By varying the reactance from zero to infinity the point B describes the semicircle ABO. The point H describes a similar arc MHO since OAB and OMH are similar triangles.

If the inductive reactance be replaced by a condensive reactance ${}_cx$, then the lines OD and DA represent the resistance and reactance drops, and OK the current I'. If the condensive reactance be varied from zero to infinity, the point D will describe the semicircle ADO. Likewise the point K will describe the semicircle MKO, since the triangles OAD and OMK are similar. The circle $OHMK$ is therefore the locus of the current in a circuit having constant resistance, variable reactance and constant voltage impressed on the terminals.

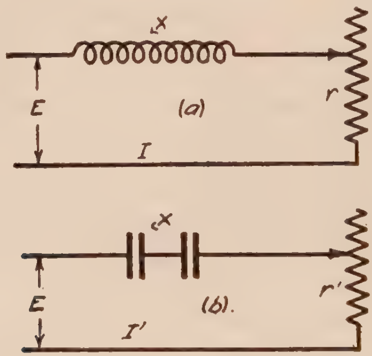


Fig. 36.7.

b. **Constant Reactance and Variable Resistance.**—The circuit diagrams for inductive or condensive reactances are shown in (*a*) and (*b*), Fig. 36.7. Let the impressed voltage E be constant.

In Fig. 37.7 lay off OA to represent E and let OF represent the current flowing through ${}_Lx$ with no resistance in series. Let ${}_Lx$ in (*a*) be equal to ${}_cx$ in (*b*), and hence OM equal to OF will represent the current flowing through ${}_cx$ in (*b*) when the resistance in series is equal to zero.

Upon OA as a diameter describe the circle $OBAD$, and upon OM and OF describe semicircles as shown in Fig. 37.7.

With a resistance r in series with ${}_Lx$ the lines OB and AB represent the voltages consumed by r and ${}_Lx$ respectively. The

current I is represented by the line OH since the triangles OAB and FOH are similar. By varying the resistance from zero to infinity the point B describes the semicircle OBA and the point H the semicircle FHO. With a resistance r' in series with the condensive reactance ${}_cx$ in (b), the voltage consumed by the resistance is represented by the vector OD, and that consumed by the reactance, by the line DA. The current I' is represented by the vector OK since the triangles MKO and ODA are similar. By varying the resistance in series with ${}_cx$ in (b) from zero to infinity the point D follows the semicircle ODA, and the point K the semicircle MKO. The locus of the current is therefore the semicircle FHO for the inductive reactance ${}_Lx$, and MKO for the condensive reactance ${}_cx$.

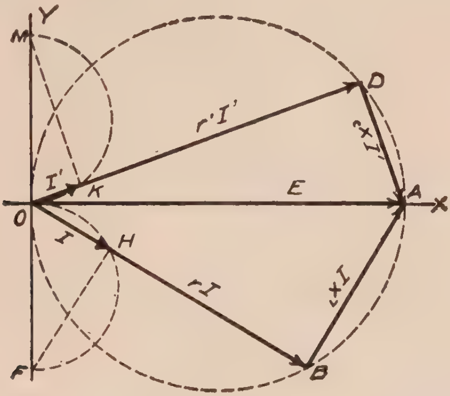


Fig. 37.7.

V. RESISTANCE IN SERIES WITH A CIRCUIT

The circuit diagram in Fig. 38.7 is typical of many simple commercial circuits; a receiver circuit having variable load and power factor, in series with a resistance. Let the generator voltage E_0, the series resistance r_2 and the receiver impedance z_1 be constant, but the receiver resistance r_1 and reactance x_1 be variable.

$$z_1 = r_1 + jx_1;\ Z_1^2 = r_1^2 + x_1^2 \tag{22.7}$$
$$z_0 = r_1 + r_2 + jx_1;\ Z_0^2 = (r_1 + r_2)^2 + x_1^2 \tag{23.7}$$

Hence the current is:

$$\dot{I} = \frac{\dot{E}_0}{z_0} = \frac{\dot{E}_0}{r_1 + r_2 + jx_1} = \frac{\dot{E}_0(r_1 + r_2 - jx_1)}{(r_1 + r_2)^2 + x_1^2} \tag{24.7}$$

The voltage at the receiver circuit:

$$\dot{E}_1 = \dot{I}z_1 = \frac{\dot{E}_0(r_1 + jx_1)}{r_1 + r_2 + jx_1} = \frac{\dot{E}_0(r_1 + jx_1)(r_1 + r_2 - jx_1)}{(r_1 + r_2)^2 + x_1^2}$$

$$= \frac{\dot{E}_0(z_1^2 + r_1r_2 + jr_2x_1)}{z_1^2 + 2r_1r_2 + r_2^2} \qquad (25.7)$$

In absolute values:

$$I = \frac{E_0}{\sqrt{(r_1 + r_2)^2 + x_1^2}} = \frac{E_0}{\sqrt{Z_1^2 + 2r_1r_2 + r_2^2}} \qquad (26.7)$$

$$E_1 = \frac{E_0 \sqrt{r_1^2 + x_1^2}}{\sqrt{(r_1 + r_2)^2 + x_1^2}} = \frac{E_0Z_1}{\sqrt{Z_1^2 + 2r_1r_2 + r_2^2}} \qquad (27.7)$$

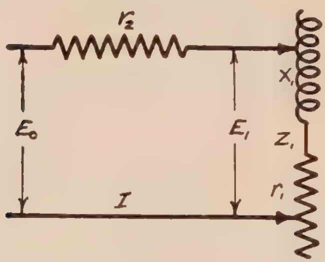


FIG. 38.7.

Current and voltage phase difference at the receiver circuit terminals:

$$\theta_1 = \tan^{-1} \frac{x_1}{r_1} \qquad (28.7)$$

and at the generator:

$$\theta_0 = \tan^{-1} \frac{x_1}{r_1 + r_2} \qquad (29.7)$$

In Fig. 39.7 is shown the vector diagram for the values: $E_0 = 100$ volts; $r_2 = 0.5$ ohm; $r_1 = 0.8$ ohm; $x_1 = 0.6$ ohm; and hence $Z_1 = 1$.

The current vector is taken along the X-axis.

The voltage vector E_0 is at an angle $\theta_0 = \tan^{-1} \dfrac{x_1}{r_1 + r_2}$ from the current and drawn to any convenient scale. The vector for the receiver voltage E_1 is at an angle of $\theta_1 = \tan^{-1} \dfrac{x_1}{r_1}$ from the current, and the magnitude may be calculated from the given data. The voltages consumed by the resistance r_1I and

r_2I are in phase with the current while the voltage taken by the reactance x_1I is in quadrature with the current.

By keeping E_0, r_2 and z_1 constant, but varying r_1 from zero to unity, and x_1 from 1-ohm inductive reactance to zero, the locus of the voltage E_0 described by the point A will be MAQ, a quadrant of a circle. If x_1 be changed from zero to 1-ohm condensive reactance with simultaneous variation in r_1 so as to keep Z_1 constant, the locus of the voltage will be the semicircle $MAQF$. The corresponding positions of the point B give the locus of the receiver voltage E_1 in the curve $HBNK$. The curve in the two quadrants is symmetrical and hence the value of the receiver voltage will be the same for any two equal inductive and condensive reactances. With the constant assumed ($Z_1 = 1$) the magnitude of the current in amperes is in each case numerically equal to E_1 in volts.

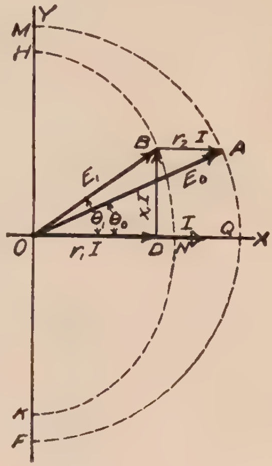


FIG. 39.7.

Problem 43.7.—Given $E_0 = 100$ volts; $r_2 = 0.8$ ohm; $Z_1 = 1$ ohm.

Let x_1 vary from $+1$ to -1 ohm. Plot in rectangular coördinates the values of E_1 and I as ordinates with x_1 as abscissæ.

VI. REACTANCE IN SERIES WITH A CIRCUIT

In a circuit having reactance instead of resistance in series, the relations of the current and receiver voltage to the constant generator voltage are derived in the same manner. The circuit diagram, Fig. 40.7 shows an inductive reactance in series with an inductive receiver circuit. To make the discussion general the reactance in series may be taken either as inductive or as condensive and the impedance in the receiver circuit may consist of resistance and either inductive or condensive reactance.

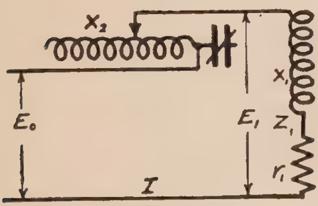


FIG. 40.7.

The receiver circuit impedance:

$$z_1 = r_1 + jx_1 \tag{30.7}$$

The total impedance:

$$z_0 = z_1 + jx_2 = r_1 + j(x_1 + x_2) \tag{31.7}$$

The current is:

$$\dot{I} = \frac{\dot{E}_0}{z_0} = \frac{\dot{E}_0}{r_1 + j(x_1 + x_2)} \tag{32.7}$$

In absolute values:

$$I = \frac{E_0}{\sqrt{r_1^2 + (x_1 + x_2)^2}} = \frac{E_0}{\sqrt{Z_1^2 + 2x_1x_2 + x_2^2}} \tag{33.7}$$

The receiver voltage:

$$\dot{E}_1 = \dot{I}z_1 = \dot{E}_0\frac{r_1 + jx_1}{r_1 + j(x_1 + x_2)} \tag{34.7}$$

$$E_1 = E_0\frac{\sqrt{r_1^2 + x_1^2}}{\sqrt{r_1^2 + (x_1 + x_2)^2}} = \frac{E_0Z_1}{\sqrt{Z_1^2 + 2x_1x_2 + x_2^2}} \tag{35.7}$$

$$\theta_1 = \tan^{-1}\frac{x_1}{r_1} \tag{36.7}$$

$$\theta_0 = \tan^{-1}\frac{x_1 + x_2}{r_1} \tag{37.7}$$

Since the reactances x_1 and x_2 may be either inductive or condensive, four combinations are possible:

(*a*) x_1 inductive, x_2 inductive.
(*b*) x_1 inductive, x_2 condensive.
(*c*) x_1 condensive, x_2 inductive.
(*d*) x_1 condensive, x_2 condensive.

From equations (33.7) and (35.7) it is evident that the numerical value depends on the square of the algebraic sum of x_1 and x_2. If both are inductive or both condensive, the algebraic sum is the same as the arithmetical sum; but if one is inductive and the other condensive, the algebraic sum is equal to the arithmetical difference of the two reactances. Hence for numerical or absolute values (*a*) and (*d*) are alike, and (*b*) and (*c*) will give similar results. The phase position of the current and voltage is, however, affected by the positive or negative sign before the reactance and these relations may be determined by equations (32.7), (33.7), (36.7) and (37.7).

If x_1 is inductive, varying x_2 from 1-ohm inductive to 1-ohm condensive reactance, with E_0 constant, it is seen from equation

(33.7) that the current will be a maximum when $x_1 - x_2 = 0$. Hence for maximum current x_2 must be inductive if x_1 is condensive and *vice versa.*

Problem 44.7.—Given $E_0 = 100$ volts; $z_1 = 1$ ohm; $x_1 = 0.4$ ohm. Circuit diagram as in Fig. 40.7. Plot in rectangular coördinates the values for E_1 and I with x_2 varying from $+1$ to -1 ohm as abscissæ.

Problem 45.7.—Given $E_0 = 100$ volts; $z_1 = 1$ ohm; $x_1 = 0.7$ ohm with circuit diagram as in Fig. 40.7. Plot curves for E_1 and I with x_2 as abscissæ as in problem 44.7.

VII. IMPEDANCE IN SERIES WITH A CIRCUIT

Regulation of the receiver voltage might be obtained by means of a reactance placed in series as discussed in the preceding paragraph. However, reactance coils always dissipate

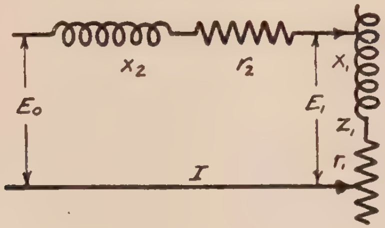


FIG. 41.7.

energy due to the ohmic resistance of the wire and the hysteresis in the iron, and hence an impedance is actually inserted instead of the desired reactance, as shown in Fig. 41.7.

$$z_1 = r_1 + jx_1;\ z_2 = r_2 + jx_2 \tag{38.7}$$

$$Z_1 = \sqrt{r_1^2 + x_1^2};\ Z_2 = \sqrt{r_2^2 + x_2^2} \tag{39.7}$$

The total impedance:

$$z_0 = r_1 + r_2 + j(x_1 + x_2) \tag{40.7}$$

$$Z_0 = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2} \tag{41.7}$$

The current:

$$\dot{I} = \frac{\dot{E}_0}{z_0} = \frac{\dot{E}_0}{(r_1 + r_2) + j(x_1 + x_2)} = \dot{E}_0 \frac{(r_1 + r_2) - j(x_1 + x_2)}{(r_1 + r_2)^2 + (x_1 + x_2)^2} \tag{42.7}$$

In absolute values:

$$I = \frac{E_0}{Z_0} = \frac{E_0}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}$$

$$= \frac{E_0}{\sqrt{(Z_1^2 + Z_2^2 + 2(r_1 r_2 + x_1 x_2)}} \tag{43.7}$$

The receiver voltage:

$$\dot{E}_1 = z_1 \dot{I} = \frac{\dot{E}_0(r_1 + jx_1)}{r_1 + r_2 + j(x_1 + x_2)} \tag{44.7}$$

$$E_1 = Z_1 I = \frac{E_0\sqrt{r_1^2 + x_1^2}}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}$$

$$= \frac{E_0 Z_1}{\sqrt{Z_1^2 + Z_2^2 + 2(r_1 r_2 + x_1 x_2)}} \tag{45.7}$$

The phase difference at the receiver circuit:

$$\theta_1 = \tan^{-1}\frac{x_1}{r_1} \tag{46.7}$$

The phase difference at the generator:

$$\theta_0 = \tan^{-1}\frac{x_1 + x_2}{r_1 + r_2} \tag{47.7}$$

As in the discussion of resistance and reactance in series let E_0, r_2, x_2 and z_1 be constant, but x_1 and r_1 be variable.

With resistance in series, E_1 is a minimum when $x_1 = 0$; and with the circuit having reactance in series the voltage at the receiver circuit is a minimum when $r_1 = 0$ and the reactances x_1 and x_2 are either both inductive or both condensive. With impedance in series, the receiver voltage is a minimum when θ_1 is equal to, and of the same sign as, θ_2, or θ_0; that is, when $\frac{x_1}{r_1} = \frac{x_2}{r_2}$.

In equation (45.7) E_1 is a minimum when the denominator is a maximum. Since Z_1, Z_2, r_2 and x_2 are constants, the denominator will be a maximum when $r_1 r_2 + x_1 x_2$ is a maximum, or with a single variable r_1 when

$$\frac{d}{dr_1}(r_1 r_2 + x_2\sqrt{z_1^2 - r_1^2}) = 0 \tag{48.7}$$

$$r_2 + \frac{1}{2}\left(\frac{x_2}{\sqrt{z_1^2 - r_1^2}}\right)(-2r_1) = 0 \tag{49.7}$$

or

$$\frac{x_1}{r_1} = \frac{x_2}{r_2} \tag{50.7}$$

This topic is discussed more fully under Regulation in Chap. XXIII.

Problem 46.7.—$E_0 = 100$ volts; $x_2 = 1$ ohm; $r_2 = 1$ ohm, with circuit diagram as in Fig. 41.7. $x_1 = kr_1$, varying from 0 to 5 ohms. Draw the loci of the current and voltages E_1 and E_2 for values of $k = 1$ and $k = 5$ when referred to E_0 as a reference vector.

VIII. COMMERCIAL CIRCUITS

In the preceding problems the resistance and reactance of each elemental circuit have been given and methods for finding the resultant equivalent impedance or admittance have been explained. In commercial systems the circuits generally consist of complex networks, in which it may be difficult or impracticable to measure the resistance and reactance of each elemental division. The total values are found directly by taking voltmeter, ammeter and wattmeter readings, and from these data the constants of the circuits may be calculated. As explained in Chap. IV, the wattmeter measures directly the power expended, which may be expressed as rI^2.

$$r = \frac{\text{(wattmeter reading in watts)}}{\text{(ammeter reading in amperes)}^2} \text{ ohms} \qquad (51.7)$$

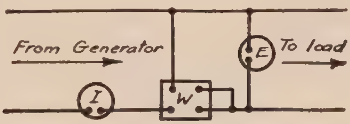


FIG. 42.7.

With E, I and r known, the numerical values of x, z, g, b, y and θ may be calculated. Thus, by the simple expedient of taking voltmeter, ammeter and wattmeter readings the equivalent resistance, reactance and impedance may be obtained for circuits of any complexity. All that part of the system farther away from the generator than the point at which the readings are taken will be included in the computed values.

Problem 47.7.—Given a circuit as shown in Fig. 42.7. $E = 224$ volts; $I = 15.2$ amp.; $W = 2{,}126$ watts; inductive load.

Find: r, x, z, g, b, y, Z, Y and θ.

Draw the impedance triangle. Draw the vector diagram.

Problem 48.7.—Same data as for problem 47.7 except the reactance is condensive.

Find: r, x, z, g, b, y, Z, Y and θ.

Draw the impedance triangle. Draw the vector diagram.

CHAPTER VIII

ELECTRIC POWER. POWER FACTOR. ELECTRIC ENERGY

Electric Power.—In direct currents the power expended in a circuit, measured in watts, is equal to the product of the volts by the amperes. In alternating currents the power in the circuit is at any instant equal to the product of the instantaneous current and voltage. In circuits having resistance only, the energy delivered to a circuit is immediately changed into heat, or light, and cannot return to the generator; *i.e.*, the process is not reversible. Therefore, the average power delivered to a non-inductive load is the sum of the instantaneous powers over a cycle divided by the time for the cycle. In circuits having magnetic or dielectric induction, part of the energy is stored in the magnetic or dielectric (electrostatic) fields in such forms while the current or voltage is increasing, that, when the current or voltage decreases, the energy is returned to the electric circuit; that is, the process is automatically reversible. Consequently, only part of the energy flowing in an alternating-current system having inductance or condensance is left in the circuit. For sine waves, as explained in Chap. I, energy is stored in the magnetic field while the current increases, and this energy returns to the circuit when the current decreases. Likewise energy is stored in the dielectric field while the voltage increases, and returns during the next quarter cycle of the voltage wave. The instantaneous value of the power in the circuit is at all times equal to the product of the corresponding instantaneous current and voltage. But the direction of energy flow for sine voltage and current waves will reverse four times in each cycle.

This is shown graphically in Figs. 1.8 and 2.8 for a current lagging $\theta°$ behind the voltage wave, and in Figs. 3.8 and 4.8 for a current leading the voltage by $\theta°$. The ordinates of the current-time curve represent the instantaneous values of the current flowing in the circuit. Likewise the ordinates of the voltage-time curve are proportional to the instantaneous values of the voltage impressed on the circuit.

Since power, $p = ei$, is the product of the instantaneous values of the current and voltage, the ordinates of the power-time curve represent the power at any instant in the cycle.

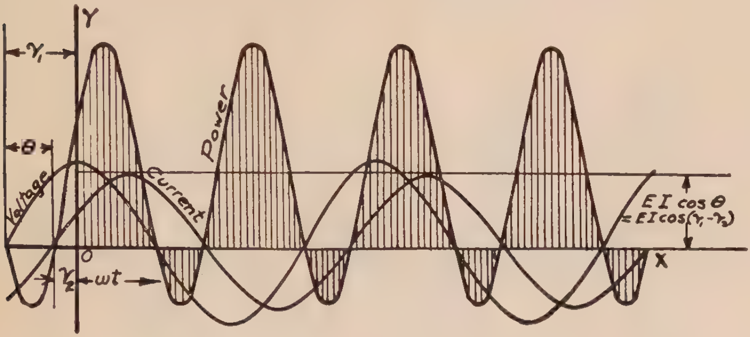


FIG. 1.8.

The shaded areas bounded by the power-time curve represent energy; the positive areas, the energy delivered by the generator; and the areas below the X-axis, the energy returned to the generator. The difference of these areas for one cycle is the

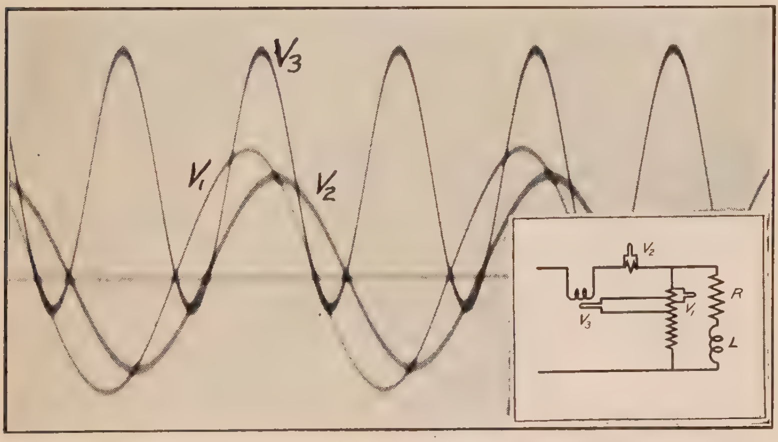


FIG. 2.8.—Oscillogram, v_1 = voltage; v_2 = current; v_3 = power.

total energy left in the circuit during one cycle. This quantity divided by the time for one cycle gives the average power delivered to the circuit.

The instantaneous value of the power,

$$p = ei \tag{1.8}$$

For sine waves:

$$e = {}^{M}E \sin (\omega t - \gamma_1) \tag{2.8}$$
$$i = {}^{M}I \sin (\omega t - \gamma_2) \tag{3.8}$$
$$p = {}^{M}E\,{}^{M}I \sin (\omega t - \gamma_1) \sin (\omega t - \gamma_2) \tag{4.8}$$

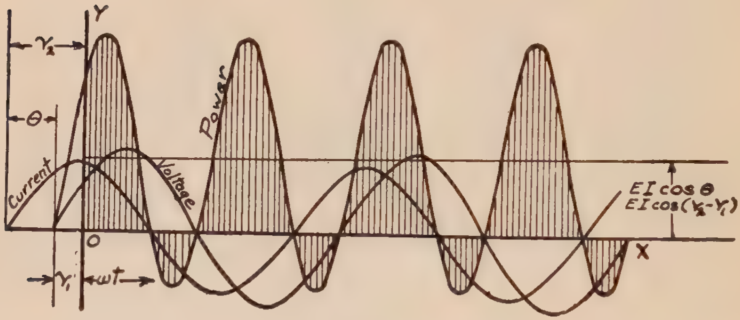


FIG. 3.8.

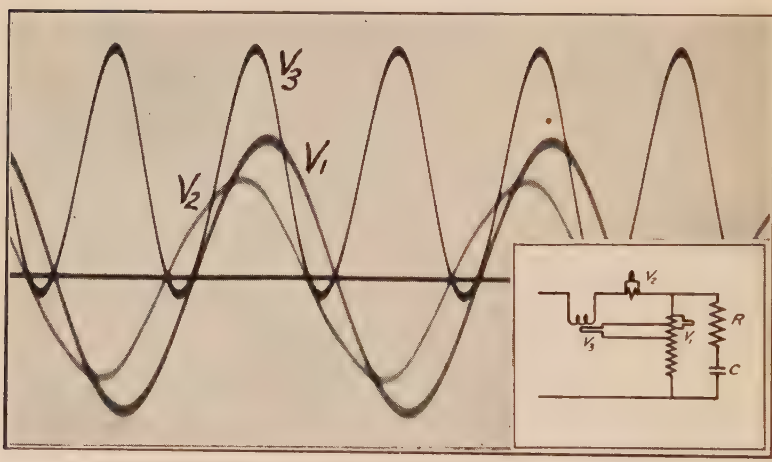


FIG. 4.8.—Oscillogram, v_1 = voltage; v_2 = current; v_3 = power.

From trigonometry:

$$\sin x \sin y = \tfrac{1}{2} \cos (x - y) - \tfrac{1}{2} \cos (x + y) \tag{5.8}$$

Let

$$x = \omega t - \gamma_1, \text{ and } y = \omega t - \gamma_2$$
$$p = EI[\cos (\gamma_2 - \gamma_1) - \cos (2\omega t - \gamma_1 - \gamma_2)] \tag{6.8}$$

From this it is seen that the instantaneous value of the power is also a sine function of the time, of double the frequency of the

current or voltage, and with its reference axis displaced by $EI \cos (\gamma_2 - \gamma_1)$ from the X-axis.

The average value of the power during a complete cycle is:

$$P = \frac{1}{T}\int_0^T eidt \tag{7.8}$$

$$= \frac{\omega}{2\pi}EI\int_0^{\frac{2\pi}{\omega}}[\cos (\gamma_2 - \gamma_1) - \cos (2\omega t - \gamma_1 - \gamma_2)]dt \tag{8.8}$$

$$= EI \cos (\gamma_2 - \gamma_1) = EI \cos \theta \tag{9.8}$$

In which

$\theta = \gamma_2 - \gamma_1 =$ phase angle between the current I and voltage E.

The product EI is called the apparent power and is often referred to as *volt-amperes* or *kilovolt-amperes* (kv.a.). Generators are usually rated in kv.a. as the heating depends on the current in the conductors and not on the power delivered.

Power Factor.—The factor *cos* θ in equation (9.8) determines what part of the power in the system is delivered to the load. For this reason it is called the *power factor*. Thus multiplying the apparent power by the power factor gives the *true power* or the *power delivered* to the circuit. In a non-reactive circuit, Figs. 5.8 and 6.8, the current and voltage are in phase, and therefore $\gamma_1 = \gamma_2$ and $\theta = 0$. Hence, $\cos \theta = 1$, or the circuit has *unity power factor*. Power factors less than unity are usually expressed in per cent. Thus 85 per cent power factor means that $\cos \theta = 0.85$; and hence that the kw. = 85 per cent of the kv.a. = 0.85 kv.a.

The shaded areas in Figs. 1.8 to 4.8 show how the energy flows to and fro when the current lags or leads the voltage. Also, that the amount of power actually delivered to the circuit is proportional to the cosine of the angle of lead or lag. Evidently the more nearly $\cos \theta$ approaches unity the less energy will be surging to and fro in the line. When the current and voltage are in phase, Figs. 5.8 and 6.8, $\cos \theta = 1$; and in this case, *and this case only*, the power is at no instant negative and hence all the energy is delivered to the load, although it is transmitted from the generator in pulsations.

The greatest amount of surging will occur when the phase-angle difference of the current and voltage is 90°, for then

$\cos \theta = 0$ and hence all the power is reactive, or returns to the generator.

Under these conditions, Figs. 7.8 to 10.8, all of the energy surges to and fro and no actual delivery of power occurs.

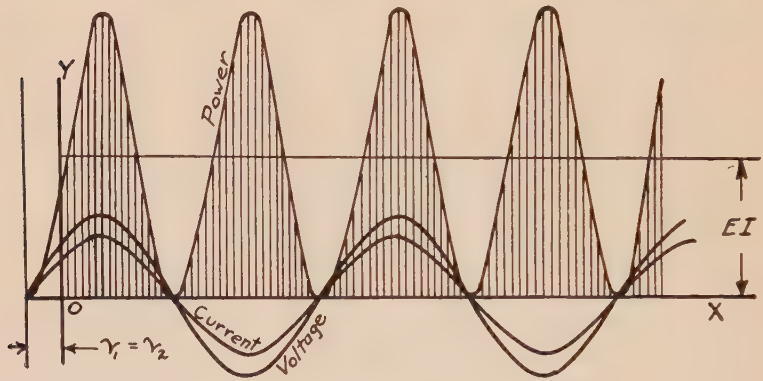


FIG. 5.8.

In this case the area above the X-axis of the power curve equals the area below the axis; or an amount of energy returns to the generator equal to that supplied by the generator to the line.

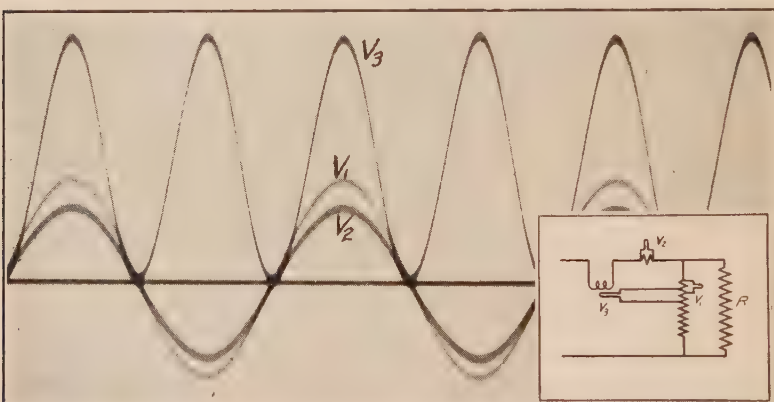


FIG. 6.8.—Oscillogram, v_1 = voltage; v_2 = current; v_3 = power.

While it is of first importance to determine the value of the power delivered in a circuit, it is often desirable to measure the *reactive energy*, or the part that surges to and fro in the line. A vector diagram of a current I leading a voltage E by $\theta°$ is shown in Fig. 11.8. If the current be divided into two components, in

phase and in quadrature with the voltage, we have the vectors $OB = {}_aI = I \cos \theta$; $OD = {}_bI = I \sin \theta$. It is evident that the product of the voltage with the component in phase with the voltage will always be positive and hence gives the real or effective

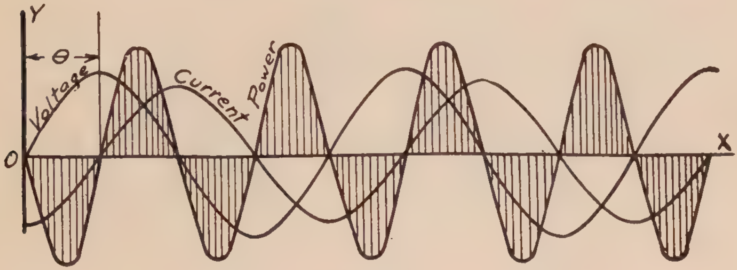


FIG. 7.8.

power. The component $I \sin \theta$ is in quadrature with the voltage and hence during each cycle it will, half the time, be on the same side of the X-axis as the voltage vector and half the time on the opposite side. Hence the product $EI \sin \theta$ will be positive during

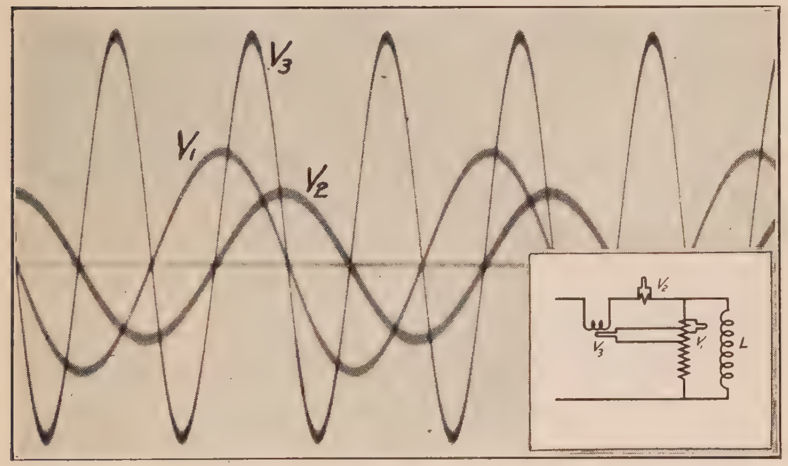


FIG. 8.8.—Oscillogram, v_1 = voltage; v_2 = current; v_3 = power.

half the cycle and negative during the other half. The average, therefore, is zero.

The two expressions give the quantitative values of the power in the circuit. The *real* or *effective power*, that is, the power

delivered to the circuit by the generator and which does not return to the generator is:

$$P = EI \cos \theta \tag{10.8}$$

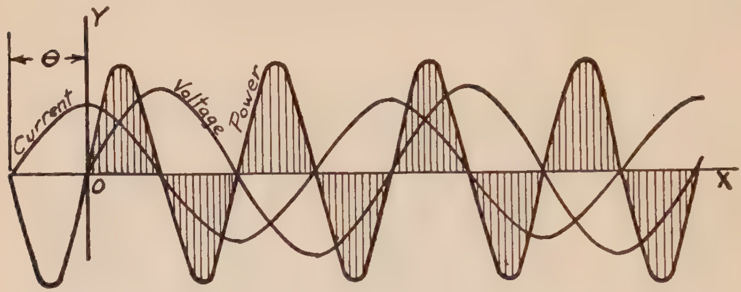


FIG. 9.8.

The *reactive power*, or the power that surges to and fro between the generator and the line is:

$${}_xP = EI \sin \theta \tag{11.8}$$

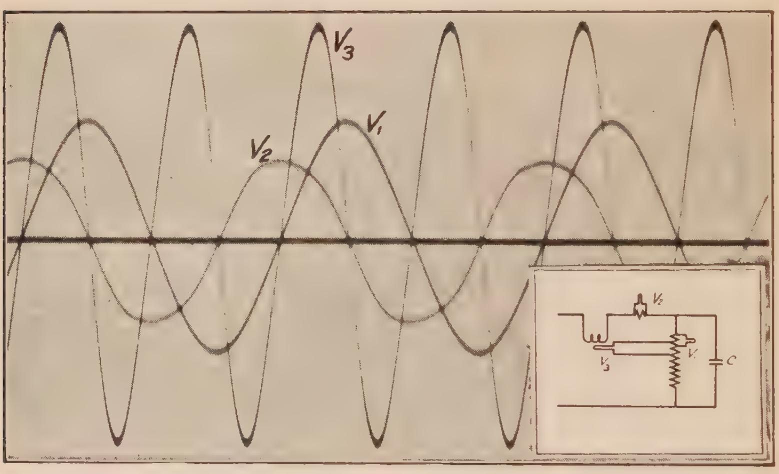


FIG. 10.8.—Oscillogram, v_1 = voltage; v_2 = current; v_3 = power.

In making a graphical representation in Fig. 11.8 the current was divided into two components in phase and in quadrature with the voltage. The same result is gained by dividing the voltage into two components, in phase and in quadrature with the current as shown in Fig. 12.8. For constant-voltage systems the current would naturally be considered as consisting of components in phase and in quadrature with the voltage as in Fig.

11.8. For similar reasons in constant-current circuits the voltage may be considered as made up of components in phase and in quadrature with the current, Fig. 12.8. Since by far the larger part of alternating-current distribution is by the constant-potential system it is customary to take the voltage as the reference vector and to speak of the *power component of the current*, the *active* or *energy current*, $I \cos \theta$; and of the *wattless*, *reactive* or *quadrature current*, $I \sin \theta$. For constant-current circuits the terms *power component of the voltage* and the *quadrature component of the voltage* would be appropriate. The term *wattless* is a misnomer as both the active and the quadrature components have power, and therefore are not without watts.

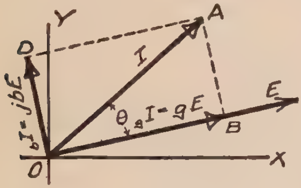


Fig. 11.8.

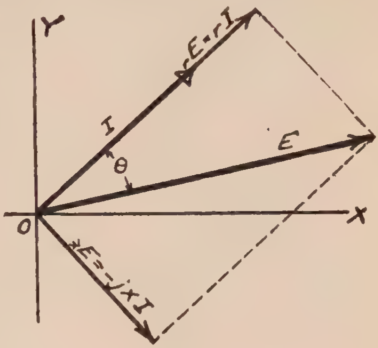


Fig. 12.8.

In the preceding paragraphs the vector diagrams have been drawn so as to have either the current I (series circuits) or the voltage E (parallel circuits) as the reference vector. The

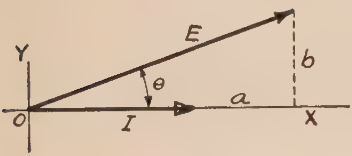


Fig. 13.8.

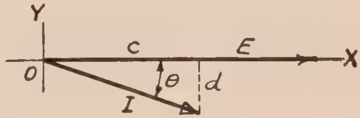


Fig. 14.8.

algebraic expressions for the power delivered and the reactive power are therefore given by the products of the voltage or current components, respectively, in phase and in quadrature with the reference vector, as illustrated by Figs. 13.8 and 14.8.

The power and the reactive power for the circuit in Fig. 13.8 are expressed by equations (14.8) and (15.8):

$$\dot{E} = a + jb \tag{12.8}$$
$$\dot{E}\dot{I} = (a + jb)\dot{I} \tag{13.8}$$
$$P = aI \tag{14.8}$$
$${}_{x}P = bI \tag{15.8}$$

Similarly for Fig. 14.8:

$$\dot{E}\dot{I} = (c - jd)\dot{E} \tag{16.8}$$
$$P = cE \tag{17.8}$$
$${}_{x}P = dE \tag{18.8}$$

If the reference axis is selected so that both the voltage and the current are expressed in complex quantities a special rule for obtaining the values of P and ${}_{x}P$ must be used, as general vector quantities are involved.

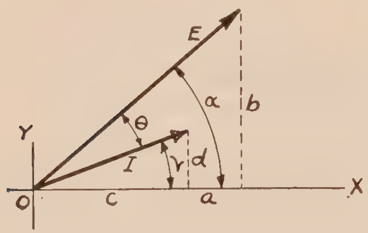


Fig. 15.8.

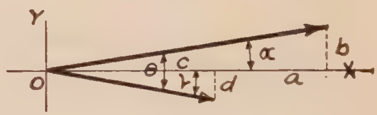


Fig. 16.8.

In Fig. 15.8, let OX be the reference axis.

$$\dot{E} = a + jb \tag{19.8}$$
$$\dot{I} = c + jd \tag{20.8}$$
$$\dot{E}\dot{I} = (a + jb)(c + jd) \tag{21.8}$$
$$P = ac + bd \tag{22.8}$$
$${}_{x}P = bc - ad \tag{23.8}$$

Similarly for Fig. 16.8:

$$\dot{E} = a + jb \tag{24.8}$$
$$\dot{I} = c - jd \tag{25.8}$$
$$\dot{E}\dot{I} = (a + jb)(c - jd) \tag{26.8}$$
$$P = ac - bd \tag{27.8}$$
$${}_{x}P = bc + ad \tag{28.8}$$

In using complex notation for both current and voltage the following rules apply for vector multiplication:

(*a*) For *power*, ignore the j^2 factor.

(*b*) For *reactive power*, reverse the sign of the j component of either the voltage or the current vector.

$\frac{c}{I}$ $\sin\theta_1 = \frac{d}{I}$

That the above rules are valid is readily proved by referring to the vector diagrams.

From Fig. 15.8:

$$P = EI\cos\theta = EI\cos(\alpha - \gamma) \tag{29.8}$$
$$= EI(\cos\alpha\cos\gamma + \sin\alpha\sin\gamma) \tag{30.8}$$
$$= \sqrt{a^2+b^2}\sqrt{c^2+d^2}\left[\frac{a}{\sqrt{a^2+b^2}}\frac{c}{\sqrt{c^2+d^2}} + \frac{b}{\sqrt{a^2+b^2}}\frac{d}{\sqrt{c^2+d^2}}\right] \tag{31.8}$$
$$= ac + bd \tag{32.8}$$
$${}_xP = EI\sin\theta = EI\sin(\alpha - \gamma) \tag{33.8}$$
$$= EI(\sin\alpha\cos\gamma - \cos\alpha\sin\gamma) \tag{34.8}$$
$$= \sqrt{a^2+b^2}\sqrt{c^2+d^2}\left[\frac{b}{\sqrt{a^2+b^2}}\frac{c}{\sqrt{c^2+d^2}} - \frac{a}{\sqrt{a^2+b^2}}\frac{d}{\sqrt{c^2+d^2}}\right] \tag{35.8}$$
$$= bc - ad \tag{36.8}$$

Similarly for Fig. 16.8:

$$P = EI\cos\theta = EI\cos(\alpha + \gamma) \tag{37.8}$$
$$= EI(\cos\alpha\cos\gamma - \sin\alpha\sin\gamma) \tag{38.8}$$
$$= ac - bd \tag{39.8}$$
$${}_xP = EI\sin\theta = EI\sin(\alpha + \gamma) \tag{40.8}$$
$$= EI(\sin\alpha\cos\gamma + \cos\alpha\cos\gamma) \tag{41.8}$$
$$= bc + ad \tag{42.8}$$

Electric Energy.—The energy transfer under unity power-factor conditions at the generator is readily shown by a power diagram. For illustration, take a system with two parallel circuits as shown in the circuit and vector diagrams in Fig. 17.8. In parallel circuits the voltage is the common factor and hence the currents should be divided into components in phase and in quadrature with the voltage.

$$\dot{I}_1 = g_1\dot{E} + j\,{}_cb_1\dot{E} \text{ and } \dot{I}_2 = g_2\dot{E} - j\,{}_Lb_2\dot{E} \tag{43.8}$$

In rectangular coördinates with time as abscissæ, we have given in Fig. 18.8 the voltage, current and power curves for the several components of the currents I_1 and I_2 and the voltage E.

In (*a*) curves for E, g_1E and the resultant power g_1E^2 or P_1.

In (*b*) curves for E, g_2E and the resultant power g_2E^2 or P_2.

In (*c*) curves for E, ${}_cb_1E$ and the resultant power (reactive) ${}_cb_1E^2$ or ${}_xP_1$.

In (*d*) curves for E, $-{}_Lb_2E$ and the resultant power (reactive) $-{}_Lb_2E^2$ or ${}_xP_2$.

The power curves in both (*a*) and (*b*) are always positive; that is, the energy transmitted is absorbed by the resistance and changed into heat. In both (*c*) and (*d*) the successive half cycles of the power wave are of opposite sign and this shows that the flow of energy reverses, or the power goes in one direction while the wave is positive and in the opposite direction during the negative half of the wave. It is of importance to note that when the power ordinate is positive in (*c*), the corresponding ordinate in (*d*) is negative, and *vice versa.* The diagram therefore shows graphically the energy transfer from the condensance in circuit 1 to the inductance in circuit 2, or in the reverse direction, for the successive half cycles of the power wave. Under resonance conditions ${}_cb_1 = {}_Lb_2$, or the corresponding instantaneous power ordinates in (*c*) and (*d*) are always equal.

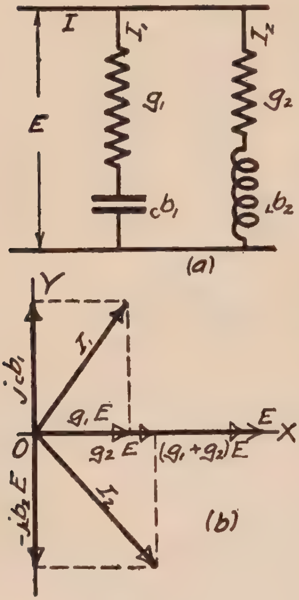


FIG. 17.8.

This means that the energy stored dielectrically in circuit 1 when the voltage is a maximum is transferred and stored magnetically in circuit 2 while the line voltage decreases to its zero value, or in one-fourth cycle. During the next quarter cycle of the voltage wave the energy in the magnetic field of circuit 2 is returned and stored in the dielectric field of circuit 1.

In the mains coming from the generator, the power transmitted is the sum of (*a*) and (*b*) as shown in (*e*). The current $I = g_1E + g_2E$ and is in phase with E. At the generator, under the given conditions, the power factor is unity.

Power-factor Meters.—It is evident that the power factor of any circuit can be computed from voltmeter, ammeter and wattmeter readings, since by definition the power factor is the ratio of the watts to the volt-amperes.

$$\text{Power factor} = \frac{\text{watts}}{\text{volt} \times \text{amperes}} = \cos\theta \qquad (44.8)$$

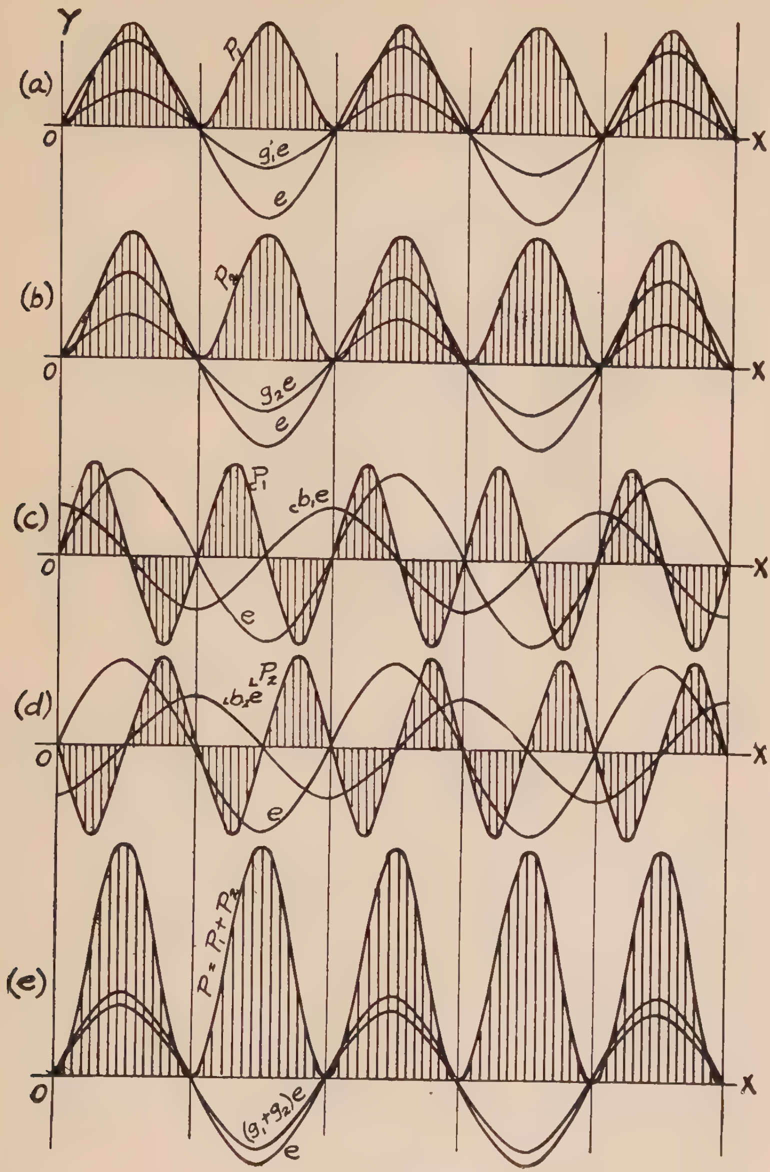

Y
(a)
O
X
P_1
g_1e
e
(b)
O
X
P_2
g_2e
e
(c)
O
X
P_1
b_1e
e
(d)
O
X
P_2
b_2e
e
(e)
O
X
$P = P_1 + P_2$
$(g_1+g_2)e$
e

FIG. 18.8.

However, in the operation of power plants it is convenient to have a special instrument that continuously indicates the power factor of the load or of any desired section of the system. Several types of power-factor meters are in commercial use. The essential features of one type of power-factor meter are illustrated in Fig. 19.8.

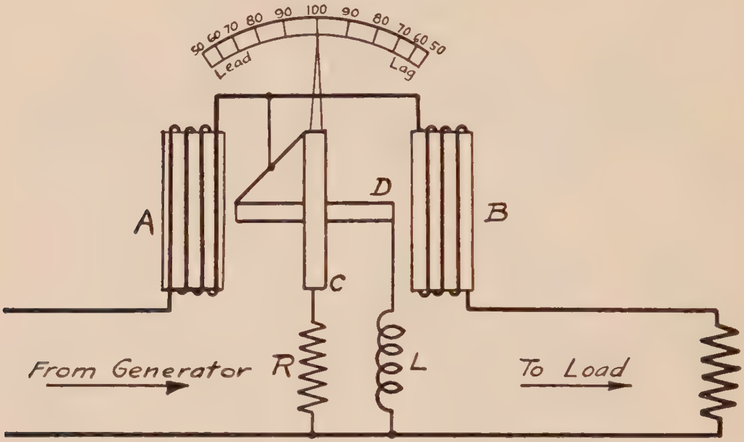


FIG. 19.8.—Circuit diagram of a power-factor meter.

The load current flows through the stationary coil AB and produces the alternating-field flux. The movable element consists of two coils, C and D, placed in space quadrature with respect to each other and mounted on a pivoted axle which also carries the indicating pointer. In series with coil C is a large resistance, R, and in series with coil D a large inductance, L. The circuit connections are evident from the diagram in Fig. 19.8. The currents in the two parallel circuits enter and leave the movable coils C and D by means of sliding contacts between fixed strips or brushes and slip rings on the rotating axle carrying the two coils.

As the resistance R is large in comparison with the inductance in coil C, the current in coil C is essentially in phase with the voltage. Likewise, the current in coil D is essentially lagging by 90°, with respect to the voltage, as the inductance L is large, in comparison with the resistance in the coil D circuit.

In explaining the operation of the meter the currents in the two coils are assumed to be in time quadrature. The actual deviation from the 90° time-phase relation, due to the small inductance in coil C, and the likewise small resistance in coil D circuit, is

compensated for by making the space position of the two coils with respect to each other correspondingly less than 90°. For loads of unity power factor the load current, and hence the field of the coil AB, is in phase with the voltage. The position of the movable element, for loads of unity power factor, must therefore be such as to bring the fields ϕ_C and ϕ_{AB} in parallel, that is, coil C will be in a plane parallel to coils A and B. Since the field, ϕ_D, in coil D is in time quadrature with ϕ_{AB} the average reaction or the resultant torque of the two fields, ϕ_D and ϕ_{AB}, is zero.

If the load power factor is less than unity the flux ϕ_C will have two components, one in time phase with ϕ_{AB} which produces a directing moment in the movable element, and the other in time quadrature with ϕ_{AB}, which becomes inactive. Similarly, flux ϕ_D will have components in time phase and time quadrature with flux ϕ_{AB}. Hence, the position of the movable element will be determined by the resultant effect produced by the components of ϕ_C and ϕ_L in time phase with the field flux ϕ_{AB}. As a consequence the movable element will change in position with changes in load power factor, to the *right* of the unity power-factor position for *lagging* currents and to the *left* for *leading* currents. The movable element will assume a stationary position (and indicate the corresponding power factor on the scale), so that the resultant flux produced by the components of ϕ_C and ϕ_D in time phase with ϕ_{AB} becomes parallel to the axis of coil AB, that is, in space phase with ϕ_{AB}.

The importance of the power factor in the operation of electric power systems can hardly be overestimated. Not only are the losses in power transmission and distribution greater for low power factors but the voltage regulation and the types of motors used and other equipment also depend to a large extent on the power factor. The many ways in which the power factor enters into the problem of electric-energy generation, distribution and utilization are discussed in Chaps. X to XXII and XXVIII as essential features in the characteristics of alternators, transformers, motors, regulators, and transmission and distribution systems.

PROBLEMS

1.8. A single-phase induction motor delivers 8.7 hp. to the belt. Efficiency of motor, 91.5 per cent; impressed voltage 240 volts; power factor, 87.5 per cent.

(*a*) Find the kv.a., the kw. or P, the reactive power ${}_xP$, and the current I.

(b) Draw the vector diagram showing E, I, θ, gE, bE, rI and xI.

(c) In rectangular coördinates draw curves for e, i, ei, ge, be, p and ${}_xp$.

2.8. Given a single-phase circuit (equivalent) having two motors operating in parallel, as shown in Fig. 20.8.

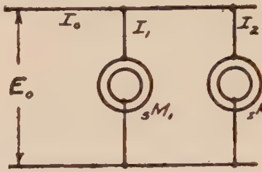


FIG. 20.8.

${}_iM_2$ = an induction motor taking 7.0 kw. at 85 per cent power factor (current lagging).

${}_sM_1$ = an overexcited synchronous motor taking 8.2 kw. at 94 per cent power factor (current leading).

E_0 = 240 volts; f = 60 cycles.

(a) Find: I_1, I_2, I_0, θ_1, θ_2, θ_0, g_1E_0, b_1E_0, g_2E_0, b_2E_0, g_0E_0 and b_0E_0.

(b) Draw vector diagram showing the quantities called for in (a).

3.8. Given the same data as for problem 2.8:

(a) Find: P_1, ${}_xP_1$, P_2, ${}_xP_2$, P_0, ${}_xP_0$, (kv.a.)$_1$, (kv.a.)$_2$, (kv.a.)$_0$.

(b) Draw curves in rectangular coördinates (assuming simple sine wave forms) for e_0, i_1, e_0i_1, g_1e_0, b_1e_0, p_1, ${}_xp_1$, i_2, e_0i_2, g_2e_0, b_2e_0, p_2, ${}_xp_2$, i_0, e_0i_0, g_0e_0, b_0e_0, p_0 and ${}_xp_0$.

CHAPTER IX

VOLTAGE AND CURRENT IN SINGLE-PHASE AND POLYPHASE SYSTEMS

(*a*) **Single-phase Systems.**—Any alternator with a single circuit in the armature has the same current, at any instant, in all the conductors. In distinction from the polyphase alternators having several circuits with currents of different values appearing simultaneously, the single circuit winding is said to produce single-phase currents. The fundamental plan of the wiring diagram for a single-phase alternator is shown in Fig. 1.9. The parts of the conductors passing under the north and south poles are so placed as to produce voltage in the same direction in the circuit at any position. In the diagram, part *A*, the direction of the current is indicated by dots and crosses and the direction of the rotation of the field by an arrow. Part *B* shows the wiring diagram on a plane instead of on the cylindrical surface of the armature. The diagram shows the fundamental arrangement necessary to produce a single-phase current.

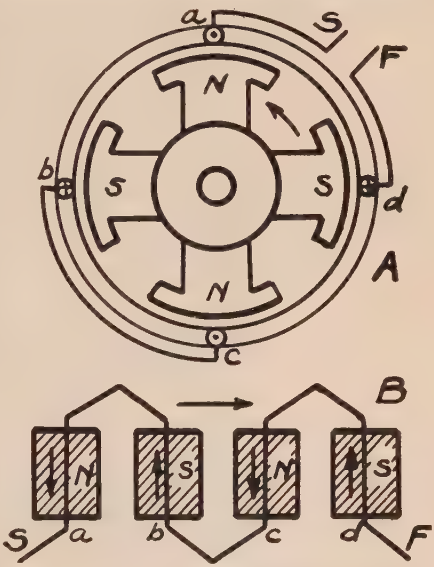


FIG. 1.9.—Fundamental single-phase winding diagram.

In order to produce the required voltage in commercial machines several layers of the simple winding are connected in series, often giving a complex appearance, but the principles involved are essentially the same as for the fundamental arrangement shown in Fig. 1.9. One or more single-phase alternators

connected to a distribution network of any complexity form a single-phase system. The circuits discussed in the preceding chapters are all single-phase and the vector diagrams at the generators give the current- and voltage-phase relations of single-phase alternators supplying the power.

(*b*) **Electrical Degree.**—The voltage and current waves from alternators complete one cycle in 2π radians, or 360°. Hence for one revolution of the generator there are produced as many complete cycles of the voltage and current waves as the alternator has pairs of poles. For a four-pole machine half a revolution of the rotating spider produces a complete voltage wave of 360°. In all cases a distance of two pole-pitches produces a complete voltage wave and for convenience this is called 360 electrical degrees.

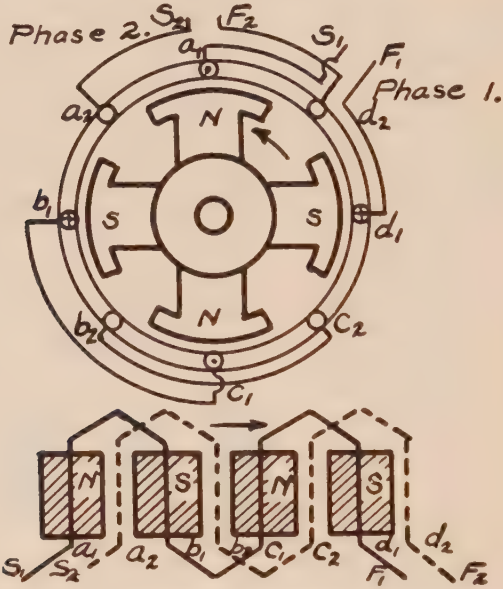


FIG. 2.9.—Fundamental two-phase winding diagram.

(*c*) **Frequency.**—Since a complete wave is produced while the conductor passes over one pair of poles, the number of cycles per revolution is $\frac{p}{2}$ when p is the number of poles.

$$\text{Hence the cycles per second} = \frac{p}{2} \times \frac{\text{r.p.m.}}{60} = f \qquad (1.9)$$

The American Institute of Electrical Engineers has adopted 60 and 25 cycles per sec. as standard frequencies.

(*d*) **Two-phase Systems.**—In a two-phase or quarter-phase alternator, two separate single-phase circuits are placed on the same armature, 90 electrical degrees apart, as illustrated in Fig. 2.9. The generator has four terminals and in a four-wire distribution system the two circuits are kept separate. Since the two circuits are alike and differ only in position on the armature, the resulting voltage waves are of the same shape but differ by 90° or a quarter phase in time position. The voltage waves for the two phases are shown in Fig. 3.9. If the generator be connected to a two-phase circuit with a balanced inductive load, the current in each phase will lag behind the voltage, as shown by the vector diagram in Fig. 4.9.

To save copper in the transmission line, two of the four terminals are connected together in the two-phase, three-wire system.

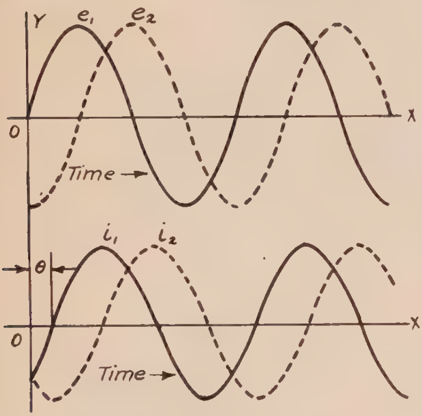


FIG. 3.9.

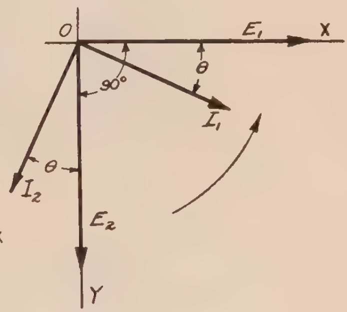


FIG. 4.9.

The circuit diagram and the corresponding voltage- and current-vector diagrams are shown in Fig. 5.9. Since the two circuits are alike and pass through the same magnetic field at the same speed, the effective voltages are equal.

$$E_1 = E_2 \tag{2.9}$$

The resultant voltage between the outside mains A and B is the vector resultant of E_1 and E_2.

$$\dot{E}_3 = \dot{E}_1 + j\dot{E}_2$$

or

$$E_3 = \sqrt{2}E_1 = \sqrt{2}E_2 \tag{3.9}$$

If the two receiving circuits are alike, the current flowing in the main A is equal to the current in main B, or $I_1 = I_2$. The

current in the common return is the vector sum (Fig. 5.9a) of I_1 and I_2.

$$\dot{I}_3 = \dot{I}_1 + j\dot{I}_2 \text{ or } I_3 = \sqrt{2}I_1 \tag{4.9}$$

Let (b) in Fig. 5.9 represent the current and voltage vector diagram corresponding to the connections in (a). If, instead of connecting terminals F_1 and F_2 together for the common return, terminal F_1 be connected to S_2, the voltage vector E_2 would be reversed. The vector diagram for the currents and voltages would then be as shown in (c), Fig. 5.9. Comparing the two

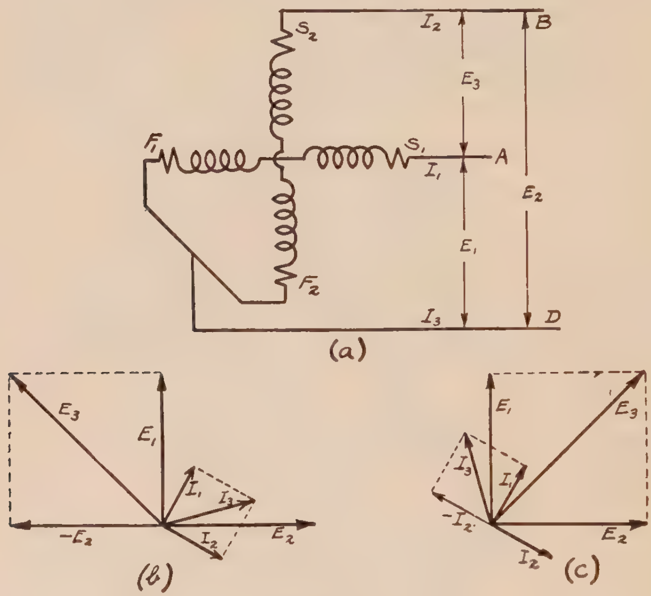


FIG. 5.9.

vector diagrams it is seen that the corresponding currents and voltages are of the same magnitude but differ in time phase by 90°.

(e) **Three-phase Systems.**—The fundamental wiring diagram is shown in Figs. 6.9 and 7.9. A three-phase winding consists of three single-phase windings spaced 120 electrical degrees, so that the voltages generated in them successively reach their maximum 120° apart. The sine voltage and current waves for the three phases are shown in Fig. 8.9 and the vector diagram of the voltages and currents for a balanced three-phase load, having

the same resistance and condensive reactance in each phase, is shown in Fig. 9.9.

(*f*) **The Delta and Y Connections.**—As seen from the circuit diagram in Fig. 7.9 the three-phase winding requires six leads or

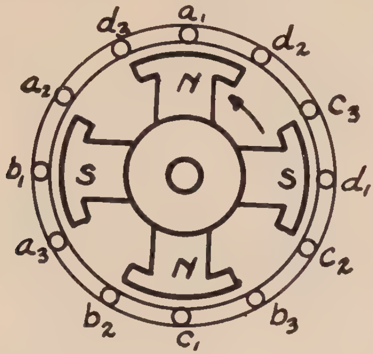


Fig. 6.9.

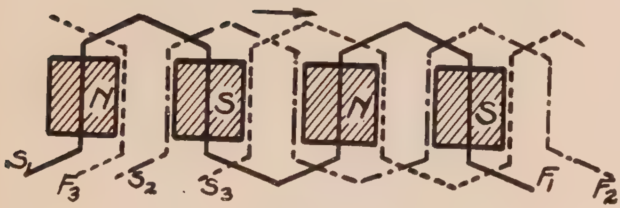


Fig. 7.9.

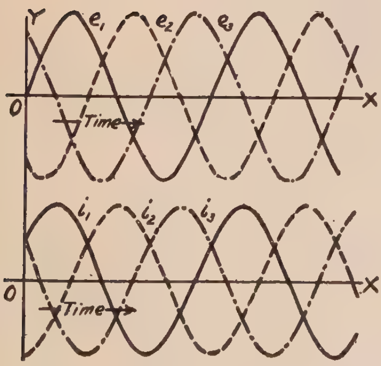


Fig. 8.9.

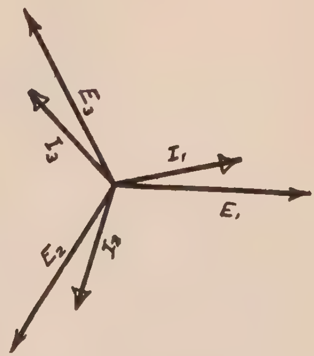


Fig. 9.9.

terminals, two for each phase. It is usual to combine these terminals by pairs so that only three leads are brought out from the alternator to the distribution system. This connection between the three phases is made in two ways. If the three phases be connected as shown by the winding diagram in Fig.

10.9 and the corresponding circuit diagram in Fig. 11.9, it is called a *delta* connection, and is represented by the letter Δ. If the three circuits be connected as shown by the winding diagram in Fig. 12.9 and the corresponding circuit diagram in Fig. 13.9, it is called a *star* or *wye* connection and is represented by the letter *Y*.

With the windings in the armature forming a closed circuit as in the delta connection, it appears likely that circulating currents would flow in the armature. By examining the three voltage

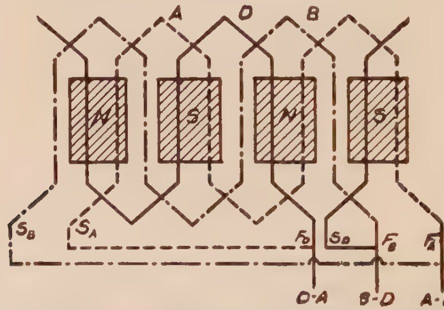


FIG. 10.9.—Fundamental three-phase, delta-connected winding.

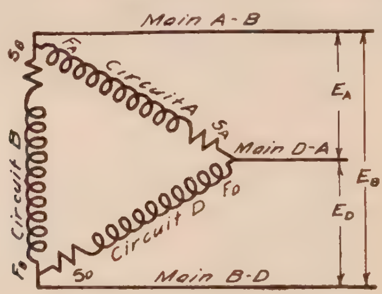


FIG. 11.9.

waves in Fig. 8.9 and the vectors in Fig. 9.9, it is seen that the sum of the voltages is at every instant equal to zero. It should be noted that, while this holds true for the three fundamental sine waves, it is not the case for waves having third or ninth harmonics. The conditions which cause circulating currents are explained in Chap. XXIV, while the discussion in this chapter deals only with simple sine waves. By comparing the three current waves in Fig. 8.9 and the corresponding vectors in Fig. 9.9 it is seen that the sum of the three currents is at any instant equal to zero, and hence the star connection will not change the flow of the currents in either phase. As neither the voltage nor

the current in either phase is affected by making the delta or Y connection, the same diagram expresses the phase relations of the *voltages and the currents in the several phases,* whether the circuit is connected in delta or in star.

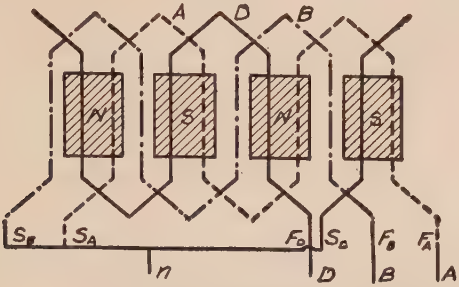


FIG. 12.9.—Fundamental three-phase, Y-connected winding.

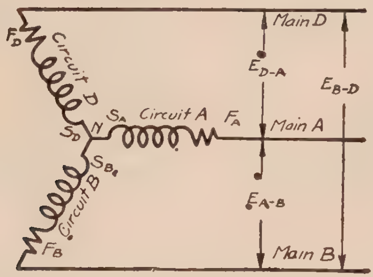


FIG. 13.9.

(*g*) **Delta Connection. Voltage and Current.**—Although in a delta connection the current in each circuit, that is, between S_A and F_A, S_B and F_B, S_D and F_D in Fig. 11.9, is the same as when the circuits were separate, the currents in the mains AB, BD and DA are formed by combining the currents in the corresponding circuits. Thus if the currents in the three circuits be I_A, I_B and I_D the current in each main is the vector difference of the currents in the corresponding circuits. This can be seen by inserting a vibrator of an oscillograph in each phase as indicated in Fig. 14.9. In order that the vibrators in the three circuits shall trace the three currents in the proper sequence, the terminals must be connected in the same direction in the three circuits. Thus in circuit A, α is nearest S_A with λ next to F_A; in circuit B, α is nearest S_B and λ next to F_B; and in circuit D, α is nearest to S_D and λ next to F_D. If, then, the vibrator be inserted in main AB

with α nearest to the junction as in Fig. 14.9, then the component of the current coming from circuit A flows in the same direction through the vibrator as it does in the first position. But the component of the current from circuit B passes through the vibrator in the reverse direction. Hence the current in the main AB is the *vector difference* of the currents in circuits A and B.

$$\dot{I}_{AB} = \dot{I}_A - \dot{I}_B \tag{5.9}$$

Similarly:

$$\dot{I}_{BD} = \dot{I}_B - \dot{I}_D \tag{6.9}$$
$$\dot{I}_{DA} = \dot{I}_D - \dot{I}_A \tag{7.9}$$

The corresponding vector diagram with the rotation in the counterclockwise direction is shown in Fig. 15.9. For the delta

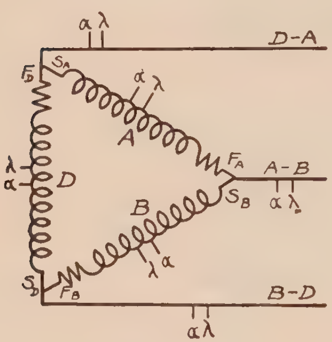


FIG. 14.9.

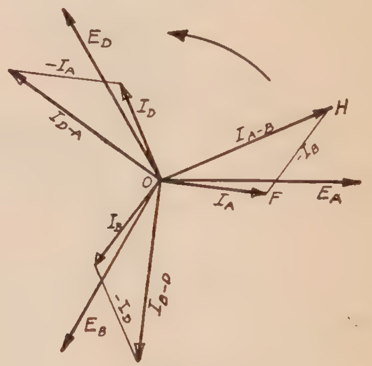


FIG. 15.9.

connection the voltage between the mains is the same as for each circuit and is represented by the same voltage vectors as if the circuits were separate. In equations (5.9) to (7.9) the currents in the mains are expressed in vector notation. To find the absolute value, the phase position as well as the magnitude of the component vectors must be known. Let the three-phase load be balanced so that in both magnitude and phase relation the currents and voltages in the three phases are alike. Or, stated in the form of equations:

$$E_A = E_B = E_D \tag{8.9}$$
$$I_A = I_B = I_D \tag{9.9}$$
$$\theta_A = \theta_B = \theta_D \tag{10.9}$$

Then the triangle *OFH*, Fig. 15.9, is isosceles and the angle *OFH* equals 120°. Hence *OH* equals $\sqrt{3}$ times *OF*, or the current in the main is equal to $\sqrt{3}$ times the current in a circuit.

$$I_{AB} = \sqrt{3}I_A = 1.73I_A \tag{11.9}$$

Similarly:

$$I_{BD} = \sqrt{3}I_B \text{ and } I_{DA} = \sqrt{3}I_D. \tag{12.9}$$

(*h*) **Star Connection. Voltage and Current.**—In the star connection the same current that flows in each circuit must pass into the corresponding main, as only one circuit is connected to each main. The voltage between each pair of mains is, however, the resultant of voltages in the two circuits in series between the

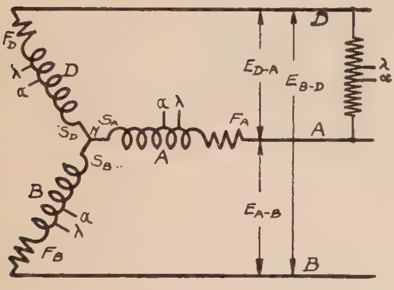


FIG. 16.9.

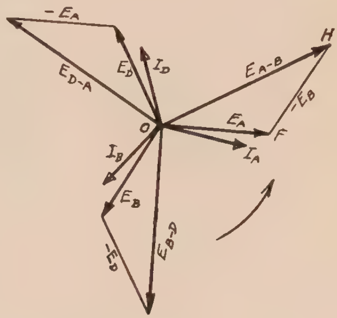


FIG. 17.9.

mains. By making connections to oscillographic vibrators similar to those explained for the currents in the preceding paragraph, it is readily seen that the voltage between each pair of mains is the *vector difference* of the voltages in the corresponding circuits. Hence for balanced loads the voltage and current relations are as shown in Fig. 17.9. Expressed in the form of equations:

$$I_A = I_B = I_D \tag{13.9}$$

$$E_A = E_B = E_D \tag{14.9}$$

$$\dot{E}_{AB} = \dot{E}_A - \dot{E}_B;\ \dot{E}_{BD} = \dot{E}_B - \dot{E}_D;\ \dot{E}_{DA} = \dot{E}_D - \dot{E}_A \tag{15.9}$$

The triangle *OFH* in Fig. 17.9 is isosceles and the angle *OFH* is equal to 120°.

Hence:

$$E_{AB} = \sqrt{3}E_A = 1.73E_A \tag{16.9}$$

Similarly:

$$E_{BD} = \sqrt{3}E_B \text{ and } E_{DA} = \sqrt{3}E_D \tag{17.9}$$

(*i*) **Equivalent or Topographic Vector Diagrams.**—To show directly the time-phase relations of the several voltages and currents in the three-phase systems, all the vectors are drawn through the origin, as in Figs. 15.9 and 17.9. It is often convenient to draw the vectors so as to indicate their position in the circuit diagram, forming triangles or polygons in which the direction and length of the sides represent the vector values. Thus in Fig. 17.9 the voltages between the mains may be moved into parallel positions so as to form the triangle shown in Fig. 18.9.

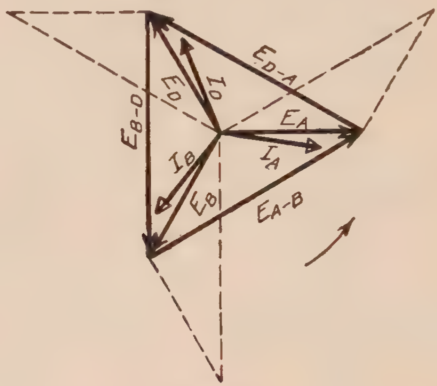


Fig. 18.9.

The sides of the triangle represent the voltages between the mains while the lines joining the vertices to the center give the voltages in the circuits. The simplicity of this form of vector diagram is of considerable importance when dealing with more complex systems, as may be seen from Figs. 5.16, 6.16 and 17.16 to 23.16 inclusive.

(*j*) **Sequence of Phases.**—In the laboratory or in commercial work it is often necessary to determine the time sequence of the several phases in a polyphase system. Thus in the three-phase system indicated by the terminals *A*, *B* and *D* in Fig. 19.9 the sequence of the phases may be either *AB*, *BD*, *DA* or *AB*, *DA*, *BD*.

A convenient voltmeter test for determining the direction of rotation in three-phase systems may be made by means of the simple circuit shown in Fig. 19.9. A resistance *R* and a condensive reactance $_cx$, approximately equal, are connected in series across phase *AB*. From the point *Q*, between *R* and $_cx$, a

voltmeter is connected to the line D. It is evident that the voltage-phase sequence will be either AB, BD, DA as represented by the vector diagram in Fig. 20.9, or AB, DA, BD as in Fig. 21.9.

In order to make the application of the test data more evident it is desirable to change Fig. 20.9 into a topographic vector diagram. Let AB be used as the reference vector. It should be noted that the topographic vector diagram for Fig. 20.9 may *seemingly* be drawn in two ways, both of which represent the three

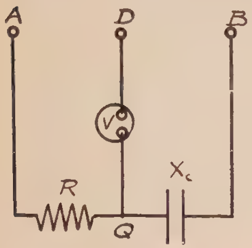


FIG. 19.9.

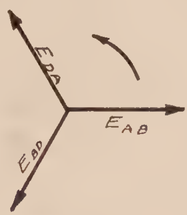


FIG. 20.9.

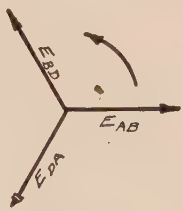


FIG. 21.9.

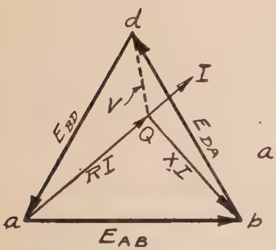


FIG. 22.9.

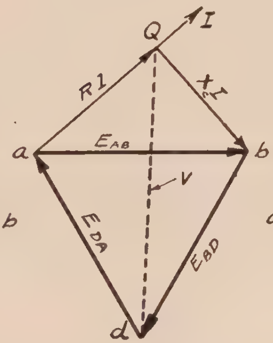


FIG. 23.9.

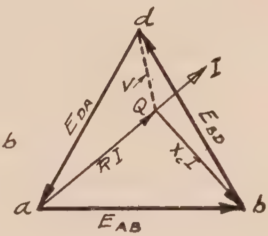


FIG. 24.9.

line voltages, but only one satisfies the voltage values for all parts of the circuit, Fig. 19.9. To make this clear, let a topographic diagram be constructed as in Fig. 22.9. The current I, in phase AB, will lead the voltage E_{AB}, as drawn in Fig. 22.9. The RI voltage drop, in phase with I, is represented by aQ, and the ${}_cxI$ drop by Qb; angle aQb being a right angle. Connect Qd. From the construction of Fig. 22.9 it is apparent that Qd represents the voltage indicated by voltmeter V in Fig. 19.9. However, the voltage vectors do not conform to the circuit in Fig. 19.9, as $R\dot{I} + \dot{V} = \dot{E}_{BD}$, in Fig. 22.9, and not $\dot{E}_{DA}$, as shown in Fig. 19.9.

Similarly, $_cx\dot{I} + \dot{V} = \dot{E}_{DA}$, in Fig. 22.9, and not $\dot{E}_{BD}$, as required by Fig. 19.9. The other possible topographic diagram for Fig. 20.9 is shown in Fig. 23.9, which conforms to all voltage relations in Fig. 19.9. That is, in both Figs. 19.9 and 23.9: $R\dot{I} + \dot{V} = \dot{E}_{DA}$ and $_cx\dot{I} + \dot{V} = \dot{E}_{BD}$. Since Qd is larger than ab the voltage V must be greater than E_{AB}. Hence, if the voltage indicated by the voltmeter V is *greater than the line voltage* the phase sequence is AB, BD, DA. If the voltmeter reading V is *less than the line voltage* the phase sequence is AB, DA, BD, as shown in Fig. 21.9. To prove this statement construct the corresponding topographic vector diagram, as shown in Fig. 24.9. Let E_{AB} be drawn as the reference vector and, in the same manner as in Fig. 22.9, construct I, RI, $_cxI$ and connect Qd. It is evident that the voltage vectors in Fig. 24.9 correspond to the circuit diagram in Fig. 19.9. That is, in both figures: $R\dot{I} + \dot{V} = \dot{E}_{DA}$, and $x\dot{I} + \dot{V} = E_{BD}$. Since Qd is less than ab the voltage indicated by voltmeter V must be less than the line voltage. Hence, the test criterion:

For phase sequence AB, BD, DA, *voltage V is greater than line voltage*, as shown in Figs. 20.9 and 23.9.

For phase sequence AB, DA, BD, *voltage V is less than line voltage*, as shown in Figs. 21.9 and 24.9.

(*k*) **Commercial Polyphase Systems.**—Besides the two-phase and three-phase systems, four-phase, six-phase and twelve-phase systems are in commercial use, although to a comparatively limited extent. For machines like the rotary converter a higher efficiency may be obtained in six- or twelve-phase designs than for the corresponding two- or three-phase types (see Chap. XVI).

PROBLEMS

1.9. Given a balanced two-phase, three-wire system (circuit diagram in Fig. 5.9). Voltage in each phase = 120 volts; current in lines A or B = 50 amp.; $\theta = 30°$ (current leading). A leads B.

(*a*) Find the current in the common wire D.

(*b*) Find the voltage between the outside wires A and B.

(*c*) Draw the vector diagram.

(*d*) Draw curves for the voltage and currents in rectangular coördinates.

2.9. Given a balanced, three-phase, delta-connected system (circuit diagram in Fig. 11.9). Voltage in each phase = 220 volts; current in each circuit = 35 amp.; $\theta = 40°$ (current lagging).

(*a*) Find the current in each main.

(*b*) Draw the vector diagram showing the voltages and currents in the mains and circuits for the three phases.

(*c*) Draw in rectangular coördinates the corresponding curves for the voltages and currents.

3.9. Given a balanced, three-phase, star-connected system (circuit diagram in Fig. 13.9). Voltage in each circuit = 125 volts; current in each main = 40 amp.; θ = 20° (current leading).

(*a*) Find the voltages between the mains.

(*b*) Draw the vector diagram showing the currents and voltages in the circuits and mains for the three phases.

(*c*) Draw curves in rectangular coördinates for the currents and voltages.

4.9. Given a balanced, three-phase, delta-connected system. Current in each main = 60 amp.; voltage between mains = 220 volts; θ = 30° (current lagging). Phase sequence *ABD*.

(*a*) Find the current in each circuit.

(*b*) Draw vector diagram for the currents and voltages.

5.9. Given a balanced, three-phase, star-connected system. Current in each main = 40 amp.; voltage between mains = 120 volts; θ = 70° (current leading). Phase sequence *ABD*.

(*a*) Find the voltage in each circuit.

(*b*) Draw vector diagram showing the currents and voltages.

6.9. An unsymmetrical, star-connected, three-phase load has the following line voltages:

$$\dot{E}_{1-2} = 100$$
$$\dot{E}_{2-3} = -45 - j90$$
$$\dot{E}_{3-1} = -55 + j90.$$

The phase impedances are:

$$z_1 = 10$$
$$z_2 = 2 - j7$$
$$z_3 = 2 + j9 \text{ ohms.}$$

Solve for vector values of phase voltages and line currents. Give a complete topographic diagram of all voltages. Show the current vectors on the same plot.

CHAPTER X

POWER IN BALANCED TWO-PHASE AND THREE-PHASE SYSTEMS

The electric power in single-phase systems was discussed in Chap. VIII. It was shown that the expression for the instantaneous power for sine waves is:

$$ei = {}^{M}E\,{}^{M}I \sin(\omega t + \gamma_1) \sin(\omega t + \gamma_2) \tag{1.10}$$

$$= EI[\cos(\gamma_1 - \gamma_2) - \cos(2\omega t + \gamma_1 + \gamma_2)] \tag{2.10}$$

The equation shows that the power curve is also a sine wave but pulsates with double the frequency of the voltage or current. It was also proved that the average power for single-phase circuits with sine-wave current and voltage is:

$$P = EI \cos\theta \tag{3.10}$$

where θ is the difference in phase of the current and voltage.

Similarly, the reactive power traveling to-and-fro in the circuit is:

$${}_{x}P = EI \sin\theta \tag{4.10}$$

(*a*) **Two-phase Circuit.**—A two-phase circuit consists of two single-phase circuits differing by 90 electrical degrees in space position and hence producing voltage and current waves 90° out of phase. Let the load be balanced and the current and voltage waves be simple harmonic.

For the first phase let the instantaneous values be:

$$i_1 = {}^{M}I \sin(\omega t) = \sqrt{2}I \sin(\omega t) \tag{5.10}$$

$$e_1 = {}^{M}E \sin(\omega t + \theta) = \sqrt{2}E \sin(\omega t + \theta) \tag{6.10}$$

$$p_1 = e_1 i_1 = 2EI \sin^2(\omega t) \cos\theta + 2EI \sin(\omega t) \cos(\omega t) \sin\theta \tag{7.10}$$

Since the second-phase voltage is 90 electrical time degrees from the first, the equations for the simultaneous instantaneous values for the second phase are:

$$i_2 = {}^{M}I \cos(\omega t) = \sqrt{2}I \cos(\omega t) \tag{8.10}$$

$$e_2 = {}^{M}E \cos(\omega t + \theta) = \sqrt{2}E \cos(\omega t + \theta) \tag{9.10}$$

$$p_2 = e_2 i_2 = 2EI \cos^2(\omega t) \cos\theta - 2EI \cos(\omega t) \sin(\omega t) \sin\theta \tag{10.10}$$

The instantaneous power for both circuits is therefore:

$$p = p_1 + p_2 = 2EI[\sin^2(\omega t) + \cos^2(\omega t)]\cos\theta = 2EI\cos\theta \quad (11.10)$$

The instantaneous power in a balanced two-phase circuit is thus found to be equal to twice the average power of one of the circuits.

If the instantaneous power be constant it must be equal to the average power. Hence for a balanced two-phase circuit and simple harmonic current and voltage:

$$P = 2EI\cos\theta \quad (12.10)$$

Power is measured by wattmeters. In single-phase circuits the current from one of the mains passes through the wattmeter and the voltage is taken between the mains. A four-wire two-

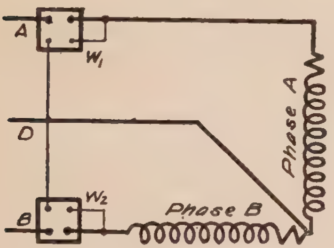


FIG. 1.10.—Two wattmeters, two-phase three-wire system.

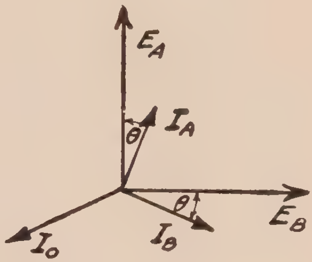


FIG. 2.10.—Vector diagram for Fig. 1.10.

phase circuit is merely two single-phase circuits and hence the power may be measured by one wattmeter in each circuit. In a three-wire, two-phase circuit the current connection on the wattmeter is made on the outside mains while the voltage is taken from the outside main to the common wire. The wiring diagram is shown in Fig. 1.10 where W_1 and W_2 represent the two wattmeters, and the corresponding vector diagram in Fig. 2.10.

The so-called *tangent method* is sometimes found convenient in determining the power factor in balanced, two-phase circuits. One wattmeter is used. The current coil is connected in the same way as for measuring power in single-phase circuits. With the voltage coil across the wires in the same circuit the wattmeter measures true watts.

$$P_1 = E_1 I_1 \cos\theta \quad (13.10)$$

Without changing the current connection, transfer the voltage coil to the second phase. The reading of the wattmeter is then:

$$P_2 = E_2 I_1 \sin \theta \tag{14.10}$$

Usually $E_1 = E_2$, and dividing the second reading by the first:

$$\frac{P_2}{P_1} = \tan \theta, \text{ or } \theta = \tan^{-1}\frac{P_2}{P_1} \tag{15.10}$$

Hence the power factor, $\cos \theta$, in a two-phase circuit can be found by the use of one wattmeter.

(*b*) **Three-phase Circuit, Delta Connection.**—Power in three-phase circuits is usually measured by two wattmeters connected as indicated in Fig. 3.10. In order to prove that the two wattmeters at any instant will measure the total power in the three circuits, it is desirable to assume some directional notation. The arrows in both the circuits and the mains correspond, in a way, to the three-phase vector diagrams. It is immaterial which direction is assumed, only, if the current arrow in the circuit be taken counterclockwise in one circuit, the arrows in the remaining two circuits must be in the same direction. Likewise the arrows for the currents in the mains may be taken from left to right as in the diagram, or in the reverse order, but the three arrows in the three mains must be in the same direction. It is evident that the arrangement of the arrows should be such that when the armature turns to a second position in which main B occupies the place previously held by main A, the arrows in B and in the adjoining circuits must then be the same as were those of main A and the adjoining circuits. The arrows, therefore, represent directional conditions at any one position in space but not for the same time instant. Like the vector diagram, the arrows give the relative relation for all positions.

With the direction of the arrows as given in Fig. 3.10 the equations for the instantaneous values of the currents in the mains are:

$$i_A = i_1 - i_2 \tag{16.10}$$
$$i_B = i_2 - i_3 \tag{17.10}$$
$$i_D = i_3 - i_1 \tag{18.10}$$

The power indicated by the wattmeter is the product of the current in the main by the corresponding voltage.

For wattmeter W_1:

$$p_1 = e_1 i_A = e_1(i_1 - i_2) \tag{19.10}$$

For wattmeter W_2:

$$p_2 = (-e_3)i_B = -e_3(i_2 - i_3) \tag{20.10}$$

The instantaneous power in both wattmeters:

$$p = p_1 + p_2 = e_1(i_1 - i_2) - e_3(i_2 - i_3)$$
$$= e_1i_1 + (-e_3 - e_1)i_2 + e_3i_3 \tag{21.10}$$
$$e_2 = -e_3 - e_1 \tag{22.10}$$

Hence,

$$p = e_1i_1 + e_2i_2 + e_3i_3 \tag{23.10}$$

The product e_1i_1 is the instantaneous power in circuit 1, e_2i_2 in circuit 2 and e_3i_3 in circuit 3. *Therefore the two wattmeters connected in this manner measure the total instantaneous power in the three circuits.*

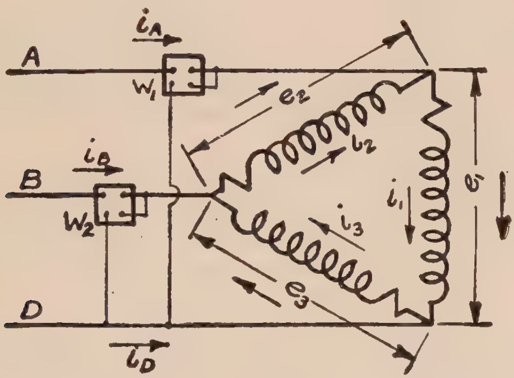


FIG. 3.10.—Two wattmeters, three-phase, delta connection.

The value of the average power will be derived both analytically and by the aid of vector diagrams.

First Method.—For sine waves with a balanced inductive load causing a lagging current, let the voltage and current equations be:

For phase 1:

$$e_1 = {}^{M}E_1 \sin(\omega t) = \sqrt{2}E_1 \sin(\omega t) \tag{24.10}$$

$$i_1 = {}^{M}I_1 \sin(\omega t - \theta) = \sqrt{2}I_1 [\sin(\omega t)\cos\theta - \sin\theta\cos(\omega t)] \tag{25.10}$$

For phase 2:

$$e_2 = {}^{M}E_2 \sin(\omega t - 120°)$$
$$= \sqrt{2}E_2\left[-\frac{1}{2}\sin(\omega t) - \frac{\sqrt{3}}{2}\cos(\omega t)\right] \tag{26.10}$$

$$i_2 = {}^{M}I_2 \sin(\omega t - 120° - \theta) = \sqrt{2}I_2\Big[-\frac{1}{2}\sin(\omega t)\cos\theta - \frac{\sqrt{3}}{2}$$
$$\cos(\omega t)\cos\theta + \frac{1}{2}\cos(\omega t)\sin\theta - \frac{\sqrt{3}}{2}\sin(\omega t)\sin\theta\Big] \qquad (27.10)$$

For phase 3:

$$e_3 = {}^{M}E_3 \sin(\omega t - 240°)$$
$$= \sqrt{2}E_3\Big[-\frac{1}{2}\sin(\omega t) + \frac{\sqrt{3}}{2}\cos(\omega t)\Big] \qquad (28.10)$$
$$i_3 = {}^{M}I_3 \sin(\omega t - 240° - \theta)$$
$$= \sqrt{2}I_3\Big[-\frac{1}{2}\sin(\omega t)\cos\theta + \frac{\sqrt{3}}{2}\cos(\omega t)\cos\theta$$
$$+\frac{1}{2}\cos(\omega t)\sin\theta + \frac{\sqrt{3}}{2}\sin(\omega t)\sin\theta\Big] \qquad (29.10)$$

For a balanced load:

$$E_1 = E_2 = E_3 = E_c \text{ and } I_1 = I_2 = I_3 = I_c$$

where E_c and I_c denote the effective voltage and current, respectively, *in each circuit.*

The instantaneous power in the three circuits is the sum of the products of the respective instantaneous voltages and currents. Taking the sum of products e_1i_1, e_2i_2 and e_3i_3 in equations (24.10) to (29.10) and reducing to the simplest form gives equation (30.10).

$$p = e_1i_1 + e_2i_2 + e_3i_3 = 3E_cI_c\cos\theta \qquad (30.10)$$

Equation (30.10) shows that the instantaneous power is a constant and hence equal to the average power.

$$p = P = 3E_cI_c\cos\theta \qquad (31.10)$$
$$I = \sqrt{3}I_c \text{ and } E = E_c \qquad (32.10)$$

Since the current in the mains I is equal to $\sqrt{3}$ times the current in the circuit I_c, and the voltage between the mains E equals the voltage in the circuit E_c, the power in a three-phase circuit expressed in terms of the current in the mains, the voltage between the mains and the power factor is:

$$P = \sqrt{3}EI\cos\theta \qquad (33.10)$$

Second Method.—In Fig. 4.10 is shown the vector diagram corresponding to the circuit diagram in Fig. 3.10. In the circuits the currents lag $\theta°$ behind the respective voltages and are represented by the vectors I_1, I_2 and I_3. The currents in the mains are the vector differences of the circuit vectors and denoted by

…he line current, and the coil voltage $E(\text{coil}) = \frac{E(\text{line})}{\sqrt{3}}$; Sub. in the equation

$P = \frac{3}{\sqrt{3}} E(\text{line})\, I\,\text{coil} = \sqrt{3}\, E(\text{line})\, I(\text{line})$

I_A, I_B and I_D in the diagram. The currents in the mains differ by 30° in phase position from the currents in the circuits and by the factor $\sqrt{3}$ in magnitude. The power indicated by wattmeter W_1, Figs. 3.10 and 4.10, is the product of the current in main A, the voltage between A and D and the cosine of the time-phase angle.

Power in wattmeter W_1:

$$P_1 = E_1 I_A \cos (30° - \theta) \tag{34.10}$$

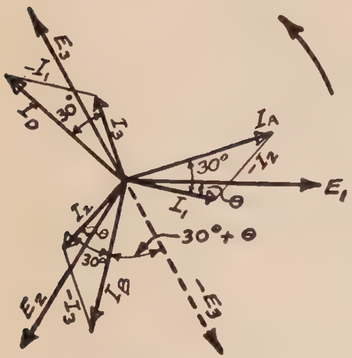


FIG. 4.10.—Vector diagram for Fig. 3.10.

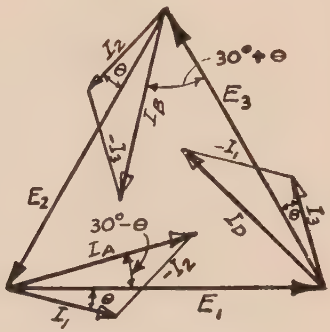


FIG. 5.10.—Topographic vector diagram for Fig. 3.10.

Similarly, the power in wattmeter W_2 is the product of the current in main B by the voltage between mains B and D and the cosine of the time-phase angle. The time-phase angle between I_B and E_3 is $(180° - 30° - \theta)$; and the $\cos (180° - 30° - \theta) = -\cos (30° + \theta)$. The voltage connection, however, is made in the reverse order from that of W_1, and hence this reverses the sign. Therefore, the power flows from the generator when $\cos (30° + \theta)$ is positive.

Power in wattmeter W_2:

$$P_2 = E_3 I_B \cos (30° + \theta) \tag{35.10}$$

In a balanced circuit

$$E_1 = E_2 = E_3 \text{ and } I_A = I_B = I_D \tag{36.10}$$

Hence the total power:

$$P = P_1 + P_2 = EI[\cos (30° - \theta) + \cos (30° + \theta)] \tag{37.10}$$
$$= 2EI \cos 30° \cos \theta = \sqrt{3}EI \cos \theta \tag{38.10}$$

$Kv.(a) = \frac{\sqrt{3}}{1,000} E_{line} I_{line}$

(*c*) **Three-phase, *Y* Connection.**—In Fig. 6.10 is shown a circuit diagram of a *Y*-connected, three-phase circuit with directional arrows symmetrically arranged as in Fig. 3.10 for the Δ connection. The corresponding vector diagram is shown in Fig. 7.10. It is seen directly from Fig. 6.10 that the instantaneous power in wattmeter W_1 is:

$$p_1 = (-e_D)i_1 = (e_1 - e_3)i_1 \tag{39.10}$$

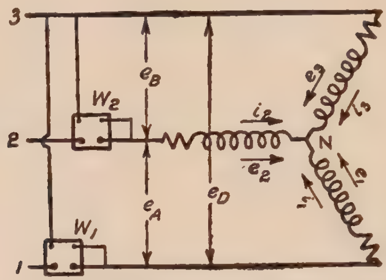


FIG. 6.10.—Two wattmeters, three-phase, *Y*-connection.

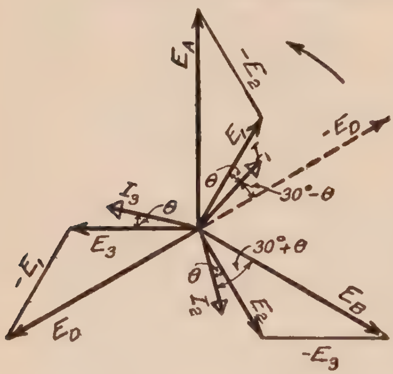


FIG. 7.10.—Vector diagram for Fig. 6.10.

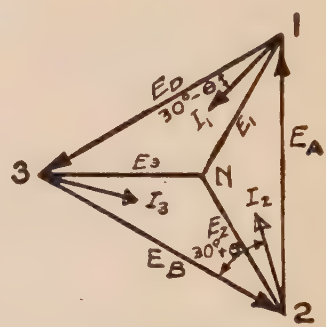


FIG. 8.10.—Topographic vector diagram for Fig. 6.10.

and for wattmeter W_2:

$$p_2 = e_B i_2 = (e_2 - e_3)i_2 \tag{40.10}$$

Hence the total instantaneous power in the wattmeters:

$$p = p_1 + p_2 = e_2 i_2 - e_3 i_2 + e_1 i_1 - e_3 i_1 \tag{41.10}$$

But

$$i_3 = -i_2 - i_1$$

and hence,

$$p = e_1 i_1 + e_2 i_2 + e_3 i_3 \tag{42.10}$$

Thus the two wattmeters together indicate the total power in the three-phase, *Y*-connected circuit at any instant.

The average power and the power factor in the Y-connected system are found in the same manner as for the Δ connection. Referring to the vector diagram in Fig. 7.10.

The power in wattmeter W_1 is:

θ = P.F. of coils.

$$P_1 = E_D I_1 \cos (30° - \theta) \tag{43.10}$$

For wattmeter W_2 the power is:

$$P_2 = E_B I_2 \cos (30° + \theta) \tag{44.10}$$

With a balanced circuit:

$E_A = E_B = E_D = E$ and $I_1 = I_2 = I_3 = I$ in effective values.

The total power is therefore:

$$P = P_1 + P_2 = 2EI \cos \theta \cos 30° = \sqrt{3}EI \cos \theta \tag{45.10}$$

(*d*) **Power Factor in Three-phase Systems.**—Several methods are used for determining power-factor in three-phase systems.

1. Power factor meters are used that indicate directly the power factor in the system or section to which the instrument is connected.

2. For both delta- and star-connected, balanced circuits the power factor can be computed from the two wattmeter readings. From equations (34.10) and (35.10) for the delta connection and equations (43.10) and (44.10) for star-connected circuits:

$$P_1 + P_2 = 2EI \cos \theta \cos 30° \tag{46.10}$$

$$P_1 - P_2 = 2EI \sin \theta \sin 30° \tag{47.10}$$

$$\tan \theta = \sqrt{3}\frac{P_1 - P_2}{P_1 + P_2} \tag{48.10}$$

$$\theta = \tan^{-1} \sqrt{3}\frac{P_1 - P_2}{P_1 + P_2} \tag{49.10}$$

$$\cos \theta = \text{the power factor}$$

3. In Fig. 9.10 is shown a curve whose abscissæ are the ratios of the two wattmeter readings, with the ordinates representing the corresponding values of the power factor, $\cos \theta$. Three points of special interest should be noted. First, if the two wattmeters read alike, $P_1 = P_2$ and hence $\theta = 0$ and $\cos \theta = 1$; that is, the circuit has unity power factor. Second, if wattmeter P_1 reads 0 then $\cos (30 + \theta) = 0$. Hence $\theta = 60,°$ and $\cos \theta = 0.5$; that is, a power factor of 50 per cent. Third, if the reading of P_1 becomes negative, then θ is greater than 60° and, therefore, the total power is equal to the difference (arithmetic) of the two readings.

In order to obtain the power factor from wattmeter readings it is important to know that the connections of the wattmeters are

$P_1 - P_2 = E_c I_c [\cos(30° - \theta) - \cos(30° + \theta)]$

in the correct sequence. For currents lagging or leading more than 60° the connections of one of the wattmeters must be reversed in order to obtain positive readings on both instruments. To determine whether the readings are both positive, so that the power transmitted is the sum of P_1 and P_2, or, if one is positive and the other negative, so that the actual power delivered is the difference of P_1 and P_2, several methods may prove useful.

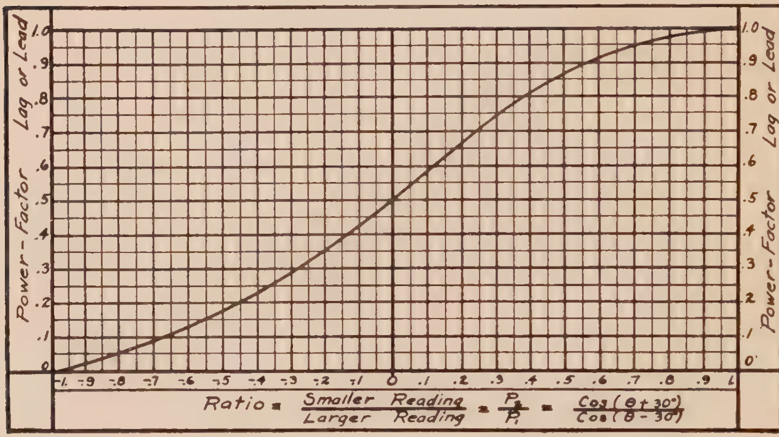


FIG. 9.10.—Curve showing the relation of the power factor to the ratio of the two wattmeter readings in balanced, three-phase circuits.

First: Suppose it is required to determine the power factor of an induction motor. The instruments are usually connected to the three-phase system, as indicated in Fig. 3.10 or Fig. 6.10, and in such order as to give positive readings on both wattmeters. If, on increasing the load on the motor, the readings on both wattmeters increase, the connections for the instruments are both positive and the total power $P = P_1 + P_2$. However, if on increasing the load the reading for P_1 increases while P_2 decreases, then the connections for P_2 are reversed and the total power $P = P_1 - P_2$.

Second: If, in testing a motor, it is permissible to open one of the leads and let the machine operate for a few moments on one phase, the check on the wattmeter connections may be made as follows: The two wattmeters are connected to the three-phase load as in Fig. 3.10 or Fig. 6.10, and in such order as to give positive readings on both instruments. If on opening line A,

Fig. 3.10, or line 1, Fig. 6.10, wattmeter W_2 gives a positive reading and, similarly, if on opening line B, Fig. 3.10, or line 2, Fig. 6.10, wattmeter W_1 gives a positive reading, then the two wattmeters were properly connected so that the total power $P = P_1 + P_2$. However, if reversal of connections for W_2 are found necessary when operating single phase, then the total power for the three-phase load is equal to the difference of P_1 and P_2.

Third: The relation of the power factor to the wattmeter readings, and whether the total power is equal to the sum or the difference of the two wattmeter readings, is illustrated for all possible power factors in Fig. 10.10.

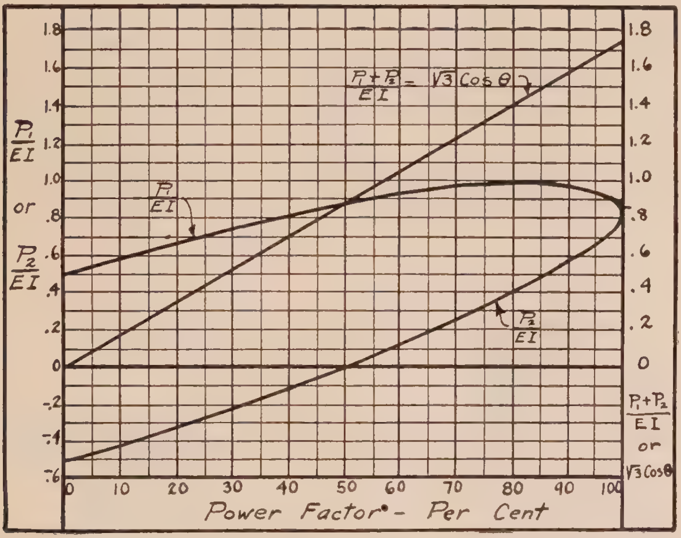


FIG. 10.10.—Curve showing the relation of the power factor to the ratio of wattmeter readings in balanced, three-phase circuits.

From the figure it is evident, that for power factors greater than 0.5, P_2 is positive and the total power $P = P_1 + P_2$. For power factors less than 0.5, the readings for P_2 are negative and the total power $P = P_1 - P_2$.

The following method[1] for checking recorded test data, as to whether the total power should be the sum or the difference of each set of wattmeter readings, may be found useful.

[1] HUMPHRIES, H. K., *J. Am. Inst. Elec. Eng.*, p. 430, May, 1926.

From equations (34.10) and (35.10):

$$P_1 + P_2 = \sqrt{3}EI \cos\theta \qquad (50.10)$$
$$P_1 - P_2 = EI \sin\theta \qquad (51.10)$$

Let

VA = volt amperes in three-phase circuit.

$$VA = \sqrt{3}EI \qquad (52.10)$$
$$\overline{VA}^2 = 3E^2I^2 = 3E^2I^2(\cos^2\theta + \sin^2\theta) \qquad (53.10)$$
$$= (P_1 + P_2)^2 + 3(P_1 - P_2)^2 \qquad (54.10)$$
$$3E^2I^2 = 4[P_1^2 + P_2^2 - P_1P_2] \qquad (55.10)$$

It is evident that for P_2 positive, equation (55.10) will give a different numerical value for $\overline{VA}^2$ than if P_2 is negative. The value of P_2 which satisfies the equation must be the true reading. Hence, equation (55.10) may be used as a criterion to determine whether P_2 was positive or negative. It should be noted that the above equations are true only on the assumption of simple sine-wave voltages and currents and for balanced load.

$\sqrt{3}(W_1 - W_2)$

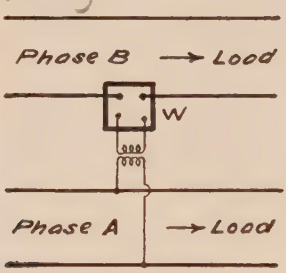


Fig. 11.10.

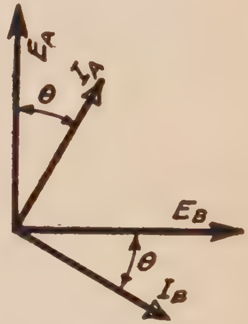


Fig. 12.10.

If neither $+P_2$ nor $-P_2$ in equation (55.10) checks the square of the observed volt-amperes this may be taken as an indication and a measure of, either,

a. Error in experimental data,
b. Unbalanced load, or
c. Harmonics in voltage and current waves.

Equation (55.10) will be found very useful as a means for checking the correctness of voltmeter, ammeter and wattmeter readings, taken simultaneously, on three-phase circuits. For

power in circuits with distorted voltage and current waves, see Chap. XXIV, and for unbalanced load, Chap. XXVI.

(*e*) **Reactive Power.**—If the power delivered and the power factor are known, the reactive power which travels to-and-fro in the circuit may be determined.

$$ {}_xP = \sqrt{3}EI \sin \theta = P \tan \theta = \sqrt{3}(P_1 - P_2) \quad (56.10) $$

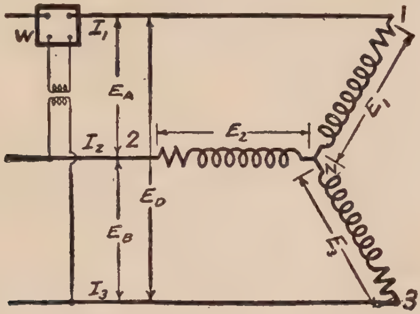


FIG. 13.10.

It is, however, sometimes more convenient to obtain the reactive power directly by means of a wattmeter. In a balanced, two-

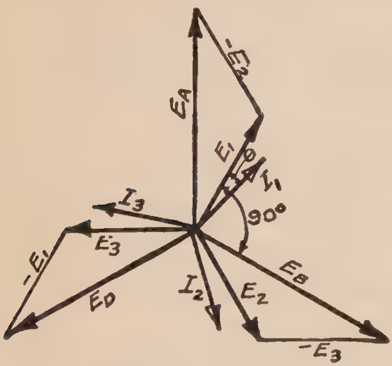


FIG. 14.10.—Vector diagram for Fig. 13.10.

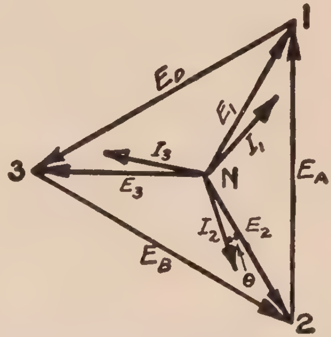


FIG. 15.10.—Topographic vector diagram for Fig. 13.10.

phase system the reactive power may be measured by a wattmeter if the instrument is connected to the circuits as in Fig. 11.10. The potential transformer is inserted to protect the wattmeter from short-circuit between the current and voltage coils in the instrument. Since the current through the wattmeter comes from phase B and the voltage from phase A, it is evident from the corresponding vector diagram in Fig. 12.10 that the wattmeter

reading is given by equation (57.10) and represents the reactive power in one phase.

$$E_A I_B \cos (90° - \theta) = E_A I_B \sin \theta = {}_xP_2 \qquad (57.10)$$

$2\,{}_xP_2$ = reactive power in the two-phase circuit when ${}_xP_2$ is the wattmeter reading. Similarly, the reactive power in a balanced, three-phase circuit is measured directly by a wattmeter connected as in Fig. 13.10. For a balanced load, either star- or delta-connected and for any power factor, the wattmeter readings are proportional to the reactive power in the three-phase system. From Fig. 14.10 or Fig. 15.10:

$$E_B I_1 \cos (90° - \theta) = E_B I_1 \sin \theta = {}_xP_3 \qquad (58.10)$$

$\sqrt{3}\,{}_xP_3 = \sqrt{3}E_B I_1 \sin \theta$ = total reactive power in a balanced, three-phase circuit when ${}_xP_3$ is the wattmeter reading.

PROBLEMS

1.10. Given a three-phase, delta-connected, balanced system with an inductive load. Voltage (between mains) $E = 230$ volts; $P_2 = 3.8$ kw.; $P_1 = 8.4$ kw. Circuit diagram as in Fig. 3.10.

(*a*) Find the power factor.

(*b*) Find the current in the mains and the current in each circuit.

(*c*) Draw vector diagram showing the current and voltage in the wattmeters and the power-factor angle.

2.10. Given a three-phase, star-connected, balanced system with a condensive load. E (between mains) = 230 volts; $P_1 = 10.5$ kw.; $P_2 = 4.8$ kw. Circuit diagram as in Fig. 6.10.

(*a*) Find the power factor.

(*b*) Find the current in the mains.

(*c*) Draw the corresponding vector diagram.

3.10. With the same system and load as in problem 1.10 let a wattmeter be connected as in Fig. 13.10.

(*a*) What is the reading of the wattmeter?

(*b*) Draw the vector diagram showing the current and voltage in the wattmeter.

4.10. With the same system and load as in problem 2.10, let a wattmeter be connected as in Fig. 13.10.

(*a*) Find the wattmeter reading.

(*b*) Draw the vector diagram showing the current and voltage in the wattmeter.

5.10. A three-phase induction motor takes 48.4 kw. at 87.5 per cent power factor. The power is measured by two wattmeters, connected as in Fig. 3.10. What are the readings indicated by W_1 and W_2? If the voltage between mains is 448 volts, what current flows through each wattmeter?

6.10. A three-phase induction motor takes 28.5 kw. at 40 per cent power factor while starting. Voltage between mains: 238 volts. Wattmeter connections as in Fig. 6.10.

(a) Find the readings of the two wattmeters.

(b) Find the currents in the mains.

(c) Draw vector diagram and indicate the currents and voltage passing through wattmeters W_1 and W_2.

7.10. In a three-phase, balanced system the load consists of a three-phase induction motor taking 45.8 kw. at 73 per cent (lagging) power factor, 22 kw. of incandescent lamps and an overexcited synchronous motor taking 72.5 kw. at 93 per cent (leading) power factor. Voltage between mains, 120 volts. The total load is measured by two wattmeters, connected as in Fig. 3.10.

(a) Find the readings of W_1 and W_2.

(b) Draw vector diagrams for the three loads and for the total load.

(c) Find the reactive power for each of the three loads and for the total load.

8.10. A three-phase, 60-cycle, 110-volt, 1,000-kv.a. electric-arc furnace has an operating power factor of 90 per cent at full load. With a constant-voltage supply, what is the maximum current that can be passed through the furnace, when a dead short exists at the electrodes, that is, when the resistance of the furnace is practically zero? The resistance of an arc furnace is decreased as the arcs are shortened by lowering the electrodes. What is the maximum amount of power that can be forced into this furnace with supply voltage constant, and under this condition what will be the operating power factor?

9.10. Three heater units each taking 1,150 watts at unity power factor are connected in delta to a 124-volt, three-phase system. What is the resistance of each unit? How many amperes of current flow in the mains? Two wattmeters are used to measure the power. How many watts does each wattmeter indicate?

10.10. Let the three heaters in problem 9.10 be connected in Y to the same three-phase system. What current flows in the line? How much power is absorbed by each heater? Let the total power be measured by two wattmeters. What is the power indicated by each meter?

11.10. Two wattmeters, P_1 and P_2, connected as in Fig. 3.10 or Fig. 6.10, to a 2,300-volt, three-phase system indicate, respectively, 161.1 kw. and 8.34 kw. It was not determined whether P_2 was connected so as to give positive or negative values; that is, whether P_2 should be added or subtracted from P_1 to obtain the total power. The line current was 83 amp. Find $\cos\theta$, θ and the total power.

12.10. Find the power factor and the total power in a three-phase, 220-volt system from the following observed readings: line current = 16.5 amp. Two wattmeter readings, P_1 = 3,260 watts; P_2 = 260 watts. It is not known whether the reading for P_2 is positive or negative with respect to P_1.

I_L = 45

$\sqrt{3}\,EI$

CHAPTER XI

MAGNETIC HYSTERESIS, EDDY CURRENTS, MUTUAL INDUCTION

When an alternating current flows in a conductor surrounded by air or other non-conducting material, energy is stored in the magnetic field surrounding the conductor, while the current increases, and is returned to the electric circuit while the current decreases to its zero value. If, on the contrary, there be conducting materials, and particularly with iron or other magnetic bodies, in the space within the range of the magnetic field of the current in the conductor, a part of the energy involved will be changed into heat. The laws for determining these losses will be discussed under magnetic hysteresis, eddy currents and mutual induction.

(*a*) **Magnetic Hysteresis.**—In Chap. II it was shown that the fundamental relation between magnetic flux Φ and induced voltage E is expressed by equation (1.11).

$$E = \sqrt{2}\pi n f \Phi 10^{-8} \text{ volts} \tag{1.11}$$

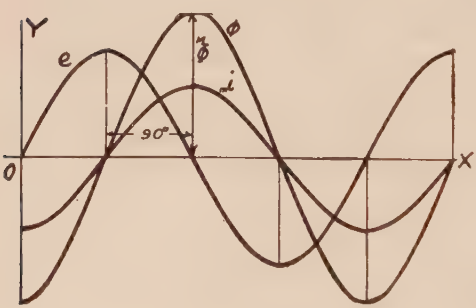


Fig. 1.11.

E is the effective volts (simple sine wave assumed), Φ the total number of lines of force, n the number of turns in series, f the frequency.

Let A = cross-sectional area, and $\mathfrak{B}$ the average magnetic flux density.

$$\Phi = \mathfrak{B}A \tag{2.11}$$

$$E = \sqrt{2}\pi n f \mathfrak{B} A 10^{-8} \text{ volts} \tag{3.11}$$

Since the induced voltage varies with the rate of cutting lines of force, the voltage wave is displaced 90° from the corresponding flux wave, as shown in Fig. 1.11. If the conductor be sur-

rounded by a material of constant permeability, such as air, the exciting current, often called the magnetizing current, is also a sine wave and in time phase with the flux. In an iron-clad circuit, as in Fig. 2.11, the same voltage of sine-wave form produces decidedly different effects:

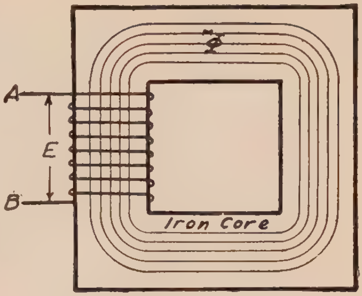


FIG. 2.11.

1. The reluctance of the iron is very low and hence a much stronger field is produced for a given exciting current.
2. While the permeability of iron and steel is large as compared to air, it is not constant for all intensities of magnetization.
3. The ratio between the exciting current and the magnetic flux is different for increasing and decreasing values of the current.
4. These ratios also depend upon the maximum value of the flux density in each cycle.

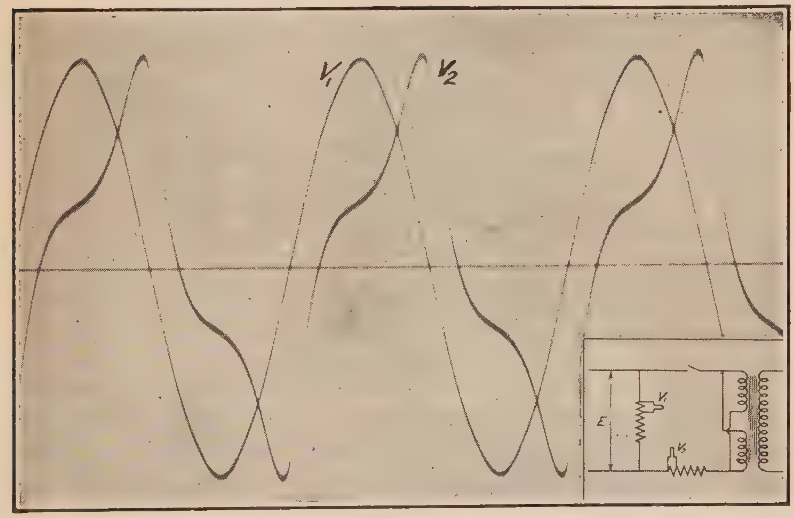


FIG. 3.11.—Oscillogram showing exciting current, V_2, of a 5 kv.a. Type H, General Electric Company's transformer. V_1, voltage = 110 volts; V_2, exciting current = 0.92 amp., f = 60 cycles.

Consider the electric circuit AB, Fig. 2.11, surrounding an iron core. The impressed voltage must at each instant be balanced by the induced voltage produced by the increasing or

decreasing magnetic flux cutting across the several turns of the electric circuit (neglecting the small ri drop in the conductor). Hence an exciting current will flow in the circuit just sufficient to produce the flux required to balance the impressed voltage. This relation is shown by the oscillogram in Fig. 3.11. The voltage is given by curve V_1, and the exciting current by curve V_2, as indicated in the circuit diagram. The voltage leads the magnetic flux wave by 90° as shown in Fig. 1.11.

In the hysteresis loops, Fig. 4.11, is shown the relation between the magnetizing current ${}_mi$, or magnetomotive force $\mathcal{H}$, and the resulting magnetic flux Φ, or flux density $\mathcal{B}$. While the relation may be expressed by the equation $\mathcal{B} = \mu\mathcal{H}$, it must be kept in mind that the permeability is a variable affected by several conditions, as already noted.

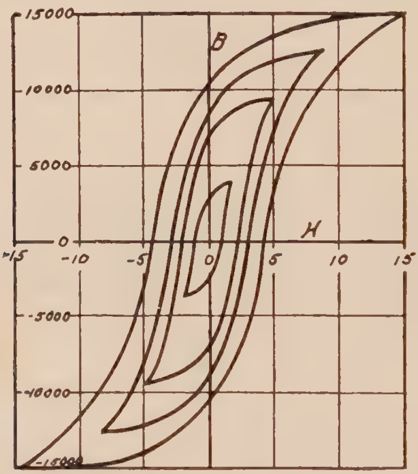


FIG. 4.11.

Plotting magnetic flux and exciting current in rectangular coördinates with time as abscissæ gives the shape of the current wave as in Fig. 5.11. With a given hysteresis loop and sine flux wave the exciting current corresponding to any point on the flux wave is found by drawing the horizontal line u-p, Fig. 5.11, and taking the distance u-s from the hysteresis curve. By laying off on the ordinate n-p the distance n-q equal to u-s, one point on the current curve is found. The maximum of the current curve will come at the same time as the maximum of the flux wave, but the current wave will not be sinusoidal in form. The wave con-

sists of a third and smaller fifth harmonic superimposed on the fundamental sine wave. Unless special accuracy is required, the exciting current may be considered equal to an equivalent sine

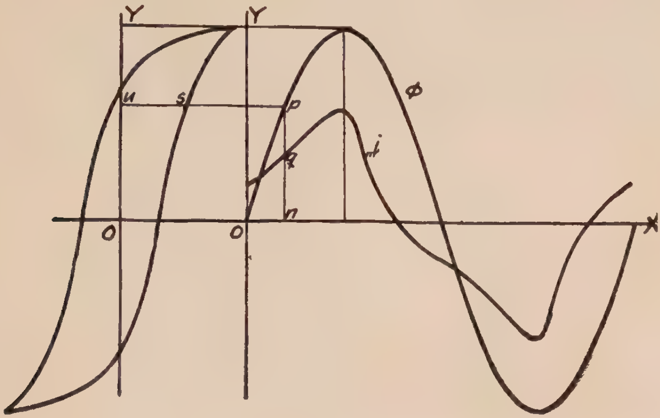


Fig. 5.11.

wave as represented by the broken line in Fig. 6.11. The equivalent current wave has the same effective intensity and power as the actual exciting current, and unless the flux reaches the saturation point in the iron, the maxima of the two current waves are

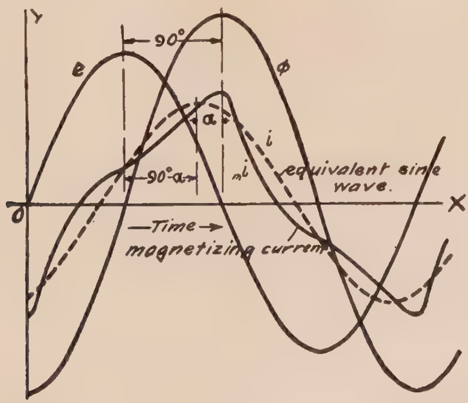


Fig. 6.11.

approximately the same. The equivalent sine wave reaches the maximum before the actual exciting current and hence before the maximum of the flux. This time-phase lead of the equivalent sine wave of the exciting current over the magnetic

flux wave is called the *hysteretic angle of advance* and is represented by the letter α. If the voltage, flux and equivalent current are sine waves their relations can be shown in a vector diagram as in Fig. 7.11. Placing the voltage along the X-axis, the corresponding vector for the magnetic flux will lie in the positive direction on the Y-axis. The vector for the equivalent exciting current will lead the flux by α or lag behind the voltage by $90° - \alpha$. The power consumed by the hysteresis will be the product of the current, voltage and the cosine of the phase angle.

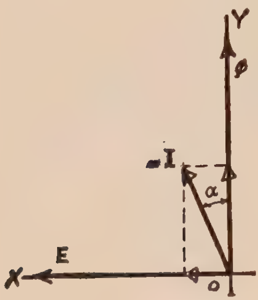


FIG. 7.11.

$$
\begin{aligned}
{}_hP &= E\,{}_mI \cos (90° - \alpha) = E\,{}_mI \sin \alpha \\
&= \text{hysteretic power loss}
\end{aligned}
\tag{4.11}
$$

The magnetizing-current component of the exciting current produces the magnetic flux.

$$
\begin{aligned}
{}_xP &= E\,{}_mI \sin (90° - \alpha) \\
&= E\,{}_mI \cos \alpha = \text{reactive power.}
\end{aligned}
\tag{5.11}
$$

$$
{}_mI \cos \alpha = \text{magnetizing current.} \tag{6.11}
$$

The exciting current can therefore be considered as consisting of the components ${}_mI \sin \alpha$ in phase with the voltage, called the *hysteretic* or *magnetic power component* of the exciting current, and ${}_mI \cos \alpha$, called the *magnetizing component* and which represents energy that is being stored in the magnetic field while the current increases and then returned to the circuit when the current decreases. Conversely, one may consider the voltage as consisting of two components: one, a power component, $E \sin \alpha$, in phase with the current, the *hysteretic power component;* and another, $E \cos \alpha$, in quadrature with the current and consisting of the voltage consumed by *self-induction.*

It is evident that the circuit producing the two components can be represented symbolically by an impedance, *i.e.*, a resistance and an inductive reactance:

$$
z = r + j\,{}_Lx = z \sin \alpha + jz \cos \alpha \tag{7.11}
$$

Similarly, by an admittance or conductance and susceptance:

$$
y = g - j\,{}_Lb = y \sin \alpha - jy \cos \alpha \tag{8.11}
$$

In equations (7.11) and (8.11) representing the exciting circuit, the quantities z, r, ${}_Lx$ and y, g, ${}_Lb$ are not constant as in circuits without iron but depend upon the maximum density of

magnetic induction $\mathcal{B}$. This introduces many complications in the analysis of circuits containing iron and necessitates approximations in calculations that otherwise could be made rigidly exact.

Dr. Steinmetz proved that the energy loss due to hysteresis in iron and many kinds of steel is proportional to the 1.6th power of the maximum magnetic induction $\mathcal{B}$ and can be expressed by the formula:

$$ {}_hw = \eta \mathcal{B}^{1.6} \tag{9.11} $$

${}_hw$ = loss of energy in ergs, per cycle, per $\overline{\text{cm.}}^3$
$\mathcal{B}$ = maximum magnetic induction in lines of force per $\overline{\text{cm.}}^2$
η = the coefficient of hysteresis.
The value of η differs depending upon the material. Dr. Steinmetz found it to vary from 0.001 to 0.0055 in iron; from 0.0032 to 0.028 in cast steel; up to 0.08 for tungsten and manganese steels; while for silicon steel values even lower than 0.001 were obtained.

From equation (9.11) the loss of power for volume V, and at frequency f, is:

$$ {}_hP = \eta f V \mathcal{B}^{1.6} 10^{-7} \text{ watts} \tag{10.11} $$

$$ = \eta f V \left(\frac{{}^M\Phi}{A} \right)^{1.6} 10^{-7} \text{ watts} \tag{11.11} $$

For certain grades of silicon steel (probably of a non-homogeneous nature) the hysteresis power loss is more closely approximated by using 1.7 instead of 1.6 as exponent for $\mathcal{B}$.

(*b*) **Eddy Currents.**—The lines of force produced by an alternating current move outward radially from the conductor while the current is increasing, and toward the conductor when the current decreases. While thus traversing the space surrounding the conductor carrying the current, the lines of force set up or induce electromotive forces in the conductor. If this voltage is induced in a conducting material like copper or iron, a local current will flow. The magnitude of this current will depend on the difference of potential induced in the conductor and the resistance in the path through which the current must flow. This induced current has also a magnetic field which, in turn, reacts upon the primary field. In solid masses of good conductors, such as iron or copper, the voltage induced would be considerable and the resistance small. As a result large currents would flow in

the mass taking curved paths much the same as eddies in a stream. In most electrical apparatus, eddy currents are undesirable; and the design must be so made as to reduce the losses from eddy currents to a minimum. This is accomplished by laminating the iron or using iron wire as conductor of the magnetic flux. This method reduces the possible difference in potential between any two points and also increases the resistance of the path through which the eddy currents must flow. Eddy currents are true electric currents but flow in very short circuits. The induced voltage causing the eddy currents is proportional to the frequency and flux density

$${}_eE \propto f\mathcal{B} \tag{12.11}$$

The resulting current is equal to the product of the voltage and conductivity of the circuit, hence:

$${}_eI = \lambda\, {}_eE \propto \lambda f\mathcal{B} \tag{13.11}$$

The power is proportional to the product of the current and voltage:

$${}_eP \propto {}_eI\, {}_eE \propto \lambda f^2\mathcal{B}^2 \tag{14.11}$$

From equation (3.11), $f\mathcal{B}$ is proportional to E, hence:

$${}_eP = kE^2\lambda \tag{15.11}$$

The loss of power by eddy currents is proportional to the square of the voltage and to the electric conductivity of the iron.

FIG. 8.11.

Like magnetic hysteresis, eddy currents produce a leading current with respect to the flux; or the eddy currents cause an *angle of advance* β, and therefore the current has components both in phase and in quadrature with the voltage. The phase relations between the impressed voltage, the induced flux and the eddy current are shown by the vector diagram in Fig. 8.11. Expressing the angle in terms of the eddy-current conductance g and the total admittance of the circuit y:

$$\sin \beta = \frac{g}{y} \tag{16.11}$$

From equation (15.11):

$$\frac{P}{E^2} \doteq k\lambda = g \tag{17.11}$$

The equivalent conductance due to the eddy currents in the iron is therefore a constant of the circuit, depending only upon the electric conductivity of the iron.

To find the power loss from eddy currents let:

ϵ = eddy-current coefficient.
V = the volume of the iron.

Dividing the expression for power in equation (14.11) by the frequency, we have the energy per cycle per $\overline{\text{cm.}}^3$.

$${}_{\epsilon}w = \epsilon\lambda f\mathcal{B}^2 \text{ ergs} \tag{18.11}$$

Hence total power:

$${}_{\epsilon}P = \epsilon\lambda V f^2\mathcal{B}^2 10^{-7} \text{ watts} \tag{19.11}$$

As the quality of the iron is included in the conductivity factor, λ, the coefficient of eddy currents ϵ depends only upon the shape of the parts in the magnetic circuit. In commercial apparatus thin plates or sheets and wires are used, and the coefficient depends upon the thickness of the sheets, or upon the diameter of the wire.

1. *Thin Plates or Laminated Iron.*—Let the thickness of the iron be very small compared to the length of the sheet, so that the

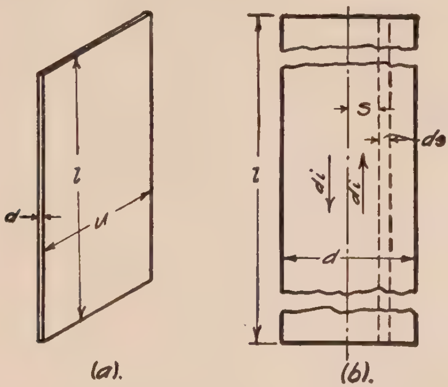


FIG. 9.11.

currents may be considered as flowing parallel to the surface and in opposite directions on the two sides and in that way forming a circuit. This is the case in most commercial machines, as transformer cores, armature cores, etc. Let Fig. 9.11*a* be an edge-

wise (greatly enlarged) section of a thin plate or sheet of iron placed in a vertical position. Let d be the thickness of the plate, l the length (up and down) and u the width of the thin plate or sheet of iron. Let ds be an elemental zone, parallel to the surface of the plate as shown in cross-section in Fig. 9.11b.

For currents flowing up and down in the plate the conductance g for the zone ds is:

$$g = \frac{\lambda u(ds)}{l} \tag{20.11}$$

The number of magnetic lines of force passing across ds in a quarter cycle is $\mathcal{B}sl$. Hence the voltage induced in this zone is:

$$_eE = \sqrt{2}\pi fl\mathcal{B}s \text{ in c.g.s. units} \tag{21.11}$$

This voltage produces a current:

$$_eI = g\,{}_eE = \sqrt{2}\pi f\mathcal{B}\lambda us(ds) \tag{22.11}$$

Hence the power consumed in this zone:

$$d\,{}_ep = {}_eE\,{}_eI = 2\pi^2 f^2\mathcal{B}^2 l\lambda us^2(ds) \tag{23.11}$$

Therefore the total power consumed by the eddy currents in the sheet of thickness d is:

$$_ep' = \int_{-\frac{d}{2}}^{+\frac{d}{2}} (d\,{}_ep) = 2\pi^2 f^2\mathcal{B}^2\lambda\, ul \int_{-\frac{d}{2}}^{+\frac{d}{2}} s^2(ds) = \frac{\pi^2 f^2\mathcal{B}^2 l\lambda ud^3}{6}$$

$$\text{ergs per sec.} \tag{24.11}$$

Power per $\overline{\text{cm.}}^3$

$$_ep = \frac{_ep'}{lud} = \frac{\pi^2 f^2\mathcal{B}^2\lambda d^2}{6} \text{ ergs per sec.} \tag{25.11}$$

Energy per cycle, per $\overline{\text{cm.}}^3$:

$$_ew = \frac{_ep}{f} = \frac{\pi^2 f\mathcal{B}^2\lambda d^2}{6} \text{ ergs} \tag{26.11}$$

From equations (18.11) and (26.11):

$$\epsilon = \frac{\pi^2 d^2}{6} = 1.645d^2, \text{ if } \lambda \text{ is given in c.g.s. units.} \tag{27.11}$$

As λ is usually given in practical units the coefficient ϵ includes the conversion factor 10^{-9} or:

$$\epsilon = 1.645d^2 10^{-9} \text{ for } \lambda \text{ in practical units.} \tag{28.11}$$

In practical units λ varies within the limits of 10^5 and 10^4. Equation (18.11) can therefore be restated:

$$ {}_\epsilon w = \epsilon\lambda f\mathfrak{B}^2 = 1.645 d^2\lambda f\mathfrak{B}^2 10^{-9} \text{ ergs} \qquad (29.11)$$
$$ = 1.645 d^2\lambda f\mathfrak{B}^2 10^{-16} \text{ joules}$$

Also equation (26.11) becomes:

$$ {}_\epsilon p = f\,{}_\epsilon w = 1.645\lambda d^2 f^2\mathfrak{B}^2 10^{-16} \text{ watts} \qquad (30.11)$$

and the total loss by eddy currents for volume V:

$$ {}_\epsilon P = {}_\epsilon p V = 1.645\lambda V d^2 f^2\mathfrak{B}^2 10^{-16} \text{ watts} \qquad (31.11)$$

2. *Iron Wire.*—The coefficient for eddy currents in an iron wire may be found in a similar manner. Let Fig. 10.11 represent the cross-section of a wire of diameter d. The magnetic lines of force lie parallel to the length of the wire and move at right angles to their direction, either toward or away from the center of the wire. Hence the lines of force pass through the elemental sheet ds and generate voltages that tend to send currents in the sheet ds around the axis of the conductor. The resistance of an elemental circular zone ds in Fig. 10.11

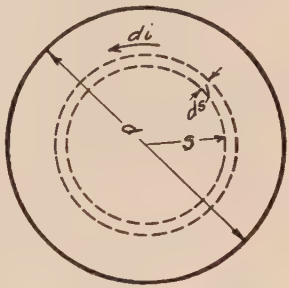


FIG. 10.11.

at a distance s from the center and l cm. in length is $\frac{2\pi s}{l\lambda(ds)}$ r and the flux crossing the zone in each quarter cycle is $\mathfrak{B}\pi s^2$.

Hence the voltage generated in the zone:

$$ {}_\epsilon E = \sqrt{2}\pi^2 f\mathfrak{B}s^2 \text{ in c.g.s. units} \qquad (32.11)$$

The current produced by this voltage:

$I = \frac{E}{r}$

$$ {}_\epsilon I = g\,{}_\epsilon E = \frac{\pi f l\lambda\mathfrak{B}s(ds)}{\sqrt{2}} \qquad (33.11)$$

The power loss in this zone:

$$ (d\,{}_\epsilon p) = {}_\epsilon E\,{}_\epsilon I = \pi^3\lambda l f^2\mathfrak{B}^2 s^3(ds) \qquad (34.11)$$

The power consumed in l cm. of wire:

$$ {}_\epsilon p' = \pi^3\lambda f^2\mathfrak{B}^2 l\int_0^{\frac{d}{2}} s^3(ds) = \frac{\pi^3\lambda f^2\mathfrak{B}^2 l d^4}{64} \qquad (35.11)$$

The power consumed in 1 $\overline{\text{cm.}}^3$ of iron:

$$_\epsilon p = \frac{_\epsilon p'}{V} = \frac{\pi^2 f^2 \lambda \mathcal{B}^2 d^2}{16} \text{ ergs per sec. (c.g.s. units)} \qquad (36.11)$$

Energy consumed by eddy currents per cycle, per $\overline{\text{cm.}}^3$ of iron:

$$_\epsilon w = \frac{p}{f} = \frac{\pi^2 f \lambda \mathcal{B}^2 d^2}{16} \text{ ergs} \qquad (37.11)$$

The coefficient of eddy currents is, therefore, from equations (18.11) and (37.11):

$$\epsilon = \frac{\pi^2 d^2}{16} = 0.617d^2, \text{ if } \lambda \text{ is expressed in c.g.s. units} \qquad (38.11)$$

For λ in practical units the conversion factor 10^{-9} must be introduced and then:

$$\epsilon = 0.617d^2 10^{-9} \qquad (39.11)$$

With this value for ϵ, equations (36.11) and (37.11) may be written as in equations (41.11) and (40.11).

Energy consumed by eddy currents in iron wire per $\overline{\text{cm.}}^3$ per cycle:

$$_\epsilon w = \frac{\pi^2 f \lambda \mathcal{B}^2 d^2 10^{-9}}{16} = 0.617 f \lambda \mathcal{B}^2 d^2 10^{-9} \text{ ergs} \qquad (40.11)$$

Power consumed at frequency f:

$$_\epsilon p = f\,_\epsilon w = 0.617 f^2 \lambda \mathcal{B}^2 d^2 10^{-16} \text{ watts} \qquad (41.11)$$

Total power consumed by volume V:

$$_\epsilon P = {_\epsilon p} V = 0.617 V f^2 \lambda \mathcal{B}^2 d^2 10^{-16} \text{ watts} \qquad (42.11)$$

3. *Comparison of Power Consumed by Eddy Currents in Wires and Laminated Iron.*—With the same quality of iron, the same flux density, frequency and volume, the power loss is directly proportional to the eddy-current coefficient ϵ.

For laminated iron:

$$\epsilon' = 1.645 d_1^2 10^{-9} \qquad (43.11)$$

For iron wire:

$$\epsilon'' = 0.617 d_2^2 10^{-9} \qquad (44.11)$$

For the same loss of power $\epsilon' = \epsilon''$ and hence:

$$\frac{\text{diameter of wire}}{\text{thickness of sheet}} = \frac{d_2}{d_1} = \sqrt{\frac{1.645}{0.617}} = 1.63 \qquad (45.11)$$

For equal power losses due to eddy currents the diameter of the iron wire is therefore 1.63 times the thickness of the sheet, provided the quality of the iron is the same.

(c) **Mutual Induction.**—The eddy currents, discussed in the previous paragraphs, are produced by the alternating field passing into the laminated iron or the iron core. If the lines of force cut another conductor, an e.m.f. will also be induced therein. If the conductor forms a circuit, a current will flow and energy will be consumed. This energy must necessarily come from the circuit that produced the alternating magnetic flux. Thus in Fig. 11.11, if an alternating current flows in circuit A and a portion of the circuit lies parallel to a conductor forming part of circuit B, an e.m.f. will be induced causing a current to flow in B. The power consumed by the current flowing in circuit B is supplied by the generator in circuit A. The voltage induced in circuit B and the resulting current are in the opposite direction to the voltage and current in A. If circuit B has both resistance and inductance, the current will lag behind the voltage, and either the current may be considered as consisting of a power component in phase with the voltage and a reactive component in quadrature with the voltage, or the voltage may be divided

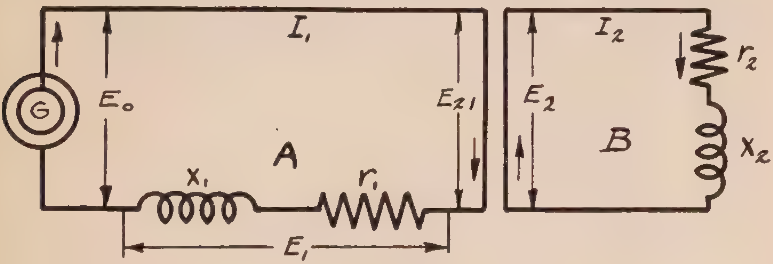


FIG. 11.11.

into components in phase and in quadrature with the current. Where the amount of energy transferred is considerable, it is usually best to take the two circuits separately, and to determine the effect of mutual induction in terms of resistance and reactance constants for both the primary and the secondary circuits. In Fig. 11.11 let r_1, x_1, E_1 and I_1 for circuit A, r_2, x_2, E_2 and I_2 for circuit B, represent the resistance, reactance, voltage and current respectively. The reactance x_1 refers to circuit A alone and similarly x_2 to circuit B without taking into consideration the existence of the other circuit.

If L_1 represents the induction due to all the interlinkages of the magnetic flux from unit current in A with the number of

turns in circuit A, and likewise L_2 the induction due to all the interlinkages of the magnetic flux from unit current in B with the number of turns in circuit B, then $x_1 = 2\pi f L_1$, and $x_2 = 2\pi f L_2$. However, part of the lines of force produced by I_1 in circuit A will also interlink with circuit B, and similarly the same proportion of the lines produced by current I_2 in circuit B will interlink with circuit A, thus forming an additional inductance between the two circuits. The mutual inductance M is the total number of interlinkages of the magnetic flux produced by unit current in circuit A with circuit B, or the total number of interlinkages of the magnetic flux produced by unit current in circuit B with circuit A. Since in either case part of the magnetic flux will lie between the two circuits, M^2 must be less than L_1L_2. If M be the mutual inductance, $x_m = 2\pi f M =$ the mutual reactance.

Let

ϕ_1 = total flux produced in circuit A by I_1.
ϕ_2 = total flux produced in circuit B by I_2.
$\phi_{12} = K\phi_1$ = flux from circuit A which interlinks B.
$\phi_{21} = K\phi_2$ = flux from circuit B which interlinks A.
K = coupling coefficient.

From equation (18.2):

$$E_2 = \sqrt{2}\pi N_2 f \phi_{12} 10^{-8}$$

2 the lines
f interlink

$$= \frac{\sqrt{2}\pi N_2 f \phi_{12} \sqrt{2} I_1}{\sqrt{2} I_1 10^8} \tag{46.11}$$

$$M_{12} = \frac{N_2 \phi_{12}}{\sqrt{2} I_1 10^8} \tag{47.11}$$

From equations (46.11) and (47.11):

$$E_2 = 2\pi f M_{12} I_1 \tag{48.11}$$

$$x_m = 2\pi f M_{12}, \text{ the mutual reactance.}$$

Substituting in equation (48.11):

$$\dot{E}_2 = -jx_m \dot{I}_1 \tag{49.11}$$

The current in the secondary is therefore expressed by equation (50.11).

$$\dot{I}_2 = \frac{\dot{E}_2}{z_2} = \frac{-jx_m \dot{I}_1}{r_2 + jx_2} \tag{50.11}$$

The current I_2 in circuit B similarly induces a voltage E_{21} in circuit A.

$$E_{21} = \frac{\sqrt{2}\pi f N \phi_{21} \sqrt{2} I_2}{\sqrt{2} I_2 10^8} \tag{51.11}$$

$$M_{21} = \frac{N_1 \phi_{21}}{\sqrt{2} I_2 10^8} = M_{12} \tag{52.11}$$

$$E_{21} = 2\pi f M_{21} I_2 \tag{53.11}$$

$$\dot{E}_{21} = -jx_m \dot{I}_2 = \frac{j^2 x_m^2 \dot{I}_1}{r_2 + jx_2} = \frac{-x_m^2 \dot{I}_1}{r_2 + jx_2}$$

$$= \left(\frac{-r_2 x_m^2}{r_2{}^2 + x_2{}^2} + \frac{j x_m^2 x_2}{r_2{}^2 + x_2{}^2} \right) \dot{I}_1 \tag{54.11}$$

induced

The impressed voltage necessary to overcome this induced counter e.m.f. is therefore: sign reversed on

$$\dot{E}_{12} = \left(\frac{r_2 x_m^2}{r_2{}^2 + x_2{}^2} - j \frac{x_m^2 x_2}{r_2{}^2 + x_2{}^2} \right) \dot{I}_1 \tag{55.11}$$

This shows that the effect of the mutual inductance is equivalent to a circuit having a resistance and condensive reactance, or that the mutual inductance increases the resistance and decreases the inductive reactance.

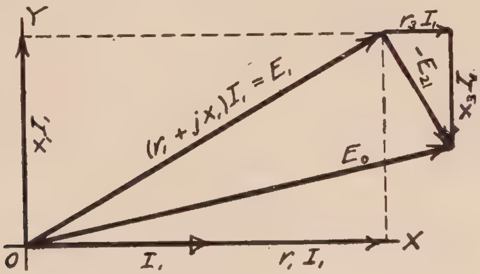


FIG. 12.11.

Let $r_3 = \dfrac{x_m^2 r_2}{r_2{}^2 + x_2{}^2}$, the equivalent resistance of mutual induction and

$x_3 = \dfrac{x_m^2 x_2}{r_2{}^2 + x_2{}^2}$, the equivalent reactance of mutual induction;

then:

$$\dot{E}_{12} = (r_3 - jx_3)\dot{I}_1 \tag{56.11}$$

Since circuit A has resistance r_1 and reactance x_1, the voltage impressed by the generator, causing a current I_1 to flow through

A, must be the vector sum of the voltages consumed by r_1I_1, x_1I_1 and E_{12}.

$$\dot{E}_0 = (r_1 + jx_1)\dot{I}_1 + \dot{E}_{12} \tag{57.11}$$
$$= [r_1 + r_3 + j(x_1 - x_3)]\dot{I}_1 \tag{58.11}$$

The equivalent resistance is therefore increased and the equivalent reactance decreased in circuit A by the mutual inductance between circuits A and B. The phase relations of the current and the components of the generated voltage consumed by the resistance and reactance in circuit A and by the equivalent resistance and reactance due to the mutual inductance between the two circuits are illustrated by the vector diagram in Fig. 12.11.

(*d*) **Effective Resistance, Reactance, Conductance and Susceptance.**—From the preceding it is evident that the phenomena occurring in the space surrounding an alternating circuit, and due to the rapid changes in the magnetic field, consume power in addition to the ordinary ri^2 losses in the circuit. Hence the transmission losses in a circuit are greater for alternating than for direct currents of equal magnitude. A similar loss is caused by dielectric hysteresis, corona, etc., as is shown in Chap. XXI.

Taking the power loss by means of a wattmeter for direct and alternating currents of equal magnitude and in the same circuit, we have two values for the power.

For direct currents:

$$_dP = {_dr}I^2 \tag{59.11}$$

For alternating currents:

$$_aP = EI \cos \theta = zI^2\frac{_ar}{z} = {_ar}I^2 \tag{60.11}$$

$$_aP > {_dP}, \text{ hence } {_ar} > {_dr} \tag{61.11}$$

The value as measured by direct currents $_dr$ is called the *true* or *ohmic resistance;* and the quantity as measured by alternating currents $_ar$, the *effective resistance*. The effective resistance is always larger than the ohmic resistance, as it is composed of the ohmic resistance plus a quantity representing the power losses due to dielectric and magnetic hysteresis, eddy currents, mutual magnetic and dielectric induction, skin effect, corona, etc.

Likewise, the reactance of a circuit is influenced by its proximity to other circuits, and the actual value determining the phase relation of the currents and voltages is the algebraic sum of the reactances due to the inductance and condensance of the circuit itself and the effects produced by the neighboring con-

ductors. This mutual induction decreases the inductive reactance while dielectric induction increases the condensive reactance of the circuit. The resultant value is called the *effective reactance* of the circuit. As several of these outside-the-conductor phenomena, affecting the value of the effective resistance and effective reactance, are directly proportional to the impressed voltage, these are in a sense parallel circuits and therefore more conveniently expressed in terms of conductance and susceptance. As the relation between resistance, reactance, conductance and susceptance is fixed by the equations given in Chap. VI, the terms *effective conductance* and *effective susceptance* correspond to *effective resistance* and *effective reactance.*

In Chap. II the term *effective* as applied to current and voltage was used to indicate $\frac{1}{\sqrt{2}}$ times the maximum value of a sine wave. In a broader sense, with waves of different shapes, it means the value actually measured by a voltmeter or ammeter indicating the square root of the average of the squared instantaneous values (see page 556). Likewise the term *effective* when applied to resistance, reactance, conductance and susceptance refers to the quantities measured in the alternating circuit whether the quantities are in the conductor of the circuit itself or in the surrounding magnetic and dielectric fields.

It is important to note that while the phenomena both in the conductor and in the space surrounding an alternating-current circuit are of great variety the mutual relations of the current, voltage and power can be expressed in a very simple manner by using two constants, either effective resistance and effective reactance, or effective conductance and effective susceptance.

$$\text{Power (real or true)} = EI \cos\theta = rI^2 = {}_rEI \qquad (62.11)$$

${}_rE$ = power component of the voltage = effective resistance by current.

$$\text{Power (real or true)} = EI \cos\theta = gE^2 = {}_gIE \qquad (63.11)$$

${}_gI$ = power component of current = effective conductance by voltage.

$$\text{Power (reactive or wattless)} = EI \sin\theta = xI^2 = {}_xEI \qquad (64.11)$$

${}_xE$ = reactive component of voltage = effective reactance by current.

$$\text{Power (reactive or wattless)} = EI \sin\theta = bE^2 = {}_bIE \qquad (65.11)$$

${}_bI$ = reactive component of current = effective susceptance by voltage.

The effective values can usually be measured in an existing circuit and may also, with a fair degree of accuracy, be predetermined by calculation. It must be kept in mind, however, that, in general, effective values of r, x, g and b are not constants but depend upon the frequency, voltage, current, flux density, etc. The notation rule for *effective* values of current and voltage applies to the resistance, reactance, conductance and susceptance; namely, that unless specifically stated otherwise the *effective* values are understood when dealing with alternating-current circuits.

PROBLEMS

1.11. Given sheet steel of thickness $d = 0.07$ cm.; $\lambda = 3 \times 10^4$ $\eta = 0.0015$; $f = 60$ cycles.

(*a*) Find ${}_hP$ and ${}_eP$ for 1 c.c., 1 cu. ft., 1 kg., and 1 lb. for $\mathcal{B} = 8{,}000$.

(*b*) Plot curves for ${}_hP$ and ${}_eP$ as ordinates and $\mathcal{B}$ as abscissæ, for 1 kg. and for values of $\mathcal{B}$ from 4,000 to 18,000.

(*c*) Find ${}_hW$ and ${}_eW$ for 24 hr. in 100 lb. of the sheet steel with $\mathcal{B} = 10{,}500$.

2.11. The iron losses in a 50-kv.a. transformer for $\mathcal{B} = 8{,}500$ are found to be ${}_eP = 56$ watts and ${}_hP = 182$ watts. $f = 60$ cycles. Find ${}_eP$ and ${}_hP$ for $\mathcal{B} = 6{,}000$, 7,500, 9,500, 11,000 and 12,000.

3.11. Two transformers have iron cores of the same kind of sheet steel of equal volumes. The windings differ so as to give values to $\mathcal{B}$ equal to 6,500 and 10,500 respectively.

Find the ratios of the hysteresis losses and the eddy-current losses.

4.11. The total volume of iron in a certain transformer = 9,850 c.c. $\eta = 0.001$; $\lambda = 5 \times 10^4$; $d = 0.035$ cm.

Plot curves for hysteresis and eddy-current losses for values of $\mathcal{B}$ from 4,000 to 16,000 for $f = 50$ and 60 cycles.

5.11. A laboratory test shows that a certain transformer has 181 watts iron loss at 30 cycles and 446 watts at 60 cycles per sec. for the same maximum flux density. (*a*) What is the hysteresis loss at each frequency? (*b*) What would be the total iron loss at double the flux density at 50 cycles per sec.?

CHAPTER XII

TRANSFORMERS

A. THE CONSTANT-POTENTIAL TRANSFORMER

The chief advantage of alternating over direct currents comes from the simple manner in which the alternating voltage may be raised or lowered. Transmission-line losses are, as stated by Joule's law, fundamentally proportional to the product of the resistance and the current squared. Since the power transmitted equals $EI \cos \theta$, it is evident that, with constant power factor, an increase in voltage means that more power can be transmitted for the same line loss. For example, if the voltage be increased

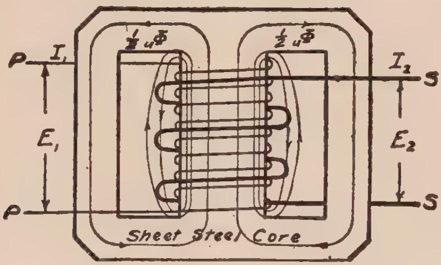


FIG. 1.12.—Shell-type transformer.

fourfold the power that can be transmitted over the same line at a given per cent line loss will be sixteen times as large as at the lower voltage. At the receiver end of the line the safety of the customer and the economical design of lamps, motors and other appliances receiving the electric power require low voltages. Hence the fundamental requirement for economical transmission and distribution of electric energy is comparatively high potential on the transmission line combined with low voltage at both ends. This requirement is admirably met in alternating-current circuits by the use of the potential transformer.

The single-phase transformer is the simplest and most efficient of alternating-current apparatus. It consists of a magnetic circuit interlinked with two electric circuits, a primary and a secondary. Two fundamental designs for the magnetic circuit

are in commercial use, the *core* and the *shell* type, as illustrated in Figs. 1.12 and 2.12. In some designs features of both the shell and core types are included. The electric circuits are insulated from each other and the iron core; hence all the energy passing from the primary to the secondary circuit is transmitted through the transformer by the magnetic flux. In both the core and shell types the reluctance of the iron or steel cores is very small, so that almost all of the magnetic flux is inside of the steel core with only a weak stray field in the surrounding space.

(*a*) **The Ideal Transformer.**—At full load both the leakage flux and the power loss in the transformer are small as compared to the total flux and the power transmitted. Therefore, in a

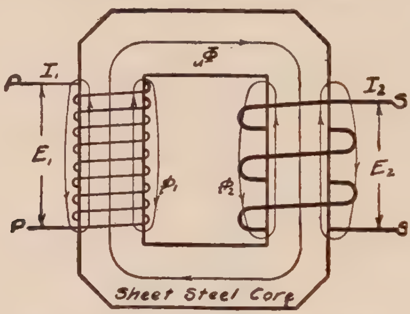


FIG. 2.12.—Core-type transformer.

preliminary discussion of the relations between the flux, impressed and induced voltages and load current it may be assumed:

1. That all the magnetic flux passes through the steel core, thus neglecting the leakage flux.

2. That all the power supplied to the primary is delivered by the secondary, thus neglecting the losses in the transformer.

Let

n_1 = number of turns in the primary.
n_2 = number of turns in the secondary.
${}^M_u\Phi$ = maximum useful magnetic flux in the steel core, expressed in lines of force.
${}_\phi E_1$ = voltage induced in the primary by the flux ${}^M_u\Phi$.
${}_\phi E_2$ = voltage induced in the secondary by the flux ${}^M_u\Phi$.
f = frequency in cycles per sec.

From the first assumption it follows that the total flux cuts all the turns in both the primary and secondary.

$$ {}_\phi E_1 = \sqrt{2}\pi n_1 f\, {}^M_u\Phi 10^{-8} \text{ volts} \tag{1.12} $$

$$ {}_\phi E_2 = \sqrt{2}\pi n_2 f\, {}^M_u\Phi 10^{-8} \text{ volts} \tag{2.12} $$

Hence,

$$\frac{_\phi E_1}{_\phi E_2} = \frac{n_1}{n_2} \tag{3.12}$$

From the second assumption,

$$\text{Power} = {}_\phi E_1 I_1 \cos\theta = {}_\phi E_2 I_2 \cos\theta, \text{ or } n_1 I_1 = n_2 I_2 \tag{4.12}$$

Hence,

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} \tag{5.12}$$

Therefore in an *ideal* transformer the voltage ratio is directly and the current ratio inversely proportional to the number of turns. The ratio between the number of turns may be any value and in commercial designs it varies within wide limits.

It is difficult to represent clearly on the same vector diagram quantities differing much in magnitude, as, for instance, the primary and secondary voltage on a 20:1 ratio transformer. Hence

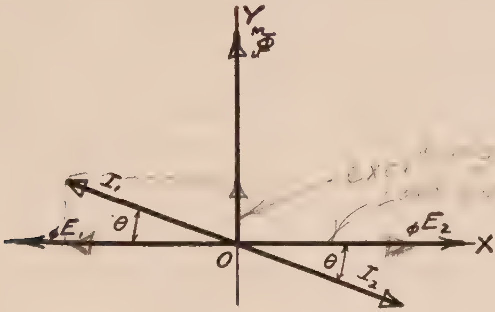


FIG. 3.12.

it is customary either to consider a transformer having a ratio of 1:1, for general illustrative purposes, or to let the vectors represent ampere-turns and voltage per turn instead of current and total voltage. This gives vectors of equal magnitude for the primary and secondary because $n_1 I_1 = n_2 I_2$, by equation (5.12), and $\frac{_\phi E_1}{n_1} = \frac{_\phi E_2}{n_2}$, by equation (3.12).

For an ideal transformer the vector diagram in Fig. 3.12 represents phase relations of the flux, currents and voltages. Let time be counted from the instant when the flux wave is passing through zero in the positive direction. Let the magnetic flux vector be drawn upward along the Y-axis, and since the induced voltage in the secondary coil lags behind the flux by 90° it is represented by the vector $O_\phi E_2$ along the X-axis. The impressed

voltage in the primary is equal and opposite to the voltage induced by the flux in the primary and is represented by the vector $O_\phi E_1$. Let the current in the secondary lag behind the voltage by $\theta°$ and be represented by the vector OI_2. Then the corresponding current flowing in the primary is the vector OI_1, lagging behind the impressed voltage by $\theta°$.

(*b*) **Exciting and Magnetizing Currents.**—While the basic relations of the magnetic flux, the voltages and currents are given in the description of the ideal transformer, even an elementary discussion of the actual transformer must take into account the exciting and magnetizing currents and the effect of the resistance and reactance in both the primary and the secondary windings. With the secondary open and hence no current flowing in the secondary winding, a small current, sometimes incorrectly called the *leakage current*, flows in the primary providing the magnetomotive force $n_1 {}_mI$, necessary to produce the flux ${}^M_u\Phi$, in the transformer. This is the *exciting current*. The quadrature component of the exciting current is called the *magnetizing current*. As explained in Chap. XI, the exciting current is not a simple sine function of the time, but can be represented approximately by an equivalent sine wave of equal effective value and equal power, leading the magnetic flux by an angle α, the hysteretic angle of advance. The exciting current lags behind the impressed voltage ${}_\phi E_1$ by $90° - \alpha$, and can be divided into components in time phase and time quadrature with the voltage.

${}_mI \cos (90° - \alpha)$ = the power component, and represents the losses from hysteresis and eddy currents, ${}_hP + {}_eP$.

${}_mI \sin (90° - \alpha)$ = the magnetizing current or wattless component, and represents the reactive power in the magnetic field.

In the symbolic notation:

$$ {}_m\dot{I} = {}_mg\, {}_\phi\dot{E}_1 - j\, {}_mb\, {}_\phi\dot{E}_1 \qquad (6.12) $$

$$ {}_mg\, {}_\phi E_1^2 = {}_hP + {}_eP, \text{ or } {}_mg = \frac{{}_hP + {}_eP}{{}_\phi E_1^2} = \frac{V\eta f \mathcal{B}^{1.6} + V\epsilon\lambda f^2 \mathcal{B}^2}{{}_\phi E_1^2 10^7} \qquad (7.12) $$

The susceptance ${}_mb$ can be found from the dimensions of the magnetic circuit, the quality of the iron and maximum flux density.

A = cross-section of iron core in square centimeters.
${}^{M}_{u}\Phi$ = maximum useful lines of force.

$$\mathcal{B} = \frac{{}^{M}_{u}\Phi}{A} = \text{maximum flux density.}$$

$\mathcal{R}$ = the reluctance of the magnetic circuit for the density $\mathcal{B}$.

The maximum magnetomotive force required to magnetize the iron core is equal to the product of the reluctance of the magnetic circuit and the maximum core flux.

$$\frac{4}{10}\pi n_1\sqrt{2}\,{}_mI \sin (90° - \alpha) = \frac{4}{10}\pi n_1\sqrt{2}\,{}_mb\,{}_\phi E_1 = {}^{M}_{u}\Phi\mathcal{R} \qquad (8.12)$$

$${}_mb = \frac{10\,{}^{M}_{u}\Phi\mathcal{R}}{4\pi\sqrt{2}n_1\,{}_\phi E_1} = 0.56\frac{{}^{M}_{u}\Phi\mathcal{R}}{n_1\,{}_\phi E_1} \qquad (9.12)$$

It should be noted that ${}_\phi E_1$ in equations (6.12) to (9.12) is that part of the total voltage E_1 impressed on the primary terminals which is balanced by the inducing action of the magnetic flux ${}^{M}_{u}\Phi$ cutting the n_1 turns of the primary winding. As will be shown later, however, the loss of voltage in the primary winding is small, and therefore in most cases the terminal voltage E_1 may be substituted for ${}_\phi E_1$ in equations dealing with the exciting current, without introducing appreciable errors.

(*c*) **Magnetic Leakage.**—In the preceding paragraph the exciting current has been discussed under no load or open secondary circuit conditions. As the maximum flux interlinking both the primary and the secondary circuits, that is, the useful flux in the iron core, is approximately the same for all load conditions, the exciting current remains practically the same under load as when the secondary is open. With the transformer loaded and hence a current flowing in the secondary circuit, magnetomotive forces equal to the product of the currents into the respective number of turns appear in both the secondary and the primary. In the primary circuit a m.m.f. equal to n_1I_1 produces a flux which surrounds the primary winding. This flux may be separated into two components, one part in the iron interlinking all the turns of both the primary and secondary circuits; and a second part, the leakage flux ${}_l\phi_1$, in Fig. 2.12, very much smaller, lies in the space outside the iron core, occupied by the insulation and interlinks part or all of the primary turns, but does not interlink with the secondary. Similarly, the secondary m.m.f. produces a magnetic flux in the opposite direction, one part in

the iron and a second part ${}_l\phi_2$, in Fig. 2.12, passing through the space occupied by the insulation and interlinking part or all of the secondary turns but not interlinking with the primary winding. In the shell type, Fig. 1.12, only that part of the flux in the air lying between the primary and secondary windings is the leakage flux. All lines of force cutting both the primary and secondary windings are useful and hence grouped as ${}_u^M\Phi$. Since the m.m.f. of the primary is opposed by the m.m.f. of the secondary, the resultant or difference is at all loads, the m.m.f. of the exciting current which produces the useful flux ${}_u^M\Phi$ interlinking both the primary and secondary windings.

$$n_2\dot{I}_2 = n_1\dot{I}_{11} \tag{10.12}$$
$$n_1\dot{I}_1 = n_1\,{}_m\dot{I} + n_1\dot{I}_{11} \tag{11.12}$$

Hence the resultant m.m.f. for both primary and secondary, in the iron core:

$$n_1\dot{I}_1 - n_2\dot{I}_2 = n_1\dot{I}_{11} + n_1\,{}_m\dot{I} - n_2\dot{I}_2 = n_1\,{}_m\dot{I} \tag{12.12}$$

The lines of force outside of the iron core and interlinking all or part of the turns in either the primary or the secondary circuit, but not both, constitute what is called the leakage flux.

${}_l\phi_1$ = primary leakage flux, proportional to I_1.
${}_l\phi_2$ = secondary leakage flux, proportional to I_2.

The leakage flux linkages are therefore a simple inductance in each circuit and can be represented by a primary and a secondary reactance.

In general, transformers are designed for minimum leakage reactance, but under certain conditions the service required necessitates the insertion of a considerable amount of reactance in the circuit, as, for example, in the operation of rotary converters on a rapidly varying load. For this service the transformer is designed with a magnetic bridge between the primary and secondary windings which gives a large leakage reactance.

(*d*) **Circuit Diagram.**—In Fig. 4.12 is shown a circuit diagram for a potential transformer receiving power over a line from a generator and supplying a load.

E_0 = generator voltage.
E_1 = voltage impressed on the transformer primary.
${}_\phi E_2$ = voltage induced in the secondary by the flux ${}_u^M\Phi$.

$_{\phi}E_1 = \dfrac{n_1\,{}_{\phi}E_2}{n_2}$, voltage in primary balancing the voltage induced by the flux ${}^{M}_{u}\Phi$ in the primary.

E_2 = terminal voltage of secondary.
r_0 = line resistance.
x_0 = line reactance.
r_1 = primary resistance.
x_1 = primary reactance.
r_2 = secondary resistance.
x_2 = secondary reactance.
r_3 = load resistance.
x_3 = load reactance.
$_{m}g$ = equivalent conductance in magnetizing circuit.
$_{m}b$ = equivalent susceptance in magnetizing circuit.
I_1 = primary current.
I_2 = secondary current.
$_{m}I$ = exciting current.

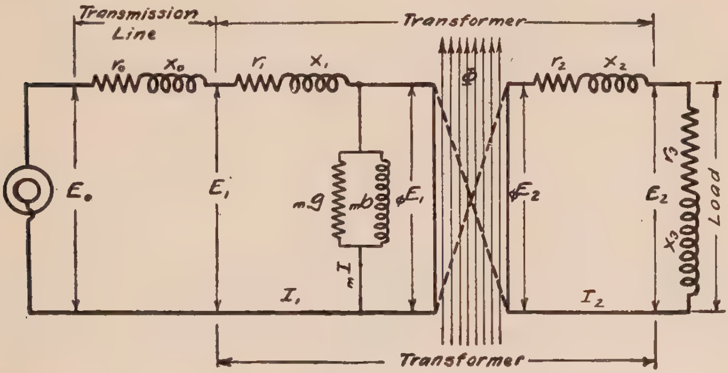


FIG. 4.12.

The broken lines across the magnetic flux lines are intended to indicate that, while the power is transmitted from the primary to the secondary winding by the flux, the directions of the current and voltage in the two windings are reversed. The exciting current flows in an equivalent parallel circuit having a conductance $_{m}g$ for the hysteresis and eddy-current losses, and a susceptance $_{m}b$ for the reactive power in the magnetic field. Starting at the load end, it is evident that the voltage $_{\phi}E_2$ induced in the secondary is equal to the vector sum of the terminal voltage E_2 and the impedance drop in the secondary caused by r_2 and x_2.

$$_{\phi}\dot{E}_2 = \dot{I}_2(r_2 + jx_2) + \dot{E}_2 \tag{13.12}$$

The voltage in the primary ${}_\phi E_1$ balanced by the induced voltage in the primary and equivalent to $-\frac{n_1}{n_2}{}_\phi E_2$ combined vectorially with the impedance drop in the primary is equal to the voltage E_1 impressed on the primary.

$$\dot{E}_1 = \dot{I}_1(r_1 + jx_1) + {}_\phi\dot{E}_1 = \dot{I}_1(r_1 + jx_1) - \frac{n_1}{n_2}{}_\phi E_2 \quad (14.12)$$

The voltage impressed on the transformer E_1 combined vectorially with the impedance drop of the line equals the generator voltage E_0.

$$\dot{E}_0 = \dot{I}_1(r_0 + jx_0) + \dot{E}_1 \quad (15.12)$$

The current in the primary I_1 consists of the equivalent primary load current $I_{11} = -\frac{n_2}{n_1}I_2$ combined vectorially with the exciting current ${}_m I$.

$$\dot{I}_1 = {}_m\dot{I} + \dot{I}_{11} = {}_m\dot{I} - \frac{n_2}{n_1}\dot{I}_2 \quad (16.12)$$

(*e*) **Vector Diagrams.**—The time is selected so that the magnetic flux vector lies in the positive direction on the Y-axis,

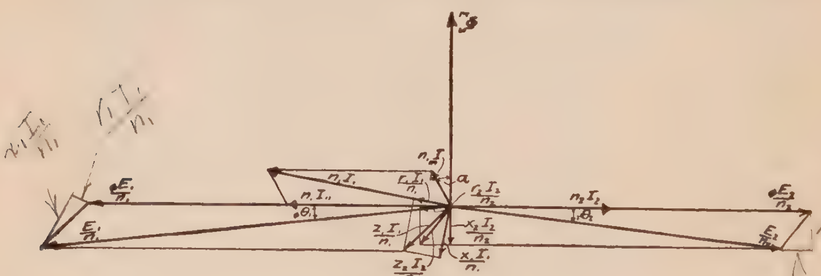


FIG. 5.12.

Fig. 5.12. The voltage $\frac{{}_\phi E_2}{n_2}$ induced per turn in the secondary, by the flux ${}^M_u\Phi$, lags behind the flux by 90° and therefore is drawn in the positive direction along the X-axis. The impressed voltage in the primary $\frac{{}_\phi E_1}{n_1}$ required to balance the voltage induced per turn in the primary by the flux ${}^M_u\Phi$ is equal to $\frac{{}_\phi E_2}{n_2}$ and opposite in direction. Hence $\frac{{}_\phi E_1}{n_1}$ is drawn along the X-axis in the negative direction. Let the current in the secondary

be in phase with the induced voltage ${}_{\phi}E_2$, then the ampere-turns are represented by the vector n_2I_2. The impedance drop per turn in the secondary is given by the vectors $\frac{r_2}{n_2}I_2$ and $\frac{x_2}{n_2}I_2$, in time phase and in time quadrature, respectively, with the current I_2. The secondary terminal or load voltage $\frac{E_2}{n_2}$ is the vector resultant of $\frac{{}_{\phi}E_2}{n_2}$, $\frac{r_2I_2}{n_2}$ and $\frac{x_2I_2}{n_2}$. Hence $\frac{E_2}{n_2}$ closes the voltage triangle in the secondary.

The current in the primary I_1 is the vector sum of the exciting current ${}_mI$ and I_{11}, the current equivalent to the secondary and differing by 180° in phase, as given by equation (16.12). The impedance drop per turn in the primary is given by $\frac{r_1}{n_1}I_1$ and $\frac{x_1}{n_1}I_1$ in time phase and in time quadrature, respectively, with the primary current. The impressed voltage $\frac{E_1}{n_1}$ per turn in the primary is therefore the vector sum of $\frac{{}_{\phi}E_1}{n_1}$, $\frac{r_1}{n_1}I_1$ and $\frac{x_1}{n_1}I_1$.

In illustrative vector diagrams for transformers the phase relations are of much more importance than the relative magnitudes of the currents and voltages. Therefore, in Figs. 6.12 to 15.12 inclusive, the resistance and reactance drops, both in the primary and secondary and the exciting currents are, relatively to the impressed voltages and load currents, represented as from ten to twenty times as large as in ordinary commercial transformers. Also the current and voltage notation represents ampere-turns and volts per turn, or the ratio of the number of turns has been taken as unity.

In the series, Figs. 6.12 to 15.12, the power factor of the load is varied so as to illustrate the effect of the transformer impedance on the terminal voltages for lagging and leading currents. The angle ${}_{\phi}\theta_2$ refers to the phase difference of the secondary current and the induced secondary voltage. It is therefore the time lag in the secondary circuit due to the total reactance, that is, to the sum of the internal reactance in the secondary circuit and the external or load reactance.

The following values are constant:

${}^{M}_{u}\Phi$, the maximum flux.

f, the frequency.

n_1 and n_2, the number of turns in primary and secondary.

r_1 and r_2, the resistance in the primary and secondary.

x_1 and x_2, the reactance in the primary and secondary.

${}_mI$, the exciting current (ampere-turns).

α, the hysteretic and eddy-current angle of advance assumed to be 30°.

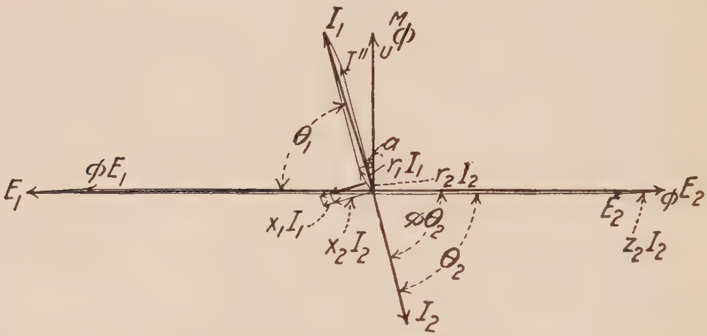


FIG. 6.12.—${}_\phi\theta_2$ = 75°, lagging current.

${}_mg$ and ${}_mb$, the conductance and susceptance of the exciting circuit.

I_2, the secondary current (ampere-turns).

I_{11}, component of primary current opposite and equal to I_2 (ampere-turns).

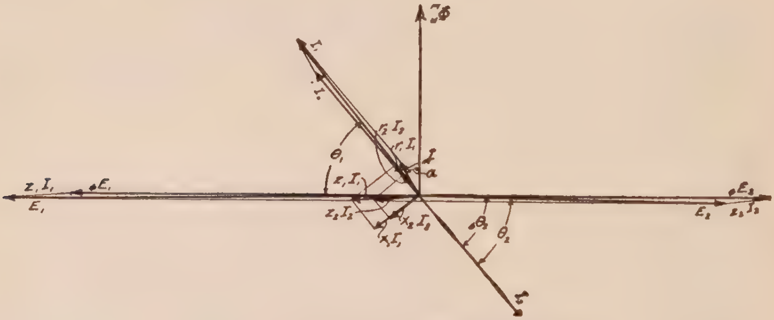


FIG. 7.12.—${}_\phi\theta_2$ = 50°, lagging current.

${}_\phi\theta_2$, the angle between the load current and the induced voltage in the secondary is assumed to have the following values:

Fig. 6.12, ${}_\phi\theta_2$ = 75°, lagging current.

Fig. 7.12, ${}_\phi\theta_2$ = 50°, lagging current.

Fig. 8.12, ${}_\phi\theta_2$ = 25°, lagging current.

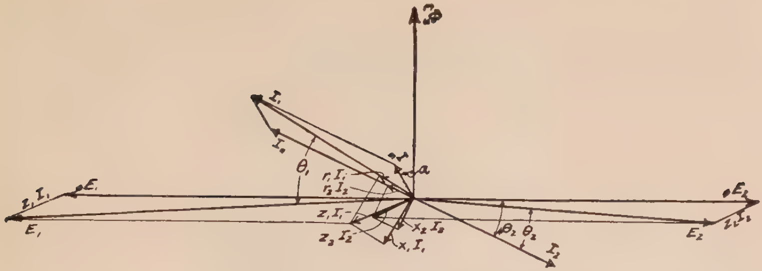


FIG. 8.12.—$_\phi\theta_2$ = 25°, lagging current.

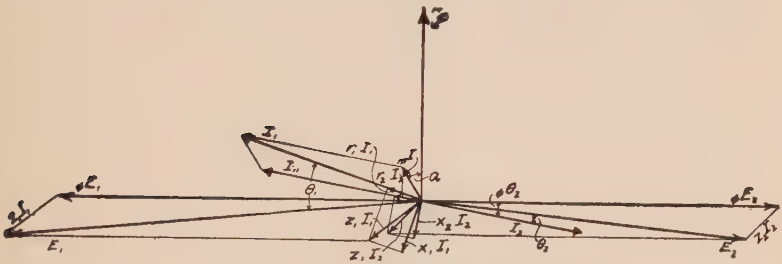


FIG. 9.12.—$_\phi\theta_2$ = 10°, lagging current.

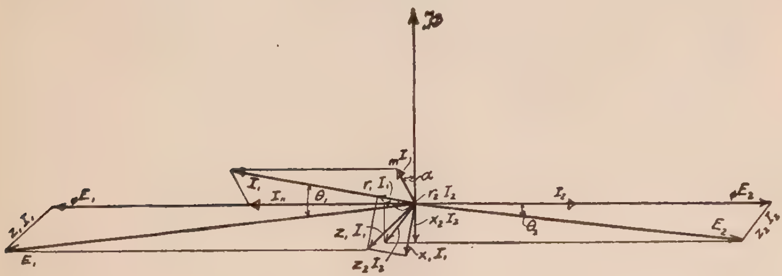


FIG. 10.12.—$_\phi\theta_2$ = 0°.

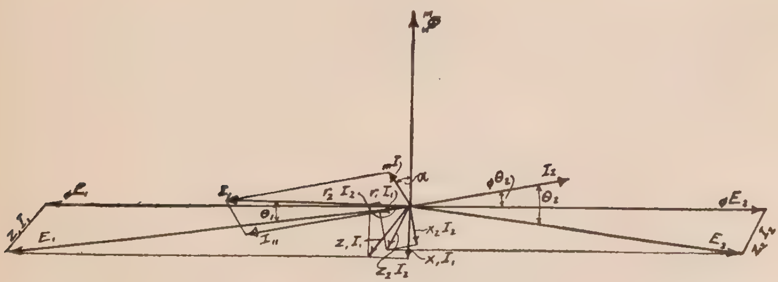


FIG. 11.12.—$_\phi\theta_2$ = 10°, leading current.

Fig. 9.12, ${}_{\phi}\theta_2 = 10°$, lagging current.
Fig. 10.12, ${}_{\phi}\theta_2 = 0°$, unity power factor.

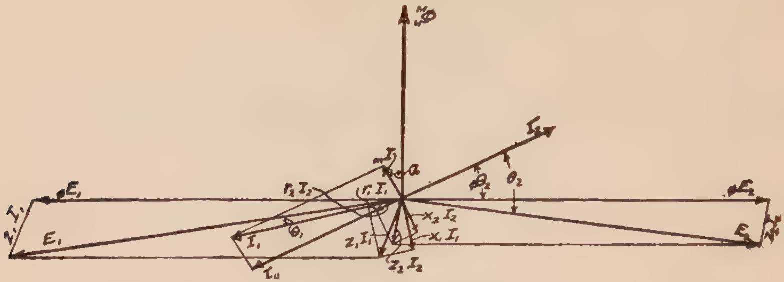


Fig. 12.12.—${}_{\phi}\theta_2 = 25°$, leading current.

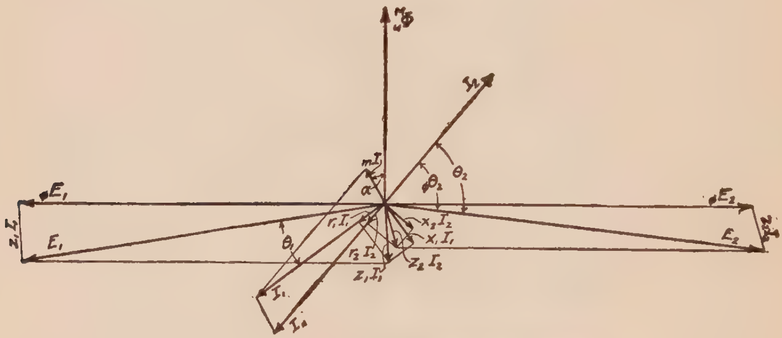


Fig. 13.12.—${}_{\phi}\theta_2 = 50°$, leading current.

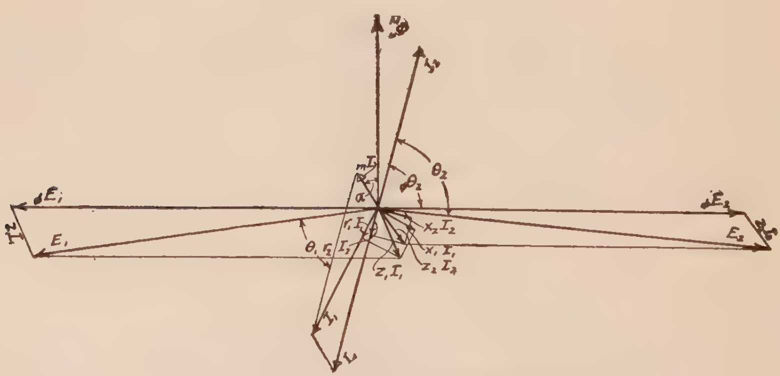


Fig. 14.12.—${}_{\phi}\theta_2 = 75°$, leading current.

Fig. 11.12, ${}_{\phi}\theta_2 = 10°$, leading current.
Fig. 12.12, ${}_{\phi}\theta_2 = 25°$, leading current.

Fig. 13.12, ${}_{\phi}\theta_2 = 50°$, leading current.
Fig. 14.12, ${}_{\phi}\theta_2 = 75°$, leading current.
Fig. 15.12, ${}_{\phi}\theta_2$ varies from 90° lagging to 90° leading.

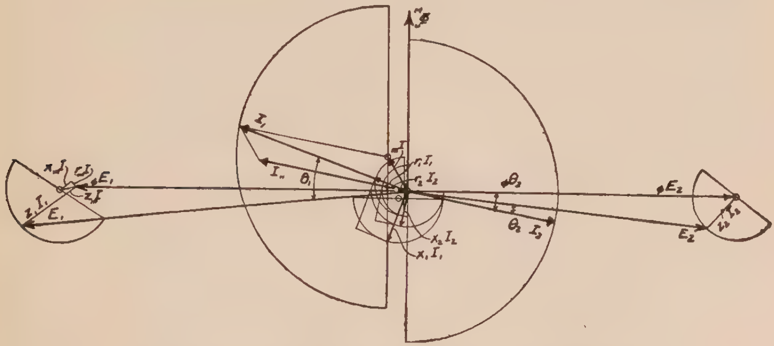


FIG. 15.12.—${}_{\phi}\theta_2$ variable, current and voltage loci.

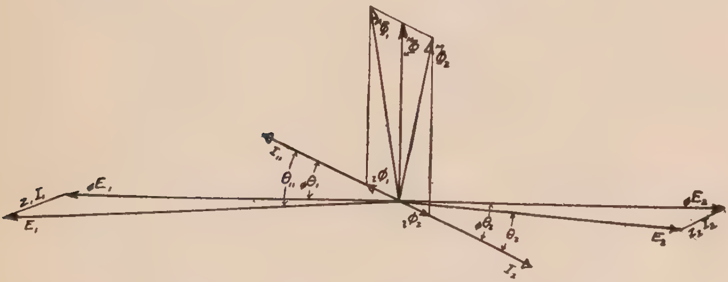


FIG. 16.12.—${}_{\phi}\theta_2 = 25°$, lagging current.

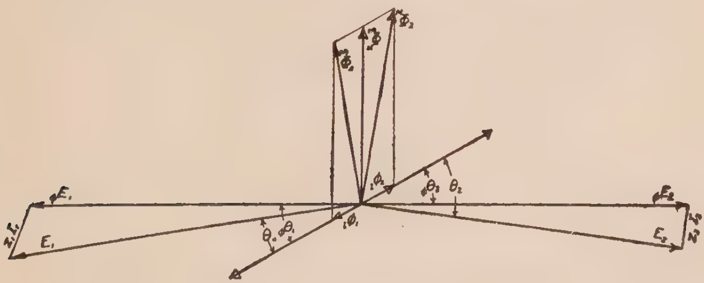


FIG. 17.12.—${}_{\phi}\theta_2 = 30°$, leading current.

In the preceding diagrams, ${}^{M}_{u}\Phi$ represents the maximum useful magnetic flux, or the lines of force passing through both the primary and secondary coils. The total flux in each coil is the vector sum of ${}^{M}_{u}\Phi$ and the leakage flux ${}_{l}\Phi$. In Figs. 1.12,

2.12, 16.12 and 17.12 are shown useful flux ${}^M_u\Phi$, the primary leakage flux ${}_l\phi_1$, the secondary leakage flux ${}_l\phi_2$, the total primary flux ${}^M\Phi_1$ and the total secondary flux ${}^M\Phi_2$. Since the leakage flux, representing the reactive power in the transformer, is necessarily in phase with the current, the total primary and secondary fluxes vary both in magnitude and phase position for changes in the angle of lead or lag of the load current.

For the assumed values of ${}_\phi\theta_2$ the following quantities are found graphically:

I_1 = the primary current (ampere turns).
E_1 = the primary impressed voltage (volts per turn).
θ_1 = the phase angle of E_1 and I_1.
E_2 = the secondary terminal voltage (volts per turn).
θ_2 = the phase angle of E_2 and I_2.

(*f*) **Equivalent Circuits.**—The transfer of power from the generator to the load through the transformer is much the same as if the primary and secondary formed a series circuit. With a change in the secondary constants the transformer circuit in Fig. 4.12 may be represented by the equivalent continuous circuit in Fig. 18.12. In the secondary circuit electric energy is changed into heat by the $r_2 I_2{}^2$ losses. In the equivalent circuit in Fig. 18.12 the same loss must be represented by the product of a resistance r' into the square of the equivalent primary current.

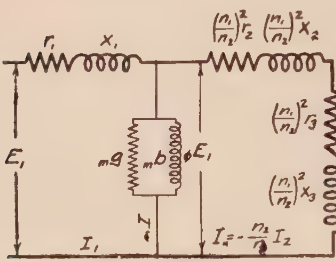


FIG. 18.12.

$$r' I_{11}^2 = r_2 I_2{}^2 \tag{17.12}$$

$$I_2 = \frac{n_1}{n_2} I_{11} \tag{18.12}$$

Hence the equivalent resistance $= \left(\frac{n_1}{n_2}\right)^2 r_2$ (19.12)

Similarly, the equivalent secondary reactance $= \left(\frac{n_1}{n_2}\right)^2 x_2$ (20.12)

Likewise the equivalent load resistance $= \left(\frac{n_1}{n_2}\right)^2 r_3$ (21.12)

And the equivalent load reactance $= \left(\frac{n_1}{n_2}\right)^2 x_3$ (22.12)

Since the resistance and reactance of the primary are small, the difference between the impressed voltage E_1 and the voltage inside the transformer ${}_{\phi}E_1$ is small compared to E_1. Hence the circuit may be simplified by transferring the exciting current circuit to the primary terminals, as shown in Fig. 19.12. A further simplification in notation may be made by letting r_t equal

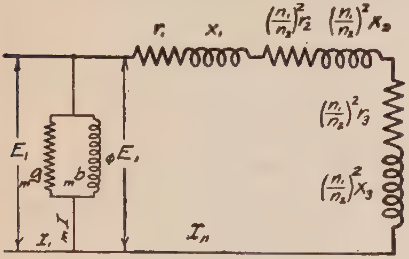


FIG. 19.12.

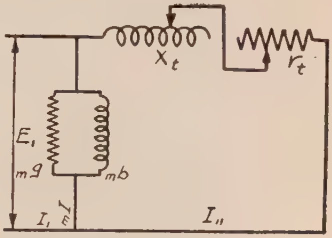


FIG. 20.12.

the equivalent total resistance and x_t the equivalent total reactance of the primary and secondary windings and of the load.

$$r_t = r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2 + \left(\frac{n_1}{n_2}\right)^2 r_3 \qquad (23.12)$$

$$x_t = x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2 + \left(\frac{n_1}{n_2}\right)^2 x_3 \qquad (24.12)$$

The equivalent simple circuit diagram which closely approximates the actual transformer is shown in Fig. 20.12.

(*g*) **The Circle Diagram.**—The conductance ${}_mg$ and susceptance ${}_mb$ are constant, and hence for constant impressed voltage E_1, the exciting current ${}_mI$ is constant. The equivalent total resistance r_t and reactance x_t vary with the load. *For noninductive load* the total reactance x_t is constant. The locus of the extremity of the vector representing the current flowing through a constant reactance x_t, and a variable resistance r_t, for a constant impressed voltage E_1, is a semicircle, as explained in Chap. VII. The corresponding vector

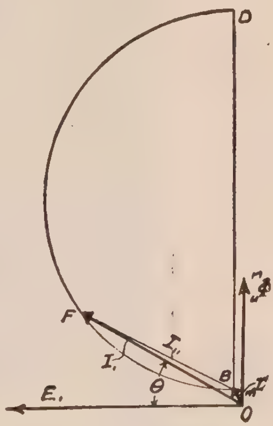


FIG. 21.12.—Circle diagram for load having unity power factor.

diagram, called the *transformer circle diagram*, is shown in Fig. 21.12. Counting time as in the previous diagrams, the flux ${}^M_u\Phi$ is drawn

upward along the Y-axis. The exciting current ${}_mI$ leads the flux by α, the hysteretic angle of advance. The impressed voltage leads

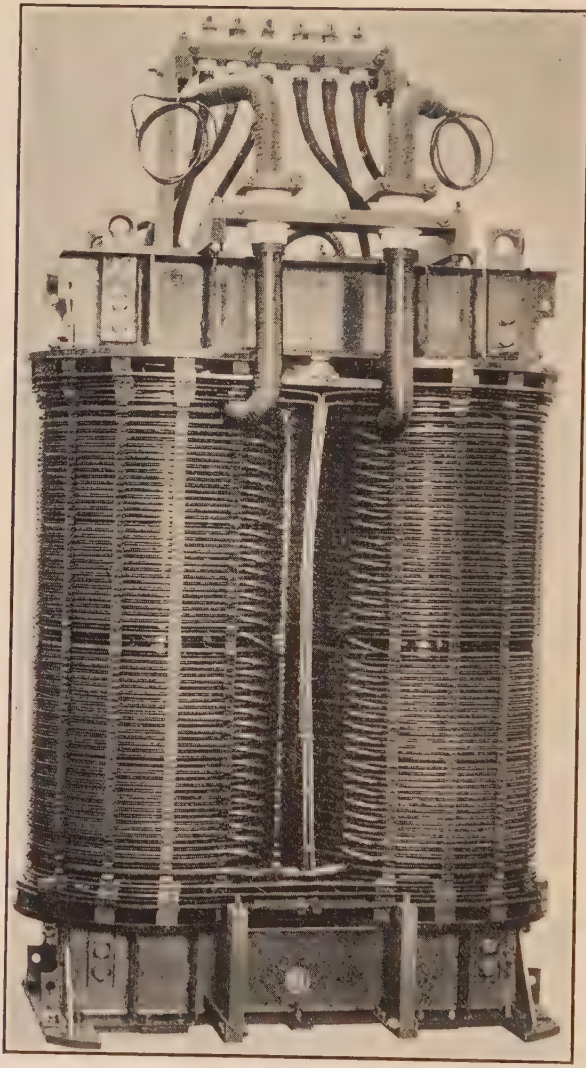


FIG. 22.12.—View from high voltage side of transformer. Type H, 50 cycles, 3,333 kw., 63,500/110,000 Y-6,600 volts. (*General Electric Company.*)

the flux by 90°. From the end of the exciting-current vector draw the line BD parallel to the magnetic flux and of length equal to

$I_1^2 R_1 = I_2^2 R_2$; $\frac{R_1}{R_2} = \frac{I_2^2}{I_1^2} = \left(\frac{N_1}{N_2}\right)^2$; Under the foregoing assumption, the primary and secondary resistances are proportional to the squares of their numbers of turns.

$\frac{E_1}{x_t}$. Upon BD as diameter describe the semicircle BFD. This is the locus of the equivalent load current I_{11}. The total primary current I_1 is the vector sum of ${}_mI$ and I_{11}.

(*h*) **Regulation.**—For constant-potential transformers, regulation is defined as the percentage of increase of secondary voltage from full load to no load, the primary impressed voltage being constant. In the vector diagrams the induced voltage has been assumed constant, thus indicating a change in both the primary and secondary terminal voltage. Assuming that the drop of voltage in the primary due to the exciting current is negligible, then at no load the terminal voltages are directly proportional to the number of turns. Hence, at no load the secondary terminal voltage is equal to $\frac{n_2}{n_1}E_1$.

Let E_2 be the secondary terminal voltage at full load, and E_1 the primary voltage.

$$\text{Regulation} = 100 \frac{\frac{n_2}{n_1}E_1 - E_2}{E_2} \qquad (25.12)$$

To determine the regulation from transformer constants either E_1 or E_2 must be found in terms of the other, the primary and secondary resistances and reactances and the power factor of the load. The vector diagram in Fig. 23.12 represents the full-load voltage E_2, and corresponding no-load voltage $\frac{n_2}{n_1}E_1$, with the load current I_2, lagging $\theta_2°$.

r_2I_2 = voltage consumed in secondary by r_2.

r_1I_1 x_2I_2 = voltage consumed in secondary by x_2.

$\left(\frac{n_2}{n_1}\right)^2 r_1I_2$ = equivalent voltage drop in secondary caused by r_1.

$\left(\frac{n_2}{n_1}\right)^2 x_1I_2$ = equivalent voltage drop in secondary caused by x_1.

For the vector diagram in Fig. 23.12, illustrating regulation of transformers, let E_2, I_2 and θ_2 be given or obtained from voltmeter, ammeter and wattmeter readings.

Let the current I_2 be used as reference vector. From the terminal of $\dot{E}_2$ lay off the voltage drops r_2I_2 and x_2I_2 parallel and at right angles, respectively, to the current I_2. Draw the induced

$I_1 n_1 = I_2 n_2$

$I_1 = I_2 \frac{n_2}{n_1}$

voltage ${}_\phi\dot{E}_2$. Similarly, from the terminal of ${}_\phi\dot{E}_2$ lay off the equivalent voltage drops $\left(\frac{n_2}{n_1}\right)^2 r_1 I_2$ and $\left(\frac{n_2}{n_1}\right)^2 x_1 I_2$ in phase and in quadrature, respectively, to the current I_2. This determines $\frac{n_2}{n_1}E_1$, in both magnitude and direction. From the given construction of Fig. 23.12 it is evident that:

$$\frac{n_2}{n_1}E_1 = \sqrt{\left[E_2\cos\theta_2 + r_2I_2 + \left(\frac{n_2}{n_1}\right)^2 r_1 I_2\right]^2 + \left[E_2\sin\theta_2 + x_2I_2 + \left(\frac{n_2}{n_1}\right)^2 x_1 I_2\right]^2} \qquad (26.12)$$

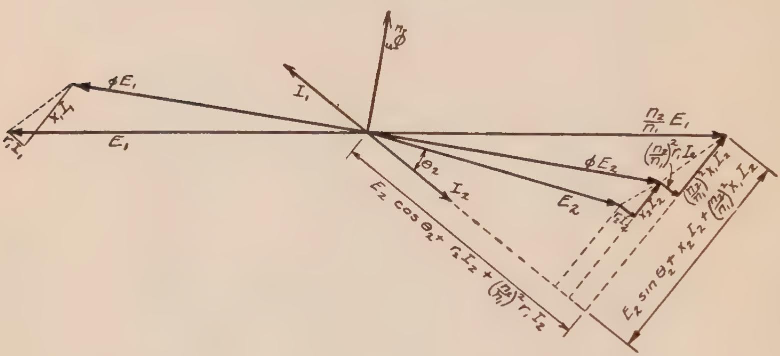


FIG. 23.12.—Transformer regulation. Load current lagging.

Hence, from equation (26.12) and (25.12), the regulation of the transformer when carrying loads in which the current is lagging is expressed by equation (27.12).

Regulation (I_2 lagging)

$$= 100\,\frac{\sqrt{\left[E_2\cos\theta_2+r_2I_2+\left(\frac{n_2}{n_1}\right)^2 r_1I_2\right]^2+\left[E_2\sin\theta_2+x_2I_2+\left(\frac{n_2}{n_1}\right)^2 x_1I_2\right]^2}-E_2}{E_2} \qquad (27.12)$$

In a similar manner the regulation is obtained for transformers carrying loads with leading power factor as illustrated in Fig. 24.12. The vector diagram is obtained in the same way as for Fig. 23.12. In the equation for regulation some of the terms change signs as shown in equation (28.12).

$$\frac{n_2}{n_1}E_1 = \sqrt{\left[E_2\cos\theta_2 + r_2I_2 + \left(\frac{n_2}{n_1}\right)^2 r_1 I_2\right]^2 + \left[E_2\sin\theta_2 - x_2I_2 - \left(\frac{n_2}{n_1}\right)^2 x_1 I_2\right]^2} \qquad (28.12)$$

Regulation (I_2 leading)

$$= 100\frac{\sqrt{\left[E_2\cos\theta_2 + r_2I_2 + \left(\frac{n_2}{n_1}\right)^2 r_1I_2\right]^2 + \left[E_2\sin\theta_2 - x_2I_2 - \left(\frac{n_2}{n_1}\right)^2 x_1I_2\right]^2} - E_2}{E_2} \quad (29.12)$$

If in the above problem the voltage $\dot{E}_2$ be selected as the reference vector, the equations become somewhat different in form but give the identical numerical results.

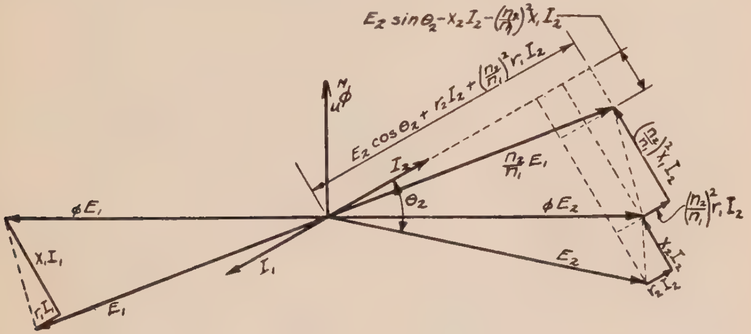


FIG. 24.12.—Transformer regulation. Load current leading.

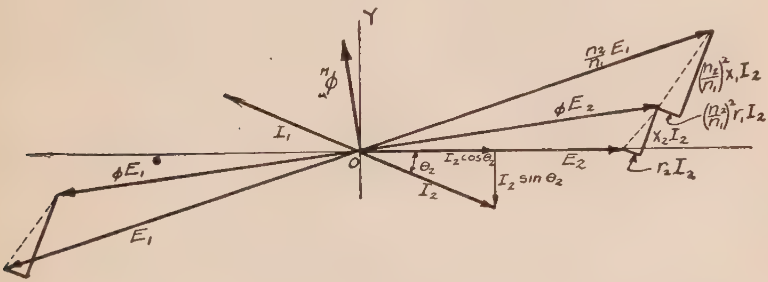


FIG. 25.12.

The construction in Fig 25.12 is similar to that of Fig. 23.12 and equation (30.12) is obtained directly from the vector diagram.

$$\begin{aligned}\frac{n_2}{n_1}\dot{E}_1 &= \dot{E}_2 + z_2\dot{I}_2 + \left(\frac{n_2}{n_1}\right)^2 z_1\dot{I}_2 \\ &= \dot{E}_2 + (r_2 + jx_2)I_2(\cos\theta_2 - j\sin\theta_2) \\ &\qquad + \left(\frac{n_2}{n_1}\right)^2 (r_1 + jx_1)I_2(\cos\theta_2 - j\sin\theta_2) \quad (30.12)\end{aligned}$$

Hence for absolute values:

$$\frac{n_2}{n_1}E_1 = \left\{\left[E_2 + r_2I_2 \cos\theta_2 + \left(\frac{n_2}{n_1}\right)^2 r_1I_2\cos\theta_2 + x_2I_2\sin\theta_2 + \left(\frac{n_2}{n_1}\right)^2 x_1I_2\sin\theta_2\right]^2 + \left[x_2I_2\cos\theta_2 + \left(\frac{n_2}{n_1}\right)^2 x_1I_2\cos\theta_2 - r_2I_2\sin\theta_2 - \left(\frac{n_2}{n_1}\right)^2 r_1I_1\sin\theta_2\right]^2\right\}^{1/2} \qquad (31.21)$$

Having determined the value of $\left(\frac{n_2}{n_1}\right)E_1$, the regulation is obtained by equation (25.12). The solution may, of course, be obtained directly from equation (30.12) using vector notation.

(*i*) **Losses.**—The transformer consists of a laminated iron core, copper conductors and insulation material or the dielectric.

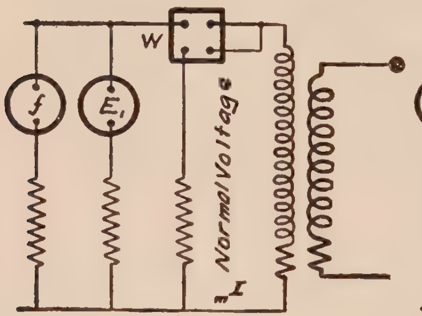


FIG. 26.12.

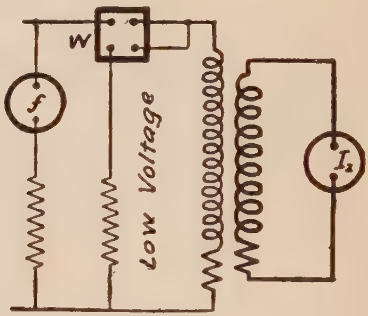


FIG. 27.12.

The iron losses were discussed in Chap. XI and found to be

$$_hP = \eta V f \mathfrak{B}^{1.6}10^{-7} \text{ watts due to hysteresis} \qquad (32.12)$$
$$_eP = \epsilon\lambda V f^2 \mathfrak{B}^2 10^{-7} \text{ watts due to eddy currents} \qquad (33.12)$$

For certain grades of silicon steel $_hP = \eta V f \mathfrak{B}^{1.7}10^{-7}$ watts.

The dielectric losses are similar to the iron losses but relatively small as explained in Chap. XX. In commercial tests they are usually included with the hysteresis and eddy-current losses of the iron. In the copper conductors the losses are given by Joule's law:

$$P_c = r_1I_1^2 + r_2I_2^2 \qquad (34.12)$$

These losses are slightly larger than for direct currents on account of eddy currents in the copper conductors. The effective values of r_1 and r_2 in equation (34.12) are therefore proportionately

larger than the ohmic resistance of the primary and secondary windings.

$$\text{Total loss} = {}_hP + {}_eP + {}_cP \tag{35.12}$$

The power lost in the transformer is changed into heat.

In commercial tests the losses are determined by wattmeter readings for the specified primary voltage, full-load current and frequency. The circuit diagram for finding the iron losses is shown in Fig. 26.12. The secondary is open. The copper loss in the primary due to the exciting current is negligible and hence the wattmeter reading gives the hysteresis and eddy-current losses in the iron and the dielectric.

The circuit diagram for finding the copper losses is shown in Fig. 27.12. The secondary is short-circuited through an ammeter. On the primary is impressed sufficient voltage, at the specified frequency, to send approximately full-load current through the ammeter in the secondary. The magnetization and voltage are so low that the iron and dielectric losses are negligible. The wattmeter reading therefore gives the copper losses. The sum of the two readings is the total power lost in the transformer; the losses in the ammeter itself are considered negligible.

(*j*) **Efficiency.**—The efficiency is the ratio between the output and the input.

$$\epsilon = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{E_2I_2 \cos\theta_2}{E_2I_2\cos\theta_2 + RI_2^2 + {}_hP + {}_eP} \tag{36.12}$$

$$\epsilon = \frac{E_2I_2\cos\theta_2}{E_2I_2\cos\theta_2 + I_1^2R_1 + I_2^2R_2 + P_0}$$

R = the equivalent resistance of the secondary and primary. The efficiency is a maximum when

$$\frac{d\epsilon}{dI_2} = 0. \tag{37.12}$$

$$(E_2I_2\cos\theta_2 + RI_2^2 + {}_hP + {}_eP)E_2\cos\theta_2 - E_2I_2\cos\theta_2(E_2\cos\theta_2 + 2RI_2) = 0$$

or

$${}_hP + {}_eP = RI_2^2 \tag{38.12}$$

That is, the iron loss is equal to the copper loss.

Transformers are usually so designed as to have the maximum efficiency between three-quarters and full load. While the efficiency at no load is zero, the losses at all loads are small. The curve rises rapidly and lies nearly horizontal from quarter to full load, as is shown in the typical efficiency curve in Fig. 29.12. Potential transformers have high efficiencies; large commercial units give up to 99.6 per cent at full load. In commercial con-

stant-potential systems the primary voltage is impressed on the transformer continuously, while power is taken from the secondary only part of the time. Thus in residence lighting the lamps are lighted only a few hours out of the twenty-four. The

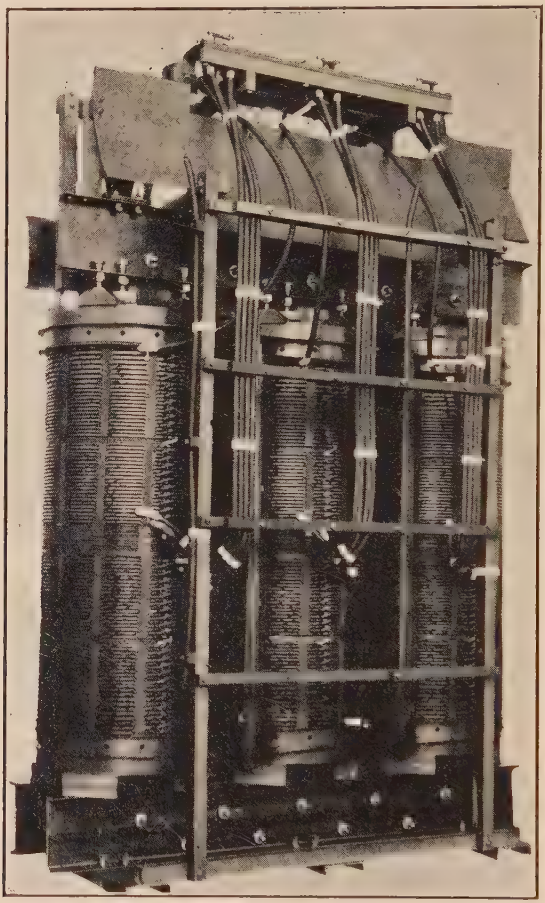


FIG. 28.12.—Core and coil assembly of a 7500-kv.a., three-phase, 60-cycle transformer, 110,000 volts star to 33,000 volts delta. View shows high voltage side. (*Westinghouse Electric and Manufacturing Company.*)

average efficiency, or all-day efficiency, as it is called, will therefore be lower, since the iron losses are constant throughout the 24 hr. If the transformer carries a steady unity power factor load, for k hr. each day, the all-day efficiency will be expressed by equation (39.12).

All-day energy efficiency:

$$\frac{E_2I_2k}{E_2I_2k + RI_2{}^2k + 24({}_hP + {}_eP)} \tag{39.12}$$

Distributing transformers are designed with a smaller iron loss than full-load copper loss, thereby increasing the average efficiency.

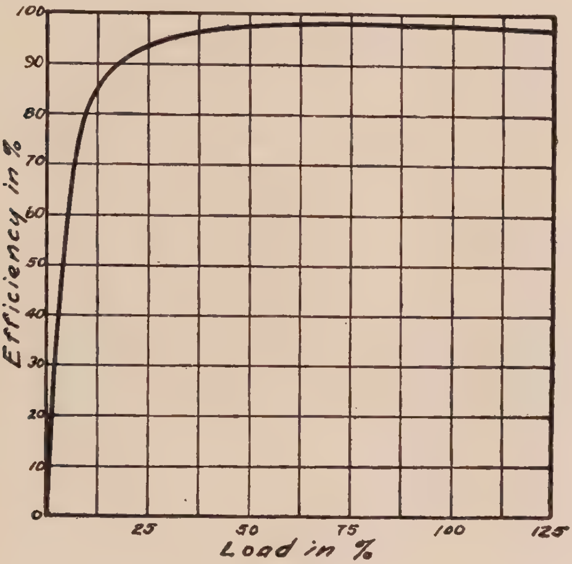


FIG. 29.12.

(*k*) **Rating.**—Transformers are designed for service at specified terminal voltage, frequency and load. For a properly designed transformer the temperature becomes the limiting factor in determining the load which the transformer can carry. The copper and iron losses are converted into heat inside the transformer. The temperature will necessarily rise until the rate of removal and generation of heat are equal. In small transformers the heat is dissipated by radiation and air conduction. The windings and core are immersed in oil to assist in the conduction of the heat to the surface of the casing. To increase the radiating surface, external tubes are provided for large-size transformers. In addition to self-cooled types large transformers are usually *water cooled*, that is, the heat is removed by means of cold water flowing through a system of pipes inside the transformer. In

late designs an air jet playing at the transformer casing produces additional radiation by removing the thin layer of air near the surface that otherwise causes heat insulation. The essential point is the fact that the temperature limit is the most important factor in determining the kw. or kv.a. rating of transformers. By the A.I.E.E. rules the temperature of the transformer should not rise more than 55°C. above the sur ounding air temperature.

(*l*) **Polarity.**—The windings of transformers are so connected to the leads extending out through the casing that the direction of

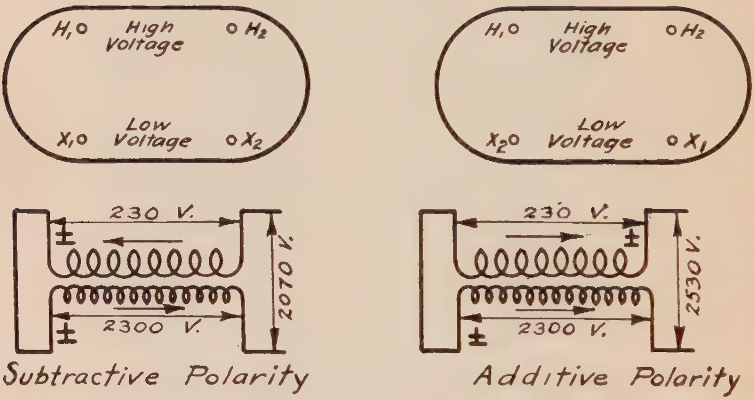


FIG. 30.12.—Polarity of single-phase transformers.

flow of current in the secondary with respect to the adjacent primary terminal is in the same direction for some types and in the opposite direction for other types, as illustrated in Fig. 30.12. This difference in bringing out the terminals[1] gives rise to the terms *subtractive* and *additive polarity* of alternating-current transformers, which may be defined as follows:

Consider a single-phase transformer having two high-voltage and two low-voltage external terminals. Connect one high-voltage terminal to the adjacent low-voltage terminal and impress an alternating-current e.m.f. across the two high-voltage terminals. If the voltage across the unconnected high-voltage and low-voltage terminals is *less* than the voltage across the two high-voltage terminals the *polarity is subtractive*, and if *greater*

[1] For rules and regulations on polarity markings for the leads of transformers on all standard types and ratings, see "Transformer Standards," published by the N.E.M.A. (National Electrical Manufacturers' Association).

than the voltage impressed on the high-voltage terminals the *polarity is additive.* In general transformers of more than 200 k.va. are subtractive and below 200 k.va. additive.

The high- and low-voltage transformer terminals should in all cases be marked so as to indicate the relative direction of flow in adjacent terminals, and in addition the name plate of single-phase transformers should have the statement *subtractive polarity* or *additive polarity* to indicate definitely the polarity of the transformer to which it is attached.

(*m*) **Voltage Ratios.**—In order to manufacture and operate transformers, lamps, motors and other electrical appliances in an economical manner, uniformity in voltage is necessary; and certain standard voltages have been adopted for commercial systems. Thus for residence lighting and for small motors 110 and 220 volts are considered standard. The distribution network between the substation and the transformer near the customer is generally operated at 2,200 volts. Hence the pole transformer transmitting energy to the lighting and small motor load in any small division of the distribution system has 10:1 and 20:1 voltage ratios. Circuits connecting the several substations in large systems are operated at voltages from 11,000 to 22,000. The substation transformers change from primary transmission or distribution voltages as 66,000, 44,000 or 22,000, to distribution feeders at 6,900, 13,900, or 23,000 volts.

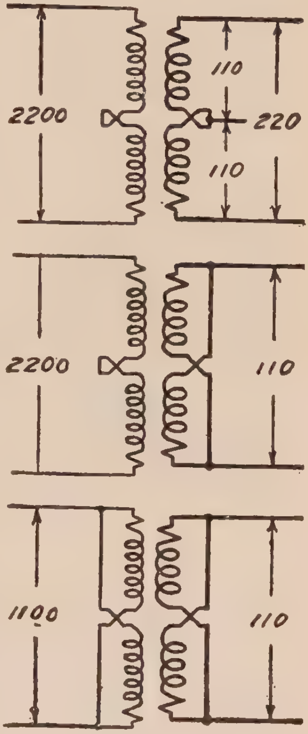


FIG. 31.12.

Long-distance transmission lines operate at higher voltages as 66,000, 110,000, 220,000, or some similar voltage, best adapted to the particular system, and this again requires transformers giving the required voltage ratio, stepping up at the generating station and stepping down at the substation end of the high-tension line. While the transformer can be built for any voltage

ratio within wide limits, most standard distribution types are designed for 10 or 20:1 ratio.

By dividing both the primary and secondary coils into two parts, different ratios may be obtained by connecting the coils in series or in parallel as shown in Fig. 31.12.

(*n*) **Connections.**—Although innumerable ways for connecting transformers may be devised, only a few forms are in general commercial use. Simplicity in wiring, efficiency in the use of apparatus, minimum insulation stresses, standardization of voltages and distributing apparatus have made uniformity in transformer connections necessary.

1. *Single-phase, Three-wire System.*—The secondary is divided into two coils, or a tap is brought out at the middle point as

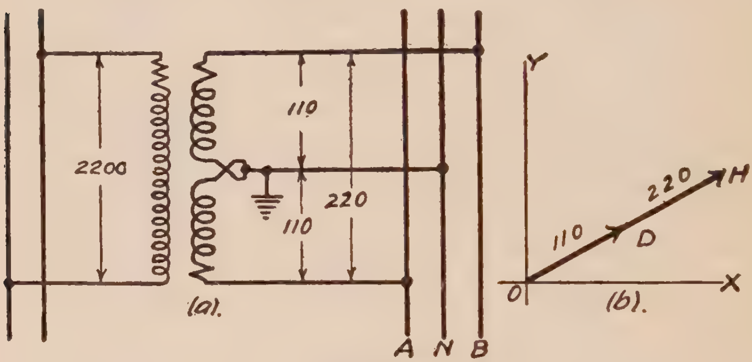


FIG. 32.12.

shown in Fig. 32.12. The middle or neutral wire is grounded in order to protect the customer as well as the apparatus. In general, the 2,200-volt side has only two wires and no neutral connection. In the vector diagram *OD* represents the voltage from either line to neutral and *OH* the voltage between the mains *A* and *B*. The system is widely used for general distribution, residence lighting, small motors, etc.

2. *Two-phase, Four-wire System.*—This system consists essentially of two separate single-phase circuits both on the primary and secondary side, Fig. 33.12. The voltages in the two phases differ by 90 electrical degrees. The secondary circuits often are three-wire systems for each phase similar to Fig. 32.12. To save copper the neutrals for both phases are combined, Fig. 34.12. The five wires on the secondary side give 110 and 220 volts in

both phases. Two-phase distribution is practically obsolete, although in some cases the equipment has not been replaced.

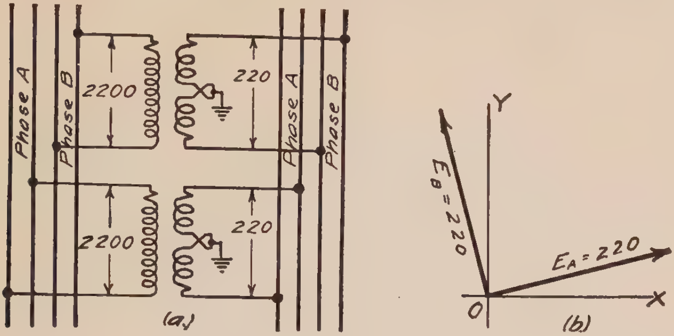


FIG. 33.12.

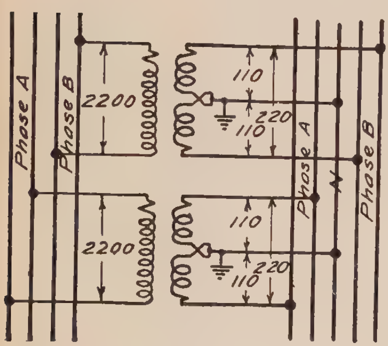


FIG. 34.12.

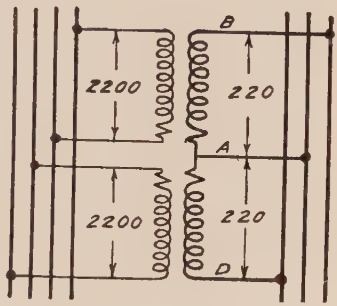


FIG. 35.12.

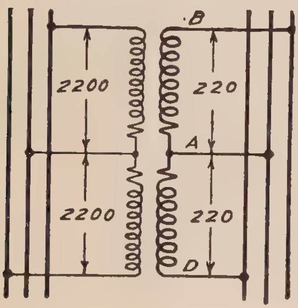


FIG. 36.12.

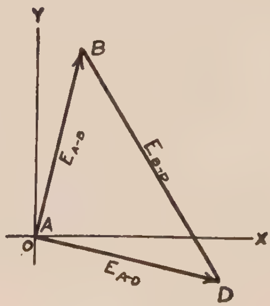


FIG. 37.12.

3. *Two-phase, Three-wire System.*—By combining one wire from each phase in a two-phase system less copper is required, since the current in the common wire is the vector sum of the currents in the two phases. In a balanced system the current

in the common wire is $\sqrt{2}$ times the current in either of the other wires. Hence, for the same voltage drop, the copper required for the third wire is in the ratio of 1.41:2 as compared to the corresponding two wires in the four-wire system. Wiring and vector diagrams are shown in Figs. 35.12., 36.12 and 37.12.

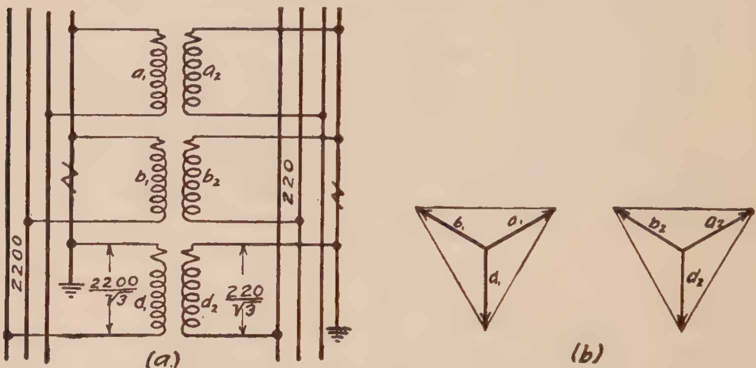


FIG. 38.12.

4. *Three-phase, Y-connection, Three-wire System.*—The connections and voltage relations are shown in Fig. 38.12. This system is well adapted for general power transmission but if one transformer is disabled the whole system is interrupted. The *Y* arrangement of the transformers provides a desirable neutral or ground connection.

5. *Three-phase, Y-connection, Four-wire System.*—This system is sometimes used for a combination of power and lighting loads, especially in connection with rotary converters in combination with the Edison three-wire system. The connections are shown in Fig. 40.12, secondary side. Alternating-current networks, generally based on the three-phase, four-wire system, are rapidly replacing the direct-current distribution formerly used in metropolitan areas.

The chief advantages of this system are: (*a*) increased flexibility, and (*b*) the neutral wire carries all unbalanced currents.

6. *Three-phase, Delta-connection, Three-wire System.*—The connections are shown in Fig. 41.12. One advantage of the delta connection is the possibility of supplying power on all three phases by the use of only two transformers, as explained in the next paragraph.

7. *Three-phase, Open-delta or V Connection.*—In three-phase circuits, single-phase transformers are used in each of the three

phases. In the delta connection continuous operation may be maintained by means of any two of the three transformers. This is sometimes desirable in a new district with light load. The

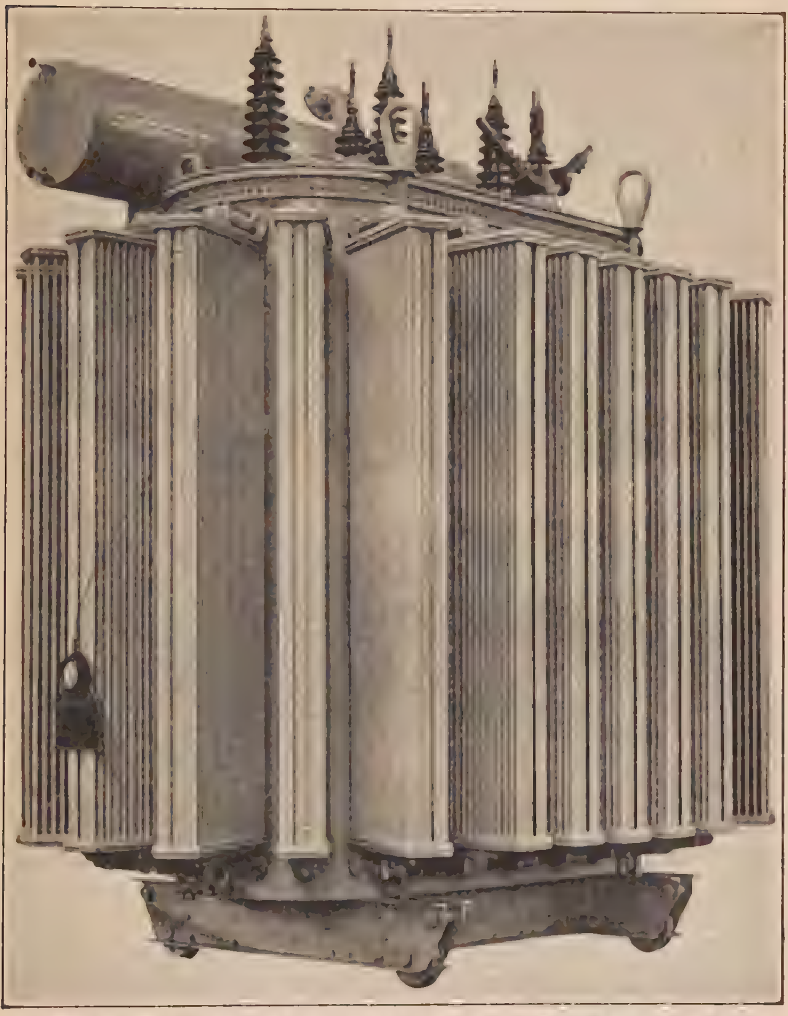


FIG. 39.12.—Three-phase transformer, air cooled. 12,000 kv-a., 44,000 volts; 60 cycle. View from the low-voltage side. (*General Electric Company.*)

third transformer may be added as soon as the load increases beyond the capacity of the two transformers. Also in case one transformer is damaged the V or open-delta connection becomes

a convenient arrangement for continuing the service until the damaged unit is repaired or replaced. However, the V connection introduces certain undesirable features that may cause dangerous disturbances in the system. It should also be noted that the two transformers in open delta will heat faster for the

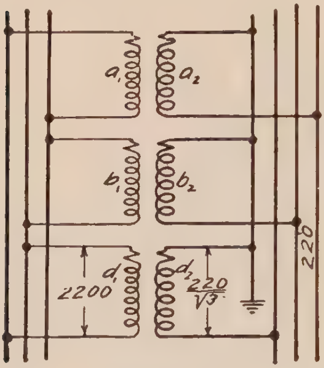


FIG. 40.12.

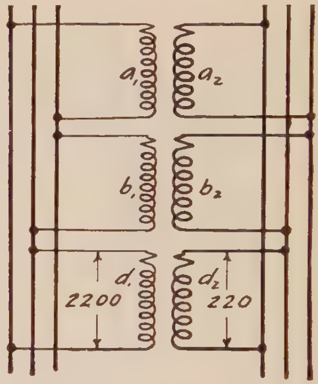


FIG. 41.12.

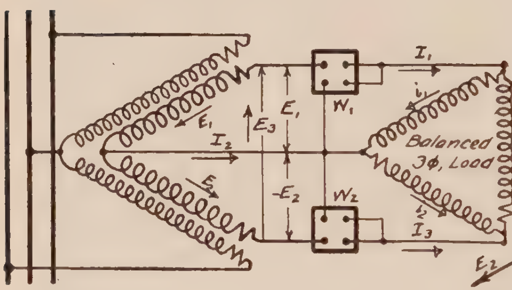


FIG. 42.12.

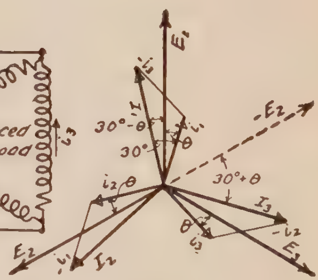


FIG. 43.12.

same load than when in the closed-delta connection. As proved below, the capacity is reduced to 86.7 per cent of the full-load rating; so that two transformers in open delta will carry, with the same rise in temperature, only $86.7 \times 2/3 = 57.8$ per cent of the load carried by the three transformers in closed delta. The circuit diagram is shown in Fig. 42.12 and the corresponding vector diagram in Fig. 43.12.

The load is balanced and hence,

$$i_1 = i_2 = i_3,\ I_1 = I_2 = I_3 \text{ and } \theta_1 = \theta_2 = \theta_3.$$

The power passing through the transformers is measured by the two wattmeters connected as shown in Fig. 42.12. The same cur-

rent flows through the transformers as in the current coil of the wattmeter, and hence the same as in the mains instead of in the circuits as would be the case with three transformers connected in delta.

If the connection of the voltage coil for wattmeter W_1 be taken as indicated by the arrows in the figure, then the connection for wattmeter W_2 is in the reverse direction.

$$W_1 = E_1 I_1 \cos (30° - \theta) \tag{40.12}$$

and

$$W_2 = E_2 I_3 \cos (30° + \theta) \tag{41.12}$$

Total power (two transformers)

$$= EI[\cos (30° - \theta) + \cos (30° + \theta)] \tag{42.12}$$

$$= 2EI \cos \theta \cos 30° = 2EI \cos \theta \times 0.867 \tag{43.12}$$

For three transformers the total power for full load would be:

$$W = 3EI \cos \theta \tag{44.12}$$

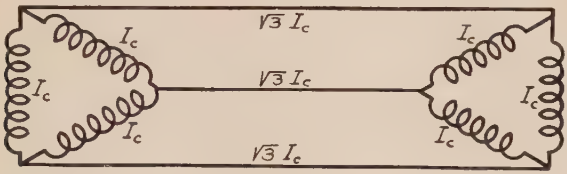


FIG. 44.12.

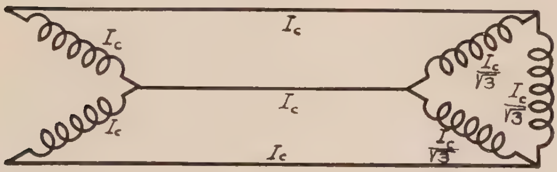


FIG. 45.12.

Therefore, the full-load capacity of two transformers in open delta is only 57.8 per cent of the three transformers in closed delta.

A more direct representation of the relative load-carrying capacities of transformers when used in *closed-delta* or *open-delta* connections may be had by referring to Figs. 44.12 and 45.12. Let the circuit diagram in Fig. 44.12 indicate a balanced, three-phase load equally divided between the three transformers in delta connection. The full-load currents in the transformers are all of the same magnitude, I_c. The line current for the delta connection is therefore $\sqrt{3}I_c$.

For the open delta the circuit diagram is shown in Fig. 45.12. In order that the transformer in open delta shall rise to the same temperature as when operating at full load in the closed-delta connections (Fig. 44.12), the current in each transformer must be of the full-load value I_c. It is evident from the circuit diagram that the line current must likewise be I_c. Therefore, the current in each phase of the load, Fig. 45.12, will be $\frac{I_c}{\sqrt{3}}$. The full load, P_3, for the three transformers in delta connection, Fig. 44.12, is expressed by equation (45.12).

$$P_3 = 3EI_c \cos \theta \tag{45.12}$$

The full load, P_2, for the two transformers in open-delta connection, Fig. 45.12, is given by equation (46.12).

$$P_2 = 3E\frac{I_c}{\sqrt{3}} \cos \theta \tag{46.12}$$

Hence:

$$\frac{P_2}{P_3} = \frac{1}{\sqrt{3}} = 57.8 \text{ per cent.} \tag{47.12}$$

The carrying capacity of each transformer in the open-delta as compared to that of the same transformer operated in the closed-delta connection is therefore as 28.9 is to 33.3, or 86.7 per cent.

8. *Distributed Y for Three-wire Edison System.*—(See Fig. 23.16, Chap. XVI.)

9. *Parallel Operation and the Delta and Y Three-wire Connections.*—If two transformer banks operate in parallel on both the primary and secondary sides, ten combinations of the delta and Y connections are possible. In six of these combinations the time-phase relations are the same for the two banks, while the remaining four will not work in parallel. This is shown in condensed form in Table VI.

TABLE VI

Combination	Primary A	Primary B	Secondary A	Secondary B
1	Δ	Δ	Δ	Δ
2	Y	Y	Y	Y
3	Y	Δ	Y	Δ
4	Δ	Y	Δ	Y
5	Y	Y	Δ	Δ
6	Y	Δ	Δ	Y
7	Δ	Δ	Δ	Y
8	Δ	Δ	Y	Δ
9	Y	Y	Δ	Y
10	Y	Y	Y	Δ

Numbers 7 to 10 will not work, since the secondary voltages will be 30° out of time phase, as is evident by inspection.

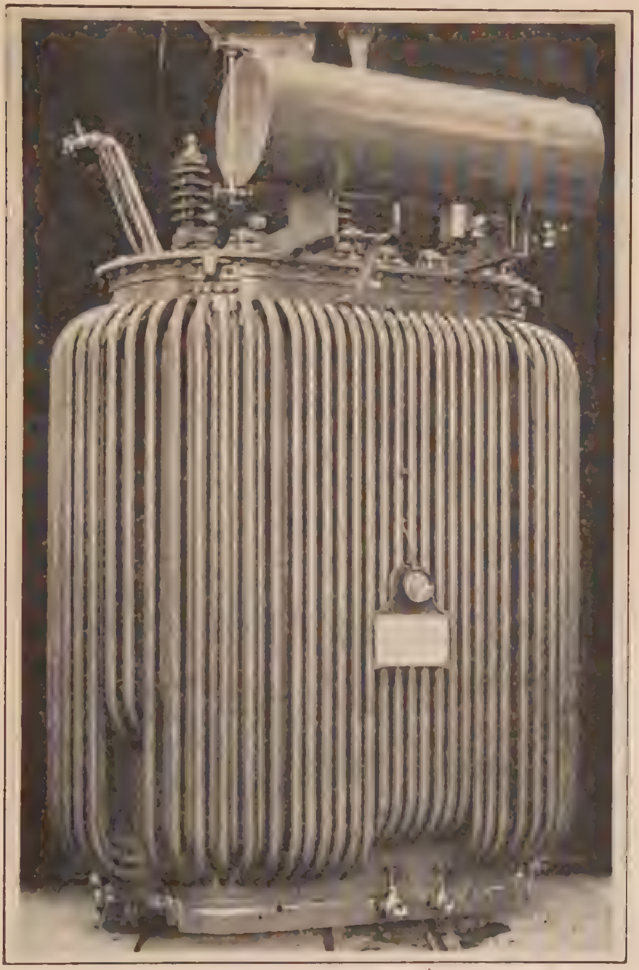


FIG. 46.12.—Outdoor transformer, view from the low voltage side. Type HT, 4,000 k.va., 13,187/26,374 – 4,150 Y volts, 60 cycle. (*General Electric Company.*)

10. *Phase Transformation. Two-phase, Three-Phase. The T or Scott Connection.*—Let the primaries of two single-phase transformers be connected to a two-phase, four-wire system as indicated in Fig. 47.12. Let the ratio of transformation in the two transformers be such that the secondary voltages are in the

ratio of $1:\frac{\sqrt{3}}{2}$; hence, if $E_A = 100$ volts, $E_B = 86.7$ volts. Connect one of the terminals of phase B to the middle point of phase A, as indicated in Fig. 47.12. The voltage between the three remaining terminals will be equal in magnitude and 120° different in phase, forming a three-phase circuit. This is readily seen from the vector diagram in Fig. 48.12 in connection with the circuit diagram in Fig. 47.12. The direction of the arrows and

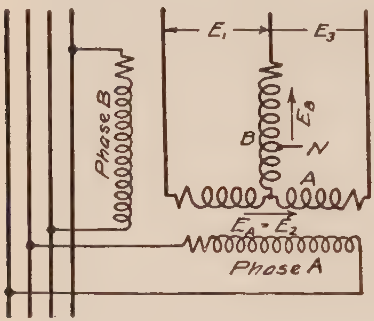


FIG. 47.12.

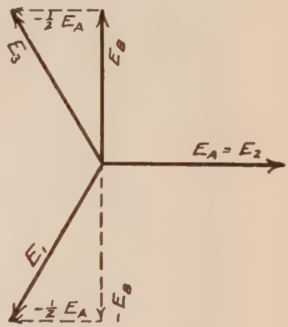


FIG. 48.12.

the notation in the two diagrams make it evident that $E_A = E_2$ in both phase and magnitude; that $\dot{E}_3$ is the vector difference of $\dot{E}_B$ and $\frac{\dot{E}_A}{2}$; and that $\dot{E}_1 = -\dot{E}_B - \frac{\dot{E}_A}{2}$. Since $E_B = \frac{\sqrt{3}}{2}E_A$, the vectors E_1, E_2 and E_3 are equal in magnitude and differ by 120° in phase. By this simple method a two-phase circuit is changed to three-phase. It is apparent that by the same arrangement a three-phase system can be changed into a two-phase. A neutral connection can be made by tapping transformer B at a point N, one-third the number of turns from the junction with the middle point of A. This excellent method for phase transformation was invented by Charles F. Scott, and is often called the *Scott connection.*

FIG. 49.12.

The diagram in Fig. 49.12 is a convenient method for showing the relation of the voltages or currents in two-phase or three-phase systems. Instead of drawing the vectors through the origin, a triangle is formed by connecting the free end of phase B to the terminals of phase A. Thus the base and perpendicular

represent the two-phase voltages and the sides of the triangle the three-phase voltages. The directions of the arrows indicate phase relations.

11. Other transformer connections in commercial use are the *ring four phase*, Fig. 50.12 and the *main and teaser*, Fig. 51.12.

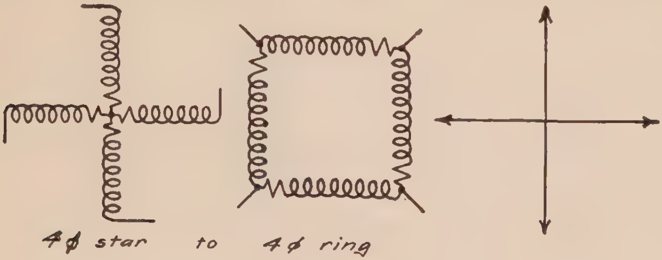


FIG. 50.12.

Several phase transformation connections from two phase to six phase and from three phase to six phase are discussed in Chap. XVI. See circuit and vector diagrams in Figs. 17.16 to 23.16 inclusive.

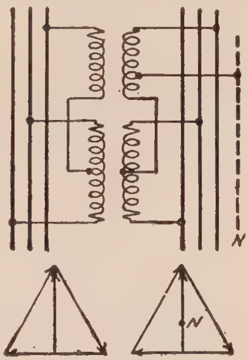


FIG. 51.12.—The main and teaser connection.

(*o*) **Autotransformer.**—If a single winding be placed around an iron core, and a tap brought out as shown in Fig. 52.12, power can be transmitted with a change in voltage in much the same way as in a transformer having two windings. The voltage on the generator and load leads will be in proportion to the number of turns. Thus in Fig. 52.12, if between the supply mains there are n_1 turns and between the service mains n_2 turns, the voltage and current relations are the same as if the transformer had two windings, a primary of n_1 turns and a secondary of n_2 turns. However, in the winding itself, and in the part carrying the current for both the supply and service circuits, the actual current flowing will be the difference between the currents in the outside mains. On this account the copper losses are less in an autotransformer for the same load than in the ordinary transformer having two windings. However, while the voltage between the wires in the service circuit may be low, the difference of potential to ground is the

same as on the supply circuit. For this reason autotransformers can be used only on circuits where the voltages on both sides are low or where the necessary protection is provided in other ways. In places where the danger factor is absent it is economical to use autotransformers, as both first cost and operating losses are less

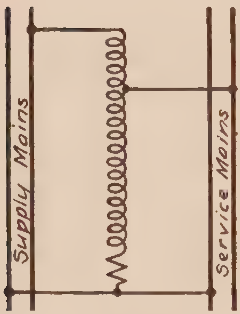


FIG. 52.12.

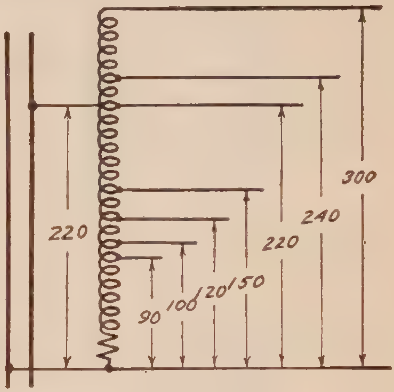


FIG. 53.12.

than for a regular transformer of equal rating. By means of several taps any number of voltages may be obtained from one autotransformer, as in Fig. 53.12. This autotransformer receives power at 220 volts and delivers at 300, 240, 220, 150, 120, 100 and 90 volts. Power may be taken from any or all of the secondary circuits at the same time.

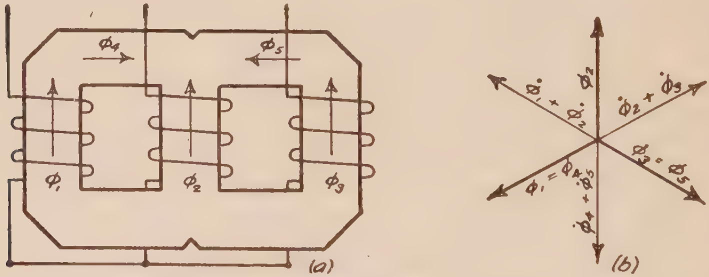


FIG. 54.12.—Three-phase, core type.

(*p*) **Three-phase Transformers.**—Since the magnetic fluxes in three single-phase transformers, connected to a three-phase system, bear the same time-phase relation as the voltages, a saving in iron may be secured by combining the three transformer cores forming one unit, called a three-phase transformer. In Fig. 54.12(*a*) are shown the electric and magnetic circuits, diagram-

matically, of a *core-type*, three-phase transformer. The corresponding magnetic-flux vector diagram is shown in Fig. 54.12(*b*). It is apparent that the vector sum of the fluxes in the cores is the same in magnitude as the resultant fluxes in the yokes. Hence for the same flux density the cross-sectional area of the iron must be the same in all parts of the magnetic circuit.

A similar diagram for the electric and magnetic circuits of a *shell-type* transformer is shown in Fig. 55.12(*b*) and the corresponding magnetic-flux vector diagram in Fig. 55.12(*c*). Note that the winding in the middle section is reversed in direction, compared to the two end windings. From the vector diagram it is evident that the fluxes in the crossbar portions of the yoke, ϕ_7, ϕ_8, ϕ_9, are equal in magnitude to ϕ_4, ϕ_5 and ϕ_6. The cross-section of the iron core, for the same flux density, is in proportion to the total magnetic flux in each of the magnetic circuits. Hence the cross-sections of the central cores for ϕ_1, ϕ_2 and ϕ_3 are twice the cross-sections of the crossbars for ϕ_7, ϕ_8 and ϕ_9, or for the yokes ϕ_4, ϕ_5 and ϕ_6.

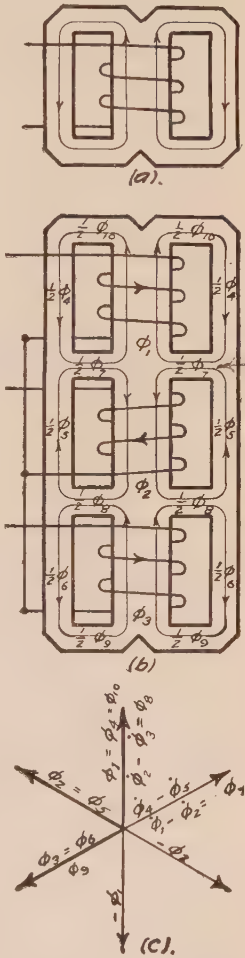


Fig. 55.12.—Three-phase, shell type.

A modified form of the *shell type*, called the *hexagonal type*, is shown in Fig. 56.12, and the corresponding magnetic-flux vector diagram in Fig. 57.12. From the diagram it is seen that the flux magnitude in each core ϕ_1, ϕ_2, or ϕ_3, is $\sqrt{3}$ times the flux in any part of the yoke ϕ_4, ϕ_5 or ϕ_6; and therefore the corresponding cross-sectional areas should be in the proportion of $\sqrt{3}$:1.

The more important advantages gained in large units by using one three-phase transformer in place of three single-phase transformers of equal rating and magnetic-flux density are:

1. The weight of the iron core is less by about 16 per cent.
2. The efficiency is increased by 0.15 to 0.4 per cent.

3. The required floor space is reduced by about 50 per cent.
4. The first cost is less by from 10 to 15 per cent.
5. The three-phase, core-type transformer has the advantage of using the third harmonic as magnetizing current in the Y-Y connection or in Y-connected autotransformers.

The chief disadvantage is in the probability of having more frequent interruptions in the service. Damage on either of the three phases in the three-phase transformer interrupts the service until the repairs have been completed. When three single-phase

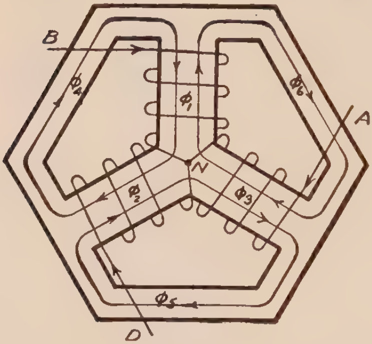


FIG. 56.12.

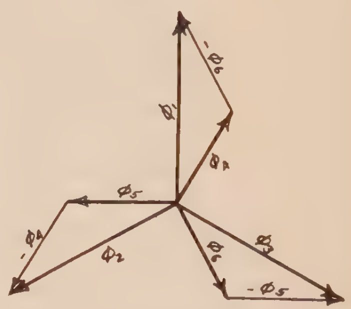


FIG. 57.12.

transformers are used, any one may be removed for repairs without interrupting the service, the two remaining transformers (open-delta connection, Fig. 42.12) supplying the three-phase load.

(*q*) **Induction Regulators.**—Induction regulators are essentially variable-ratio transformers of special design used to regulate the voltage on distribution feeders in alternating-current systems or the impressed voltage on rotary converters.

The primary is connected across the feeder to be controlled while the secondary winding is connected in series as illustrated in Fig. 58.12. The most important feature in the design of the induction regulator, considered as a transformer, is that the primary winding may be turned in space position, with respect to the secondary winding; that is, the primary winding is placed on a pivoted core or armature which can be turned with respect to the secondary or stator winding. Both the primary and the secondary windings are assembled in slots similar to those of an induction motor. The arrangement of the windings in the stator

and rotor cores of a single-phase induction regulator is shown in Fig. 59.12.

Since the excitation of the core is single phase, an alternating magnetic flux links both the primary and the secondary windings. When the primary winding is in the same, or a parallel, plane of

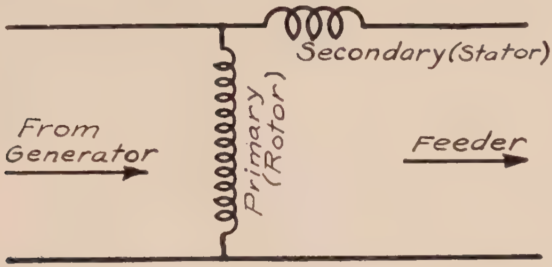


FIG. 58.12.—Regulator connections on single-phase feeder.

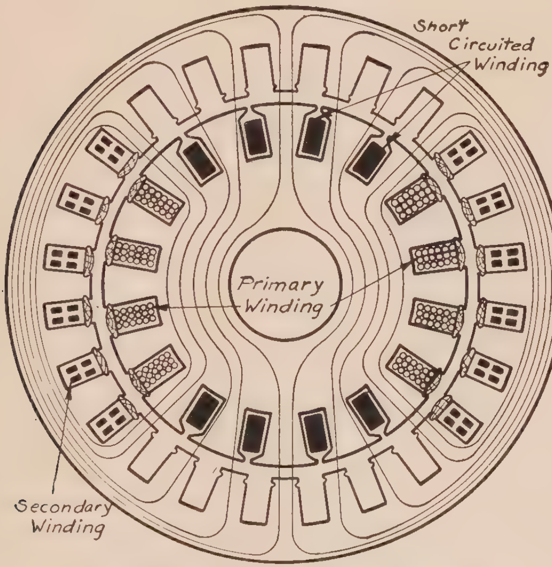


FIG. 59.12.—Primary (rotor) and secondary (stator) cores and windings of a single-phase induction regulator.

the secondary or stator winding the maximum flux will thread the stator and hence a maximum voltage, under the given conditions, would be induced in the secondary winding. By turning the rotor with respect to the stator, the number of magnetic lines threading the secondary winding will decrease until zero value is

reached, when the two coils are in space quadrature. Hence, by turning the primary coil through an angle of 90°, the induced voltage in the secondary is reduced from an initial maximum to zero value.

By continuing the rotation of the primary in the same direction, an increasing number of the lines of force produced by the current in the primary winding will thread the secondary, until a maximum is reached when the rotor and stator windings again become parallel to each other. Necessarily, a corresponding increase takes place in the induced secondary voltage until a maximum, equal in magnitude to the initial maximum, is obtained, but reversed in time-phase direction.

Since the secondary winding is connected in series with the load, it is evident that by turning the induction regulator's primary winding, or rotor, through an angle of 180°, the feeder voltage may be either *boosted* or *lowered* by an amount varying from zero to the maximum value for which the regulator is designed.

In addition to the primary and secondary windings there is also a *third* or *short-circuited winding*, as shown in Fig. 59.12. The purpose of the third winding is to equalize the reactance of the series or secondary winding of the regulator for the various positions of the rotor winding. With only the stator and rotor windings in the regulator, the reactance of the series coil to the load current depends directly on the leakage flux between the primary and secondary windings. When the primary is in the same plane as the secondary, the leakage flux is comparatively small; the two coils being iron clad and in close proximity. However, when the coils are in quadrature, the total flux in both coils become leakage and, hence, the reactance would be greatly increased. To force the load current through the series winding under the above stated conditions would require an appreciable percentage of the line voltage, and also reduce the load power factor. This undesirable feature is eliminated by the short-circuited winding which is placed on the rotor in space quadrature to the primary winding. When the primary winding is in space position parallel to the stator, no voltage is induced in the short-circuited winding. For any other position of the rotor, voltage will be induced in the short-circuited winding, which causes currents to flow that automatically keep the reactance of the secondary or series coil at or near its minimum value. The

addition of the short-circuited coil does not increase the losses in the regulator, but rather tends to hold them constant for any given line current.

In *three-phase regulators* each phase has a primary and secondary winding connected in shunt and in series, respectively, in the same way as in the single-phase regulator. The windings are spaced 120° apart, on both the rotor and the stator. The current in each winding is single phase but the magnetization of the core is produced by the combined effect of the primary

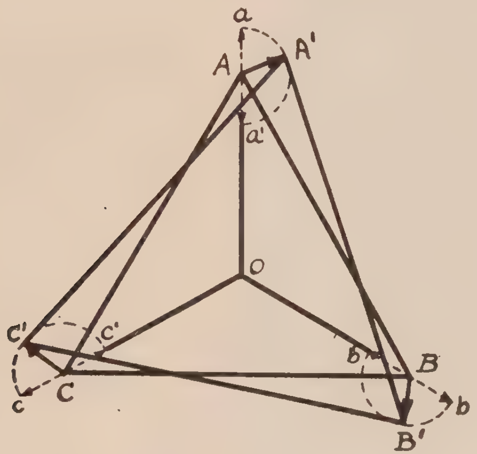


FIG. 60.12.—Three-phase regulator, vector diagram.

windings of the three phases. Hence, the resulting flux passing through or cutting the series circuits is constant in magnitude but rotates at a speed corresponding to the frequency of the circuit. Similarly, the line current in the secondary, or three-phase stator circuits, produces a rotating field constant in magnitude for any given load current.

The effect produced by the constant rotating flux from the primary windings on the three-phase line voltage depends on the time-phase relation of the voltages; that is, on the space-phase or relative space positions of the rotor and stator windings. The voltage induced in the series circuit by the rotating primary flux has vector properties, and hence, although constant in magnitude, it can be made to either raise or lower the three-phase voltages by varying the position of the rotor with respect to the stator.

The effect produced by the regulator is shown graphically by the vector diagram in Fig. 60.12. Let AB, BC and CA represent the circuit voltages on entering the regulator. The voltages induced by the regulator are represented by the vector AA', BB' and CC'. The resulting voltages in the three phases on the load side of the regulator are represented by $A'B'$, $B'C'$ and $C'A'$. By turning the rotor with respect to the stator the time-phase relation of AA', BB' and CC' can be varied over the arcs aa', bb' and cc' with a consequent range in the terminal voltage from ab, bc, ca to $a'b'$, $b'c'$ and $c'a'$.

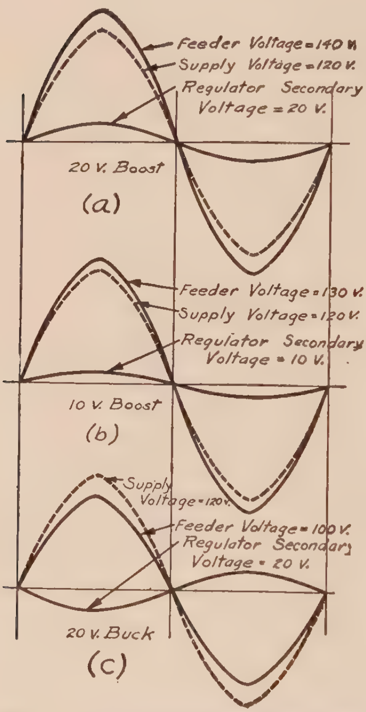


FIG. 61.12.—Single-phase regulator.

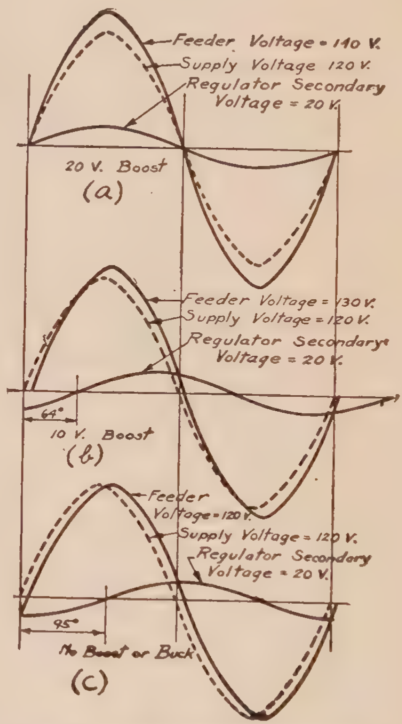


FIG. 62.12.—Three-phase regulator.

A further means of showing how the single-phase regulator differs in principle of operation from the three-phase regulator is found in Figs. 61.12 and 62.12. In the *single-phase regulator*, as indicated by the diagram in Fig. 61.12, the secondary voltage for any condition of boost or lower is either directly in phase or in opposition to the supply voltage. Changes in feeder voltage are

thus obtained by means of an induced secondary voltage which varies in magnitude only.

In the *three-phase regulator* the induced secondary voltage will remain constant for all positions of the primary windings, since it is produced by the constant rotating field set up by the currents in the three-phase primary windings. In Fig. 62.12 are shown the phase relations of a three-phase regulator in which the primary voltage is 120 volts and the induced secondary voltage 20 volts. For maximum booster action the secondary voltage must be directly in phase as shown in Fig. 62.12(*a*). To produce a 10-volt boost the rotor must be turned approximately 64° as shown in Fig. 62.12(*b*). For no booster action the rotor must be turned approximately 95° from the maximum position, as shown in Fig. 62.12(*c*). However, in all positions of the rotor, the induced secondary voltage remains at 20 volts. Lowering action will obviously be obtained by turning the rotor an angle greater than 95° from the maximum boosting position.

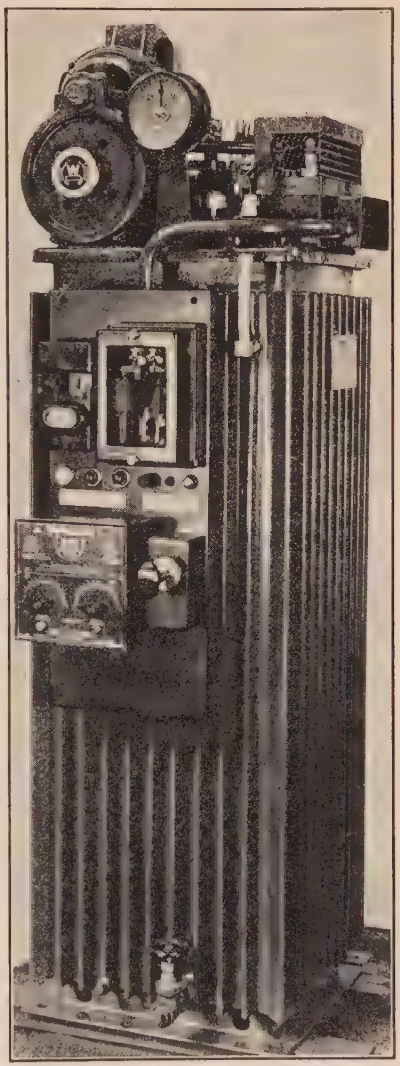


FIG. 63.12.—Induction regulator. 48 kv.a.; 2,400 volt, single phase; 60 cycle; 200 amp.; 10 per cent regulation lower or boost. Indoor type. (*Westinghouse Electric and Manufacturing Company.*)

It should be noted that although the rotor of the regulator is mounted on bearings it does not rotate like the armature of a motor. The motion of the rotor is usually produced by

turning a worm screw. Movement takes place only when the feeder voltage changes or tends to change. A motor attached to the worm screw causes the primary to change in position with respect to the stator windings, so as to automatically hold the feeder voltage at predetermined values for all loads. Under constant-load conditions the rotor of the regulator is stationary with respect to the stator windings.

Two important advantages are gained by using single-phase regulators instead of three-phase units for voltage regulation in three-phase feeder systems.

1. The voltage of each phase can be regulated independently.
2. The time-phase shift in the voltages inherent in the three-phase regulator, as illustrated from the vector diagram, Fig. 60.12, is avoided by using single-phase units. As a consequence, comparatively few three-phase regulators are in commercial operation.

It is customary to use two single-phase regulators in open-delta connection for voltage regulation in three-phase systems.

B. THE CONSTANT-CURRENT TRANSFORMER

In constant-potential systems variations in the load are accompanied by corresponding changes in the current. Similarly, in constant-current systems the voltage varies with the load. In America practically all electric systems operate on the constant-potential basis, and the current varies with the load. The important exceptions are the series arc and incandescent lamp circuits for street lighting. In most cases the street lighting is only a small part of the total load, and hence the electric energy is generated and distributed by constant-potential circuits, except the particular feeders supplying the street-lighting circuits. The connecting link between the constant-potential busbars and the constant-current series lamp circuit is the constant-current transformer. In design this transformer differs from the constant-potential types chiefly by having the secondary winding movable with respect to the primary, as shown in Figs. 64.12 and

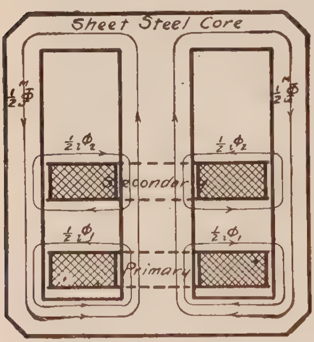


Fig. 64.12.

65.12. Since the currents in the primary and secondary coils flow in opposite directions the magnetic fluxes between the two coils pass in the same direction. The two magnetic fields therefore produce a force tending to push the coils apart. By properly

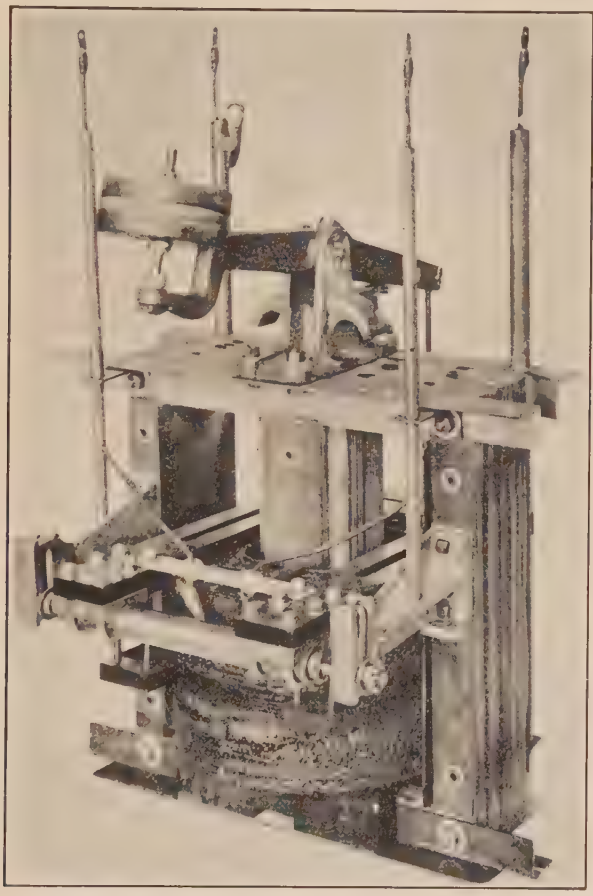


FIG. 65.12.—Constant-current transformer, Type RO, 20 kv.a.; 2,300 volt; 6.6 amp.; 60 cycle. (*General Electric Company.*)

counterbalancing most of the weight of the secondary coil the magnetic repulsion may be balanced against gravity so as to control the position of the coil by small changes in the current.

If the resistance in the series lamp circuit is decreased, the current will increase; this causes an increase in the magnetic repulsion and the secondary coil is pushed farther away from

the primary. As a consequence, the leakage flux is increased and the induced voltage in the secondary is decreased. Thus the voltage in the secondary may be made very nearly proportional to the resistance in the series lamp circuit and the current kept approximately constant. In the primary coil the increased leakage flux increases the reactance and hence supplies the required increase in the primary impedance. For an increase in the resistance in the series lamp circuit the current tends to decrease, the secondary coil is drawn nearer to the primary by the force of gravity and the secondary induced voltage increases so as to keep the current practically constant.

By this simple device the energy from the constant-potential busbars is transformed and delivered to the series lamp circuit under constant-current conditions. The transformer is automatically self-regulating but its efficiency is low as compared to the constant-potential transformer.

Pole and Manhole Types.—The excellent service of the station-type regulating transformer led to a demand, as lighting circuits spread farther and farther, for a similar transformer which could be mounted on a pole or in a manhole chamber and operated by a time switch. In service of this kind, the transformer may not always be mounted exactly level, and it is therefore necessary to guide the motion of the movable coil so as to prevent friction. One way of doing this is to mount the coil in a frame which is hinged at one end. The other end is supported by links and a lever, and the counterweight is movable on the lever. A later form of construction uses a parallel-motion mechanism to guide the moving coil. This decreases the size of the transformer slightly, and gives slightly better performance than the hinged-coil design. Both the hinged-coil and the parallel-motion designs are mounted in distribution transformer types of pole mounting tanks, and in manhole-type tanks for subway service.

C. INSTRUMENT TRANSFORMERS

By means of current and voltage transformers the range of alternating-current ammeters, voltmeters and wattmeters can be extended and these meters used upon circuits having larger currents and higher voltages than could be applied to the terminals of the instruments. The voltage that may be applied to most types of switchboard voltmeters is from 0 to 150 volts, and the range of the current in the ammeters is from 0 to 5 amp.

In principle the instrument transformers are similar to the larger power units, but the dimensions must be proportioned so as to comply with the required service. The transformer is a source

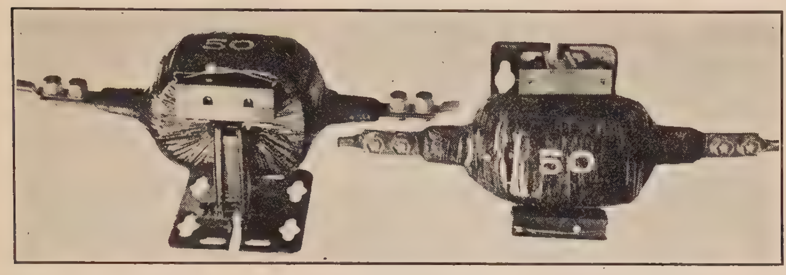


Fig. 66.12.—Current transformers. Type NA. (*Westinghouse Electric and Manufacturing Company.*)

of error in the instrument readings and must be carefully calibrated. The voltage drop in the transformer introduces a time-phase lag which may affect the current and voltage-phase displacement in the wattmeter. In properly designed trans-

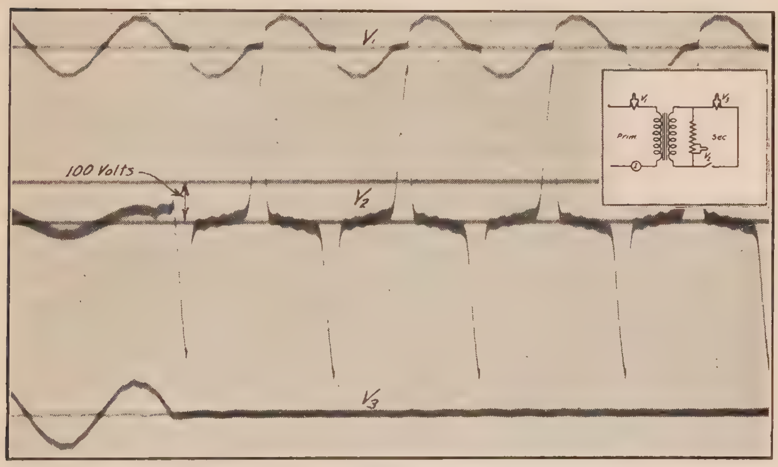


Fig. 67.12.—Oscillogram of current in an instrument current transformer before and after secondary circuit is opened.

formers the impedance drop is very small, generally less than one-tenth of 1 per cent of the voltmeter terminal voltage.

The current transformer consists of one or more turns on the primary and a secondary of as many turns as the desired ratio requires. The resistance and leakage reactance are very small.

The secondary must at all times be short-circuited, either through the ammeter or through a short circuiting switch. If the secondary should be opened the voltage across the secondary terminals becomes very high, as illustrated by the oscillogram in Fig. 67.12, endangering the insulation. Moreover, the abnormal magnetic flux would produce excessive core losses, which would in a short time overheat and ruin the transformer.

PROBLEMS

1.12. In a 50-kv.a., 2,200/220 transformer the hysteresis loss is 198 watts and the eddy-current loss 34 watts. At full load the copper loss is 508 watts.

(*a*) Find the efficiency for 25, 50, 75, 100, 125 and 150 per cent of full load. Load having unity power factor.

(*b*) Same as (*a*) but for a load having 85 per cent power factor, lagging current.

(*c*) Plot the efficiency curves for (*a*) and (*b*).

2.12. At full load the equivalent reactance drop of the transformer in problem 1.12 is 3.1 per cent and the resistance drop 1.1 per cent of the rated terminal voltage. Find the regulation.

(*a*) For $\cos \theta_2 = 1.00$.

(*b*) For $\cos \theta_2 = 0.85$, current lagging; also with current leading.

3.12. The average flux density of the transformer in problem 1.12 is 64 kilo-lines per sq. in.

Find the core loss for $\mathcal{B} = 30$, 40, 50, 60 and 70 kilo-lines.

4.12. Given a 5-kv.a., 2,200/220 – 110-volt transformer.

$_hP = 31$ watts; $_eP = 14$ watts; RI^2 (full load) $= 85$ watts.

Average $\mathcal{B} = 8{,}500$ per $\overline{\text{cm.}}^2$; equivalent resistance drop 1.7 per cent and reactance drop 2.8 per cent of the rated terminal voltage. Core weight 92 lb.

(*a*) Plot efficiency curve from 0 to 150 per cent load; $\cos \theta_2 = 95$ per cent.

(*b*) Find the regulation for loads having power factors = 100, 95, 90, 85 and 80 per cent. Lagging current.

(*c*) Find core loss for $\mathcal{B} = 6{,}000$, 7,000, 8,000, 9,000, 9,500, 10,000 and 10,500 per $\overline{\text{cm.}}^2$.

5.12. Let the following data apply to a transformer:

$$_\phi E_1 = 1{,}100 \text{ volts; ratio of } {_\phi E_1} : {_\phi E_2} = 5:1.$$
$$r_1 = 3.3 \text{ ohms; } r_2 = 0.13 \text{ ohm; } x_1 = 6.5 \text{ ohms; } x_2 = 0.26 \text{ ohm.}$$
$$_mI = 0.3 \text{ amp.; } I_2 \text{ (full load) } = 125 \text{ amp. (lagging); } {_\phi \theta_2} = 30°;$$
$$_hP + {_eP} = 148 \text{ watts.}$$

(*a*) Find E_1, E_2, θ_1, θ_2 and I_1 for full load.

(*b*) Draw the vector diagram to scale.

6.12. With the same data as in problem 5.12 except the current leads the voltage, $_\phi \theta_2 = 30°$, obtain answers for (*a*) and (*b*).

7.12. Two transformers are used in changing 2,200 two-phase to 220-volt three-phase. (*a*) Neglecting losses, find the ratios of the number of turns in the two transformers.

(*b*) Draw in rectangular coördinates the sine waves for the voltages between mains in the two-phase and three-phase circuits.

(*c*) Draw vector diagram of the voltages and currents in the three-phase and two-phase circuits with the load current lagging 40°.

8.12. Draw a vector diagram for a constant-current transformer taking power from constant-potential mains. (*a*) When under full load (all the series lamps in circuit); (*b*) for one-fourth load (one-fourth of the lamps in circuit).

9.12. Three 2,200/220-volt transformers, connected in delta, supply a lighting load of 150 kw. One of the transformers is damaged and removed for repairs.

(*a*) What current was flowing in each transformer when the three transformers were in service?

(*b*) What current flows in each of the two transformers after the third has been removed?

(*c*) What is the rating of each transformer if the two transformers on open delta carry full load?

10.12. In a 500-kv.a. 60~ transformer, ${}_eP$ = 820 watts; ${}_hP$ = 1,780 watts; RI^2 (full load) = 3,250 watts; equivalent resistance drop = 0.65 per cent and equivalent reactance drop 5 per cent of the terminal voltage (full load). $\mathcal{B}$ = 10,800 per $\overline{\text{cm.}}^2$. E_2 = 220 volts.

(*a*) Plot the efficiency curve from 0 to 150 per cent full load. Unity power factor.

(*b*) Find the regulation for power factors = 1.00, 0.90, 0.85 and 0.75. Leading and lagging currents.

(*c*) Find core loss for $\mathcal{B}$ = 5,000, 7,000, 9,000, 10,000, 11,000 and 12,000 per cm.².

11.12. Two transformers having constants as given in problem 5.12 are connected in *V*. Balanced three-phase voltage (1,100 volts) is supplied to the primary and three equal load resistances of 1 ohm each are connected in delta on the secondary. Neglect the exciting current and calculate the equivalent resistance and reactance of each transformer. Calculate the secondary voltages and currents and plot a complete vector diagram.

12.12. Determine the current in the windings of a three-phase, two-phase autotransformer bank when transforming 100-kv.a. balanced load.

(*a*) 2,300 volts three-phase to 2,300 volts two-phase.

(*b*) 2,300 volts three-phase to 3,000 volts two-phase.

(*c*) 2,300 volts three-phase to 1,800 volts two-phase.

13.12. Determine the currents in the windings of a 100-kv.a., three-phase, delta-connected autotransformer transforming from 4,600 to 2,300 volts by means of taps from the centers of the phases.

14.12. Determine the currents in the primary and secondary windings of a delta to delta-connected bank of transformers 22,000 to 2,200 volts with taps out from centers of low-voltage phases to give 1,100 volts. (*a*) When supplying 500 kv.a. 80 per cent power factor balanced load at 1,100 volts. (*b*) Same as (*a*) with an additional balanced load of 500 kv.a., 100 per cent power factor at 2,200 volts.

15.12. In oil-insulated, water-cooled transformers, coils of pipe are located in the oil through which water is circulated. The loss in the form of heat is

removed from the transformer by heat flow from transformer to oil, then from oil to water through the walls of the cooling coil. The normal heat flow through the walls of the cooling coil is approximately 1½ watts per sq. in. of surface. What difference in temperature rise would be produced if iron cooling coils be substituted for copper, both having the same dimensions?

16.12. Two 250-kv.a., single-phase transformers operate in parallel. Unit 1 has 1 per cent resistance, 5 per cent reactance. 2,200 to 22,000 volt ratio (ratio of number of turns).

Unit 2 has 1 per cent resistance, 4½ per cent reactance. 2,200 to 22,100 volts ratio (ratio of number of turns).

When the bank is delivering 500 kw., 80 per cent power factor, operating in parallel on the low-tension side, what currents will flow in each unit?

CHAPTER XIII

INDUCTION MOTORS

As already noted, the magnetic field passing between the primary and secondary windings of a transformer produces mechanical forces tending to push the coils apart. In the potential transformer the two windings are rigidly fixed in position and hence, although the forces exist, no motion is produced and no energy changed into the mechanical form. On short-circuits with excessive currents flowing, these forces may tear the transformers to pieces. In the constant-current transformer the secondary winding is, to a certain extent, movable, and the reaction between the two fields furnishes the means for automatic regulation. In the induction motor the windings of the successive phases are so arranged in space around a rotating spider that a continuous torque is produced in one direction. The induction motor was invented by Ferrari and Tesla in 1887 and is essentially an apparatus for transforming *electric energy into mechanical energy.* Some forms of the induction motor are extremely simple in design, require very little attention, and still give excellent service. Like the transformer, the induction motor has a primary and a secondary. Either may be placed on the rotating spider, but as the outside offers more space, it is generally desirable to wind the primary with the highest voltage on the stationary drum or *stator*, and place the secondary, low-voltage winding on the rotating spider, or *rotor*. The basic principles of the induction motor are more nearly like those of the transformer than those of the direct-current motor, and hence it is customary to refer to the *primary* and *secondary*, or the *stator* and *rotor* instead of the *armature* and *field* windings.

(*a*) **The Revolving Field in Polyphase Motors.**—Let two coils be placed with their sides parallel in the position shown in Fig. 1.13. If they be connected in parallel from the same pair of busbars the magnetic fluxes are in the vertical direction and vary from a positive to a negative maximum, equal in magnitude to the sum of the maxima for the separate coils. If the two coils

be connected to a two-phase circuit, the maxima of the separate coils come 90° apart in time-phase and the total resultant maximum is equal in magnitude to the square root of the sum of the squares of the component, maximum fluxes. If the maxi-

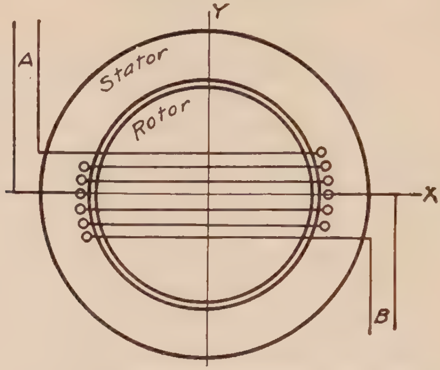


Fig. 1.13.—Coils in space-phase.

mum fluxes in the two coils are equal, the direction of the flux in space is always either up or down. In Fig. 2.13 is shown the vector diagram for the two coils on the same phase, and in Fig. 3.13 the corresponding diagram with the two coils connected to a two-

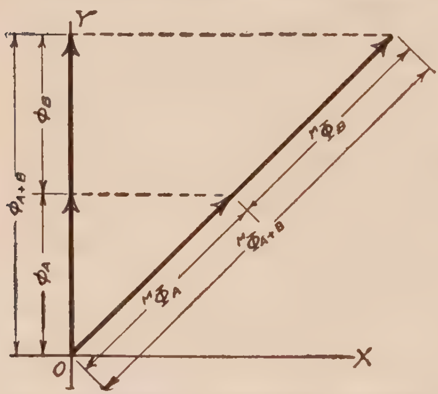


Fig. 2.13.—Coils in space-phase. Currents in time-phase. Flux stationary, pulsating.

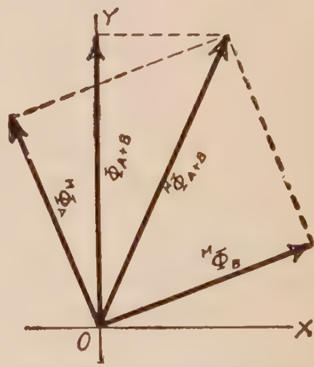


Fig. 3.13.—Coils in space-phase. Currents in time-quadrature. Flux stationary, pulsating.

phase circuit. The instantaneous flux in both cases is the projection of the resultant vector on the Y-axis. If the two coils be placed mechanically at right angles, as in a two-pole, two-phase induction motor, shown diagrammatically in Fig. 4.13, the flux in

coil A will be along the Y-axis as before, but for coil B the direction of the flux will be horizontal. If the two coils are connected in parallel on the same phase, or to a single-phase circuit, the instantaneous values for the flux in A are the projections along the Y-axis and for coil B along the X-axis, as shown in Fig. 5.13. If the two coils are alike the resultant flux Φ_r lies at 45 space degrees, or midway between the two axes. Since the fluxes in both coils are in time-phase and therefore reach their maximum and zero values simultaneously, the resultant flux also reaches its maximum and zero values coincident with the two components. The positions in space of the three fluxes are,

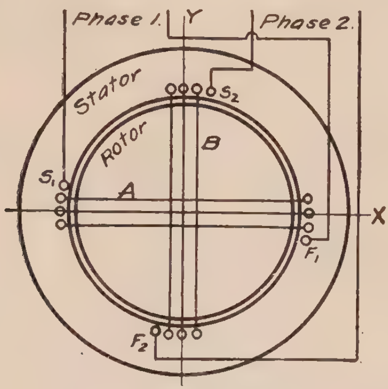


FIG. 4.13.—Coils in space-quadrature.

however, constant; for coil A along the Y-axis, for coil B along the X-axis and for the resultant at an angle of 45 space degrees.

Let the coils be connected to a two-phase system; then the two fluxes are in *time-quadrature* since the currents in the two phases are a quarter cycle out of phase, and in *space-quadrature* because the two windings are at right angles. The instantaneous values of the flux for coil A are long the Y-axis and similarly for coil B along the X-axis, on account of the *space-quadrdture* of the two coils. Since the fluxes are also in *time-quadrature* the instantaneous values are as the sine and cosine of the same time angle because the maximum values of the two coils come a quarter cycle apart. The resultant flux therefore moves in space position from the X-axis when the flux in coil B is a maximum to the Y-axis in a quarter cycle. Thus the resultant flux is constant in magnitude but rotates in space 360 electrical degrees

for each cycle of the two-phase current. This is shown in Fig. 6.13. The field for phase 1 lies vertically and for phase 2 in

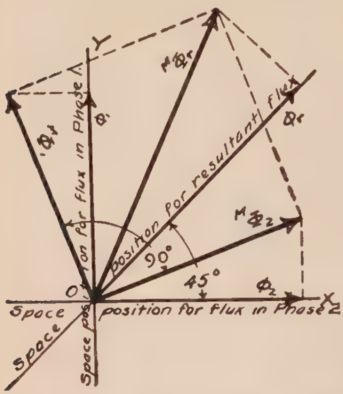


Fig. 5.13.—Coils in space-quadrature. Currents in time-phase. Flux stationary, pulsating.

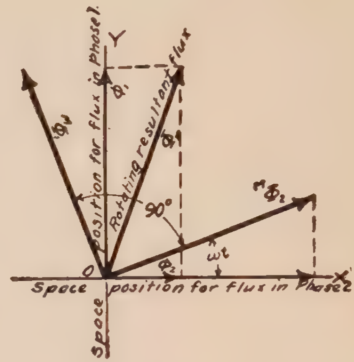


Fig. 6.13.—Coils in space-quadrature. Currents in time-quadrature. Flux rotating, constant.

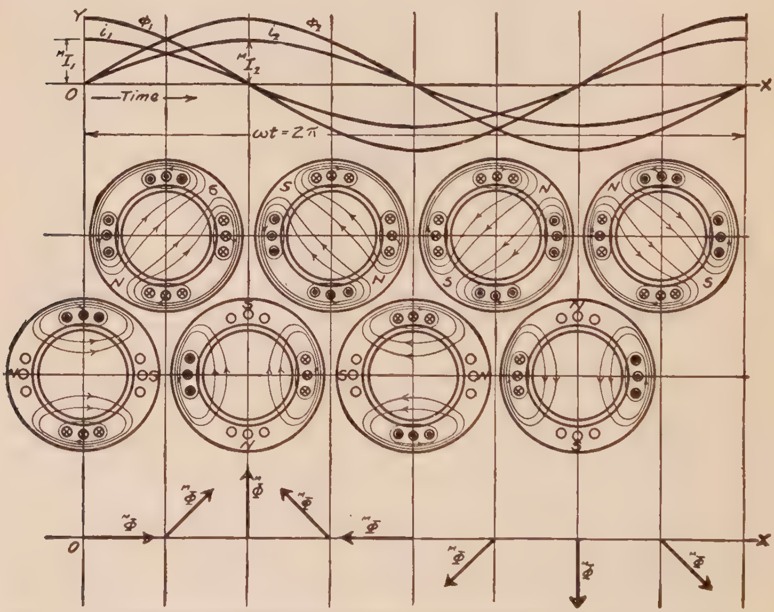


Fig. 7.13.—Diagrammatical and vector representation of revolving field in two-phase, two-pole induction motor.

the horizontal position. The magnitude of the instantaneous values depends on the time elapsed.

$$\phi_2 = {}^M\Phi_2 \sin \omega t \tag{1.13}$$

$$\phi_1 = {}^M\Phi_1 \sin (\omega t + 90°) = {}^M\Phi_1 \cos \omega t \tag{2.13}$$

Equations (1.13) and (2.13) give the instantaneous values and the positions of the coils determine the directions. In Fig. 6.13, ϕ_1 and ϕ_2 are drawn along the Y- and X-axes, respectively. For two The resultant flux is constant in magnitude and continually changes its position with ωt.

In the two-phase induction motor the coils are arranged in space as shown in Fig. 4.13. A revolving field is produced by the combination of *space-phase* and *time-phase* factors as determined by the relative space positions on the motor windings and the conditions of the circuit from which power is derived. This is shown diagrammatically in Fig. 7.13 for a two-phase, two-pole

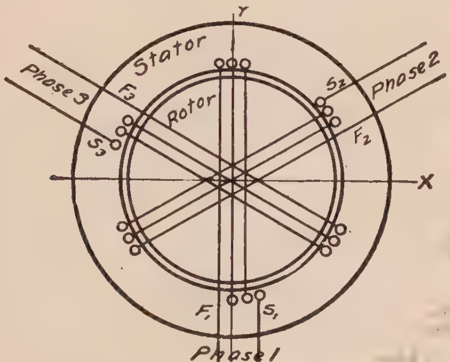


FIG. 8.13.

induction motor. The vertical lines mark the time and are spaced one-eighth of a cycle apart. Neglecting losses, the magnetic flux is in phase with the primary current in each phase. The cross and dot indicate the direction of the currents, and the arrows show the corresponding directions of the magnetic fields. The rotation of the resultant field is shown by the series of magnetic-flux vectors.

The stator-winding diagram for a three-phase, two-pole induction motor is shown in Fig. 8.13 and the corresponding revolving field in Fig. 9.13. The resultant flux is constant in magnitude and rotates in space position as in the two-phase motor. In a four-pole, two-phase motor two pairs of north and south poles are formed and a complete revolution in space requires two cycles

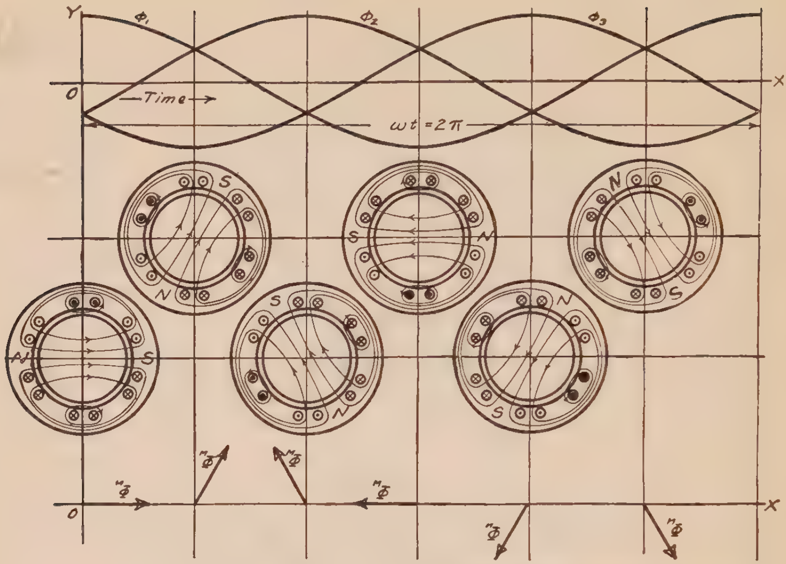


FIG. 9.13.—Diagrammatical and vector representation of revolving field in three-phase, two-pole induction motor.

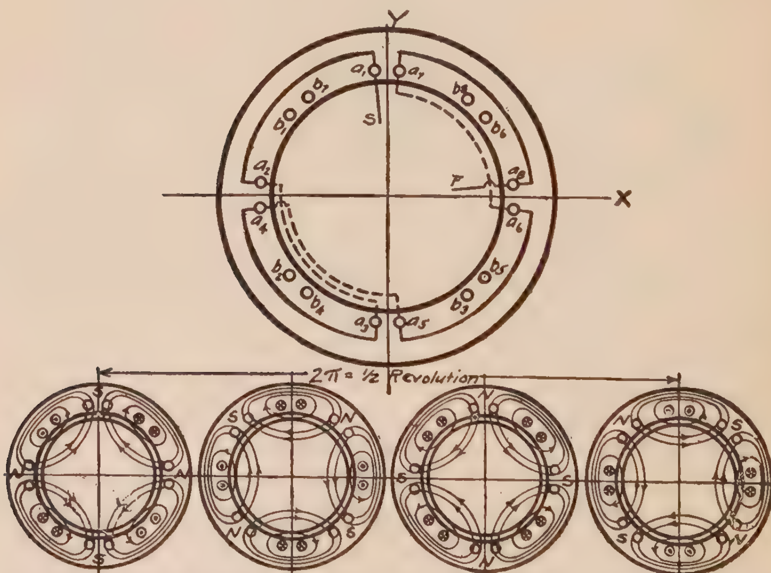


FIG. 10.13.—Diagrammatical representation of revolving field of a two-phase, four-pole induction motor.

of the magnetic flux as shown in Fig. 10.13. Precisely similar action takes place in the three-phase, four-pole motor, as indicated in Fig. 11.13. The rotation of the magnetic field should therefore be measured in electrical degrees, as each pair of poles represents a complete wave in the magnetic circuit. The primary windings on the stator must, therefore, also be spaced in electrical degrees, 90° for two-phase circuits and 120° for three-phase systems. In order to reduce the magnetic reluctance, the

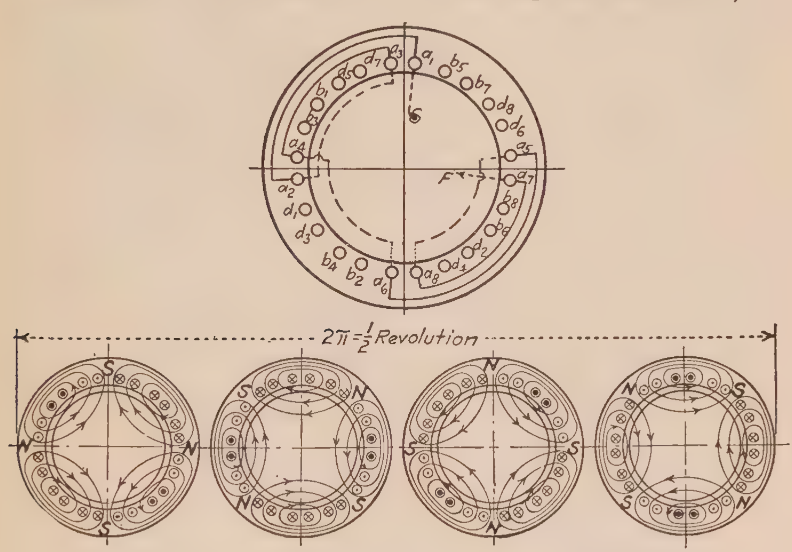


Fig. 11.13.—Diagrammatical representation of revolving field of a three-phase, four-pole induction motor.

primary winding is placed in slots on the inside of a hollow cylindrical core made of sheet steel.

(*b*) **Stator Windings for Two-phase Motor.**—The stator windings of two-phase motors occupy slots arranged essentially for a four-phase design. By merely rearranging the connections between the conductors inside the machine all the advantages of four-phase slot distribution may be gained for the two-phase motor. The fundamental principles involved are more easily explained by first replacing the rotor by a two-pole, rotating field, as in Fig. 12.13(*a*) and considering the direction of the induced voltages in the several stator conductors. In the figure, four conductors are shown, spaced 90° apart. The voltage induced when the bipolar field rotates may be represented by the vector diagram in Fig. 12.13(*b*), four vectors, differing successively by 90

time degrees and hence four phase. With four conductors in each group connected into four phases as in Fig. 12.13(*c*), the stationary part of the machine can be used either as the armature of a four-phase generator or as the stator of a four-phase induction motor. The voltages in the four phases, taken in order, are in

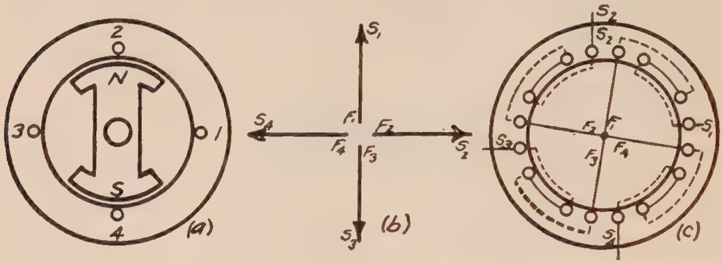


FIG. 12.13.—Four-phase.

time-quadrature or 90 time degrees apart, and the corresponding vector diagram is similar to Fig. 12.13(*b*). In multipolar machines the windings are spaced 90 electrical degrees, that is, 360 electrical degrees divided by 4, the number of phases.

By reversing the connections of coils S_3F_3 and S_4F_4, as in Fig. 13.13, or using connections as in Fig. 4.13, the directions of the

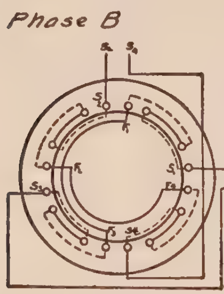


FIG. 13.13.—Two-phase.

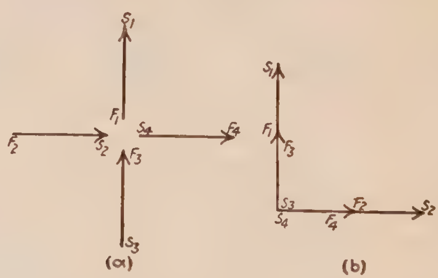


FIG. 14.13.

voltage vectors for the third and fourth phases are reversed. The corresponding vector diagram is shown in Fig. 14.13(*a*). From the connections it is seen that the first and third, and the second and fourth phases are in series between the corresponding pairs of leads. The vector diagram for the voltages between the leads is therefore two phase, as shown in Fig. 14.13(*b*). Hence by the simple expedient of changing the winding connections inside the machine, the four-phase slot structure can be used in the stator windings of the two-phase motor.

A variety of other slot and coil combinations are used in commercial motors. Two-phase distribution systems are rapidly being replaced by three-phase systems and the use of two-phase motors is decreasing.

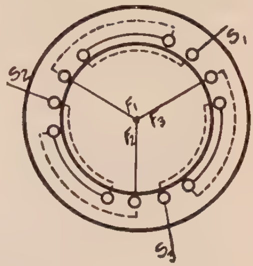


FIG. 15.13.—Three phase.

(c) **Stator Windings for Three-phase Motors.**—The most elementary three-phase motor has the windings for the three phases spaced 120 electrical degrees, with the conductors connected in some such way as indicated in Fig. 15.13. The stator windings of a three-phase induction motor, so extensively used in industrial power applications, occupy a six-phase slot structure with the conductors so connected inside the machine that with three leads the machine operates on three-phase systems. In a six-phase motor design the conductors for the six phases are

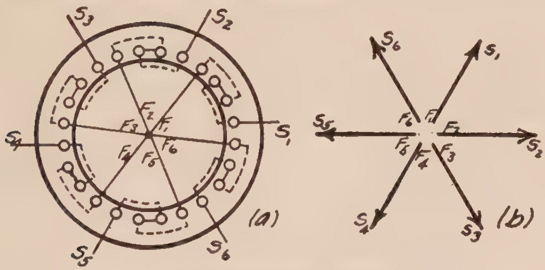


FIG. 16.13.—Six phase.

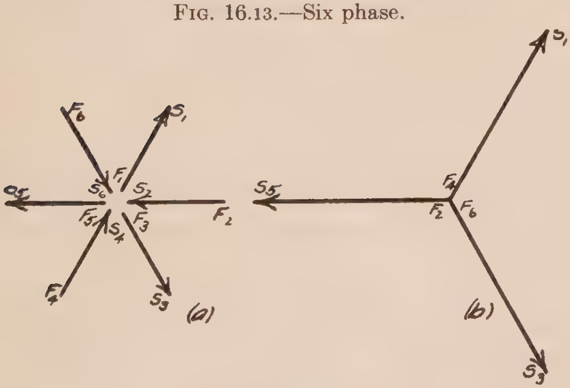


FIG. 17.13.

spaced 60 electrical degrees; that is, 360 electrical degrees divided by 6, the number of phases. The windings may be connected to six leads, as indicated in Fig. 16.13(a) with a com-

mon neutral. The corresponding vector diagram for the voltages between the leads is shown in Fig. 16.13(b). From the vector diagram it is apparent that, if the connections for the conductors in phases 2, 4 and 6 be reversed, the voltage vector diagram is represented by Fig. 17.13(a). If the windings for phases 1 and 4, 3 and 6, and 5 and 2 be connected in series as indicated in Fig. 18.13 (vector diagram Fig. 17.13(b)), and the terminals F_2, F_4 and F_6 joined at a common point, then the terminals S_1, S_3 and S_5, form the three leads of the so-called three-phase motor. Phase 1 is produced by coils S_1F_1 and S_4F_4 reversed, phase 2 by coils S_3F_3 and S_6F_6 reversed and phase 3 by coils S_5F_5 and S_2F_2 reversed. The three resultant phases with the terminals S_1, S_3 and S_5 connected to the leads are spaced 120 electrical degrees, but the individual coils are only 60 electrical degrees apart. A variety of other slot and coil combinations are used in commercial designs.

Fig. 18.13.—Three phase.

(d) **The Rotor.**—The rotor, or secondary, consists of a grooved cylinder of laminated steel mounted on a spider. The conductors lie in grooves in the steel core and the nature of the rotor winding largely determines the type of motor. In order to keep the magnetic reluctance as low as possible, the air gap between the stator and rotor must be a minimum, consistent with safe clearance. For this reason the shaft is extra stiff and the bearings are comparatively large.

The secondary may be wound on the rotor in much the same way as the armature of a direct-current motor or may consist of parallel copper bars short-circuited by brass rings at both ends or merely a cast cylindrical grid. The latter design, known as the *squirrel-cage* type, has a wide application and is in most general use. In the wound rotor the terminals are brought out to collector rings in order that outside variable resistance may be inserted in the secondary. The conditions requiring additional resistance in the secondary will be discussed in the paragraph on Torque. Whether the rotor be of the *squirrel-cage* or *wound-rotor* types, it consists of a resistance and an inductive reactance and its electrical properties may be stated quantitatively in the form of an equation.

(*e*) **Rotor Voltage and Current. Slip.**—With the rotor at standstill, the revolving field produced by the primary induces voltages in the secondary of the same frequency as in the primary. This induced voltage causes a secondary current to flow in the same way as in a transformer.

If the rotor runs in synchronism with the revolving field, called *the synchronous speed,* no lines of force cut the secondary conductors and hence no secondary voltage is generated.

The operating speeds of the rotor and of the revolving field are usually expressed in terms of the *slip* and are often given in per cent. *Slip* is defined as the ratio of the difference between the synchronous and rotor speeds to the synchronous speed. Thus, if the synchronous speed is 1,800 and the rotor speed 1,764 r.p.m., the slip is:

$$s = \frac{1{,}800 - 1{,}764}{1{,}800} = 0.02 = 2 \text{ per cent} \tag{3.13}$$

Since the voltage generated depends upon the rate of cutting lines of force, the secondary voltage depends on the rotor slip. Likewise the reactance in the rotor circuit is directly propor tional to the frequency of the secondary current, and hence also dependent upon the slip.

Let

f = frequency of primary current.
E_2 = secondary voltage at standstill.
${}_{ss}x_2$ = secondary reactance at standstill.
r_2 = secondary resistance.

Then

$$s\,{}_{ss}x_2 = \text{secondary reactance at slip } s \tag{4.13}$$

$$\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2} = \text{secondary impedance at slip } s \tag{5.13}$$

$$sE_2 = \text{secondary voltage at slip } s \tag{6.13}$$

$$I_2 = \frac{sE_2}{\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2}} = \text{secondary current at slip } s \tag{7.13}$$

$$\cos\theta_2 = \frac{r_2}{\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2}} = \text{power factor in secondary at slip } s \tag{8.13}$$

The resistance of the secondary is small when compared to ${}_{ss}x_2$, the reactance at standstill, but large enough to make the reactance $s_{ss}x_2$ negligible near synchronism. It is therefore evident that the secondary current depends very largely upon the slip. Near synchronism the secondary current varies directly with the slip; and near standstill, aside from any changes in the primary flux, the secondary current is practically independent of the speed,

since both the secondary impedance and the induced secondary voltage are affected in the same proportion by changes in the speed. The secondary current in *squirrel-cage* motors is about 5.5 times as large at standstill as at full load.

(*f*) **Torque in Polyphase Motors.**—Torque is produced by the reaction of the secondary current upon the primary field. The quantitative value of both the instantaneous and average values depends upon both the magnitude and phase relations of the current and field. From equation (8.13) the power factor of the secondary circuit depends upon the slip, being approximately unity near synchronism and decreasing with an increase of the slip. The secondary voltage is in time phase with the flux and hence both the time- and space-phase angles between the primary flux and secondary current increase with the slip. It is evident that, while the secondary current increases from zero at synchronous speed to a maximum at standstill, the torque has its maximum at a point between synchronous and standstill conditions. A typical torque curve for a squirrel-cage induction motor is shown in Fig. 19.13.

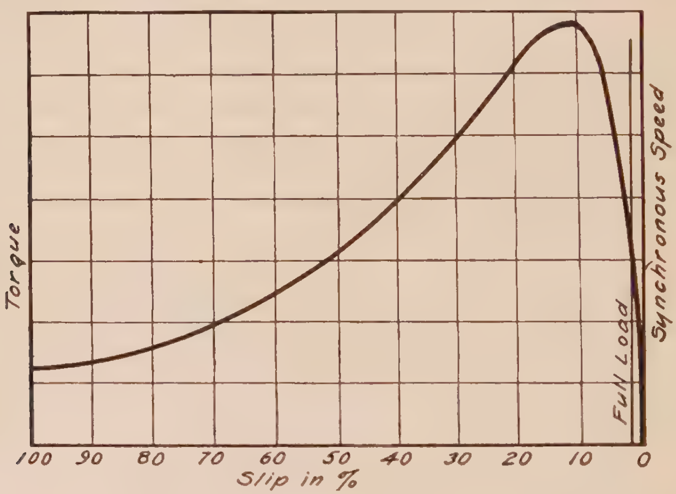


Fig. 19.13.—Torque—slip curve. Squirrel-cage induction motor.

The torque, while zero at synchronous speed, rapidly rises to a maximum value with the increase of slip, and then falls to a comparatively low value at standstill. This decrease is due to the change in the power factor in the secondary circuit and to a decrease of the primary field with the slip. The action is the

same as already discussed for the decrease in secondary voltage for inductive load in transformers. The variation is, however, small as compared to the change in secondary current and power factor, and will be neglected in the preliminary discussion. Assuming, then, that a constant impressed voltage of constant frequency on the primary produces a constant primary flux Φ_1, the equation for the torque T_1 at slip s is:

$$T_1 = k\Phi_1 I_2 \cos\theta_2 = \frac{k\Phi_1 sE_2}{\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2}} \times \frac{r_2}{\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2}}$$
$$= k\,\frac{sE_2\, r_2\, \Phi_1}{r_2^2 + s^2\,{}_{ss}x_2^2} \tag{9.13}$$

For variable slip the torque is a maximum when the rotor resistance and reactance are equal.

For maximum torque,

$$r_2 = s\,{}_{ss}x_2 \tag{10.13}$$

In the wound rotor, shown diagrammatically in Fig. 20.13, the secondary resistance may be varied by means of the outside rheostat. By continually keeping the total resistance in the secondary equal to $s\,{}_{ss}x_2$ the same maximum torque may be obtained at all speeds. A series of torque curves for the wound-rotor motor having variable secondary resistance is shown in Fig. 21.13. From equations (9.13) and (10.13) it is evident that all the maximum values are of the same magnitude; and hence the maximum torque is, within limits, independent of the secondary resistance.

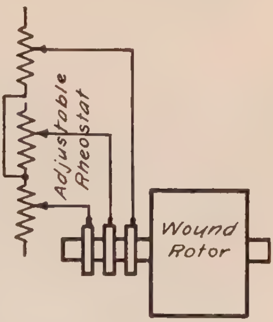


FIG. 20.13.

Substituting the value of the resistance for maximum torque in equation (9.13):

$${}^{M}T = k\frac{\Phi_1 s^2 E_2\,{}_{ss}x_2}{2s^2\,{}_{ss}x_2^2} = k\frac{\Phi_1 E_2}{2\,{}_{ss}x_2} \tag{11.13}$$

From equations (7.13) and (10.13) the secondary current for maximum torque is constant and independent of the resistance.

$$I_2 = \frac{sE_2}{\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2}} = \frac{sE_2}{\sqrt{2s^2\,{}_{ss}x_2^2}} = \frac{E_2}{\sqrt{2}\,{}_{ss}x_2} \tag{12.13}$$

The secondary power factor for maximum torque is a constant.

$$\cos \theta_2 = \frac{r_2}{\sqrt{r_2^2 + s^2 {}_{ss}x_2^2}} = 0.707 \qquad (13.13)$$

At standstill $s = 1$, and hence $r_2 = {}_{ss}x_2$ for maximum torque.

$$\text{Maximum torque at standstill} = k\frac{\Phi_1 E_2 r_2}{r_2^2 + {}_{ss}x_2} = k\frac{\Phi_1 E_2}{2r_2} \qquad (14.13)$$

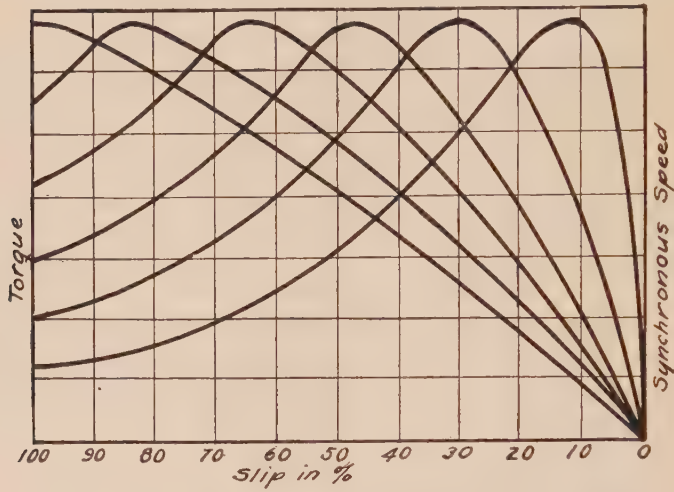


FIG. 21.13.—Torque—slip curves. Induction motor. Wound rotor. Secondary resistance variable.

If r_2 is less than ${}_{ss}x_2$ the starting torque can be increased by increasing the resistance. The maximum possible torque is reached with r_2 equal to ${}_{ss}x_2$. Any further increase of the resistance decreases the torque. Load conditions determine the choice between the squirrel-cage and wound-rotor types of induction motors. As the cost of construction of the squirrel-cage type is less than for wound-rotor design, the cheaper type is used for all cases where the load conditions fall within the torque-speed limitations. For loads having a large starting torque the more expensive wound-rotor type must be used. From Fig. 21.13 it is apparent that by inserting a variable resistance in the rotor circuit the maximum starting torque can be made available from standstill up to full speed.

The running-position curve lies between the synchronous speed and the maximum point on the torque curve. Ordinarily the full load requires less than one-third of the maximum torque. If the load be increased, the speed decreases slightly with a rapid

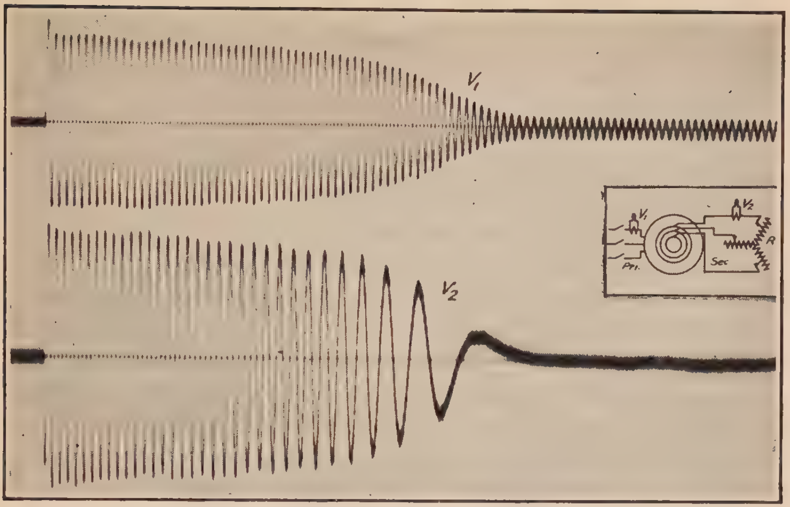


FIG. 22.13.—Oscillogram showing starting currents of an induction motor having a wound rotor with slip-ring connections for external starting resistance. E = 95.5 volts; primary starting current, V_1 = 14 amp.; rotor current, starting, V_2 = 95 amp. External resistance, R = 0.

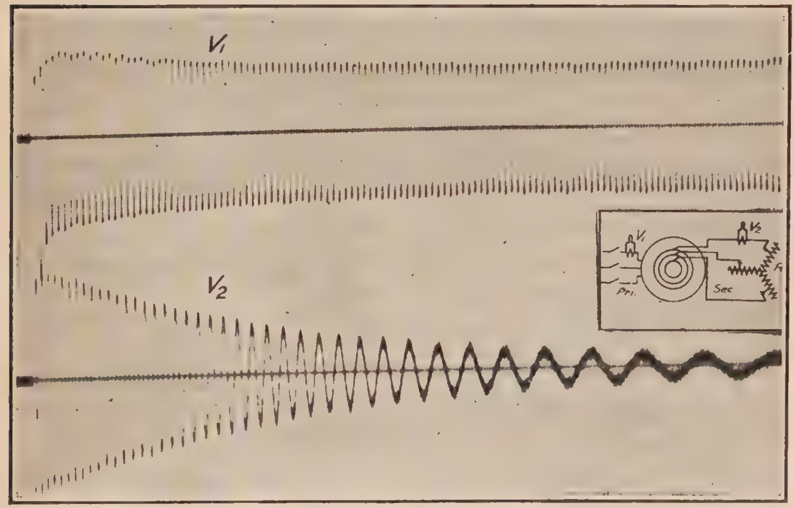


FIG. 23.13.—Oscillogram showing starting currents of an induction motor having a wound rotor with slip-ring connections for external starting resistance. E = 159 volts; primary current V_1 = 43.5 amp.; secondary current, starting, V_2 = 22 amp.; resistance, R, all in.

increase in the torque. If the load be increased past the maximum value of the torque, the motor stops, or *breaks down.* While an induction motor will carry, momentarily, overloads up to the *breakdown* point, it cannot do so continuously without overheating. For overloads the efficiency is slightly lower and the losses are greatly increased, producing more heat than can be dissipated at safe operating temperatures. As a consequence, the temperature rises and if the excessive overload be continued the insulation may be damaged and the motor *burned out.* For excessive overloads the rotor temperature may rise rapidly until the soldered joints give way. Motors having the copper bars welded or brazed to the short-circuiting rings on the rotor, Fig. 28.13, or cast-iron rotors can operate at higher temperatures, for short periods, without serious damage.

(*g*) **Power and Efficiency.**—The power output of the motor is proportional to the product of the torque and the speed.

$$P_2 \propto (1 - s)T \propto \frac{(1 - s)sE_2 r_2 \Phi_1}{r_2^2 + s^2 {}_{ss}x_2^2} \qquad (15.13)$$

The power is a maximum at a slip less than that at maximum torque, since it is the product of speed and torque. When the speed is near synchronism, as is the case under normal loads, the iron losses in the rotor are small due to the low frequency of the secondary currents and may be neglected. The copper losses in the secondary are:

$$r_2 I_2^2 = \frac{s^2 E_2^2 r_2}{r_2^2 + s^2 {}_{ss}x_2^2} = \frac{sE_2 T}{k\Phi_1} \qquad (16.13)$$

The copper loss is therefore proportional to the product of the slip and the torque; or for a given torque the slip is proportional to the rotor copper loss and independent of the secondary reactance. The copper losses in the secondary may also be expressed in terms of the voltage, current and power factor.

$$\text{Copper losses in secondary} = sE_2 I_2 \cos \theta_2 \qquad (17.13)$$

The electrical input into the secondary in terms of the secondary current, secondary voltage and power factor:

$$\text{Input for rotor} = E_2 I_2 \cos \theta_2 \qquad (18.13)$$

The mechanical output is equal to the electrical input minus the electrical losses in the secondary.

$$\begin{aligned} \text{Mechanical output} &= E_2 I_2 \cos \theta_2 - sE_2 I_2 \cos \theta_2 \\ &= E_2 I_2 (1 - s) \cos \theta_2 \qquad (19.13) \end{aligned}$$

Considering only the copper losses in the rotor and neglecting the primary losses and iron losses in the secondary, a partial efficiency may be obtained from equations (18.13) and (19.13).

$$\text{Approximate efficiency} = \frac{\text{mechanical output}}{\text{rotor input}} \qquad (20.13)$$

$$= \frac{E_2 I_2 (1 - s) \cos \theta_2}{E_2 I_2 \cos \theta_2} = 1 - s \qquad (21.13)$$

Hence the approximate efficiency, under the stated restrictions, is equal to the speed in per cent of synchronism. However, to obtain the actual efficiency of the motor all the losses must be taken into account. In addition to the copper losses in the secondary, there are iron losses in both the primary and secondary, copper losses in the primary, rotor windage and friction losses. The expression in equation (21.13) is of some interest, as it shows that the actual efficiency must always be less than the speed expressed in per cent of synchronism.

(*h*) **Transformer Features of Polyphase Motors.**—After the above preliminary discussion it is evident that, by inserting in the secondary a resistance like the load resistance in the secondary of the transformer, sufficient to consume the same power as the mechanical load, the circuits for the induction motor are the same as for the transformer. The circuit diagram for one phase is shown in Fig. 24.13.

E_1 = impressed voltage on the primary.
r_1 = primary resistance.
x_1 = primary reactance, due to the leakage flux.
r_{21} = equivalent of the secondary resistance in the primary circuit.
x_{21} = equivalent of the secondary reactance in the primary circuit due to leakage flux.
r_{31} = equivalent load resistance. This is assumed to be of such magnitude as to make $r_{31}I^2_{21}$ equal to the mechanical load.
E_{21} = equivalent of secondary voltage.
I_1 = primary current.
${}_mI$ = exciting current.
I_{21} = equivalent of secondary current.
${}_mg$ = conductance for ${}_mI$.
${}_mb$ = susceptance for ${}_mI$.

The exciting current is much larger in the induction motor than in a transformer of like capacity, due to the larger reluctance caused by the air gap in the magnetic circuit. With the rotor at synchronous speed, the total current flowing in the primary is the exciting current.

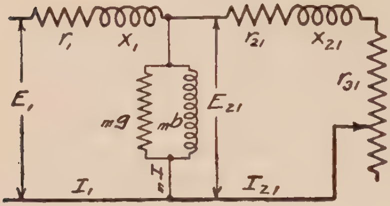


FIG. 24.13.

$$_m\dot{I} = {_mg}\dot{E}_{21} - j\,{_mb}E_{21} \quad (22.13)$$

The component taken by the conductance $_mg$ represents the part supplying the core losses and is in time phase with the voltage, while the magnetizing current is in time quadrature with the voltage and in the diagram is indicated by the susceptance $_mb$. On account of the air gap between the stator and rotor the magnetic leakage is much larger in the induction motor than in the transformer. The secondary reactance is proportional to the frequency of the secondary current, hence to the slip, and it therefore varies with the load. As the slip, under operating conditions of squirrel-cage induction motors, is only a small percentage of the speed, the reactance of the secondary is necessarily small as compared to the reactance of the primary. It may therefore be assumed that the equivalent total reactance for both stator and rotor is a constant and equal to $x_1 + x_{21}$. By moving the exciting-current circuit from the position between the primary and secondary to the position shown in Fig. 25.13 the solution of the problem is greatly simplified. The errors introduced in the calculations by this change are negligible.

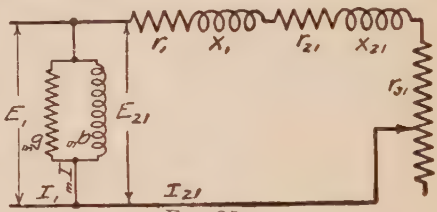


FIG. 25.13.

In the equivalent circuit diagram, Fig. 24.13, E_1, r_1, x_1, r_{21}, x_{21}, $_mg$, $_mb$ and $_mI$ are constant. The load resistance r_{31} varies, and consequently I_1 and I_{21}. The simplified equivalent circuit diagram in Fig. 25.13 gives a very close approximation of the actual conditions.

The symbolic equations for the circuit in Fig. 25.13 are:

$$ {}_m\dot{I} = {}_m g\dot{E}_1 - j\,{}_m b\dot{E}_1 \tag{23.13} $$

$$ \dot{I}_1 = {}_m\dot{I} + \dot{I}_{21} \tag{24.13} $$

$$ \dot{E}_1 = (r_1 + r_{21} + r_{31})\dot{I}_{21} + j(x_1 + x_{21})\dot{I}_{21} \tag{25.13} $$

(*i*) **The Circle Diagram for Polyphase Motors.**—From the circuit diagram in Fig. 25.13 it is evident that, aside from the exciting current, the induction motor with variable load is equivalent to a circuit having constant reactance and variable resistance. This may be shown with equation (12.13), dividing by the slip s.

$$ I_2 = \frac{sE_2}{\sqrt{r_2^2 + s^2\,{}_{ss}x_2^2}} = \frac{E_2}{\sqrt{\frac{r_2^2}{s^2} + {}_{ss}x_2^2}} \tag{26.13} $$

limit as

For variable load the locus for I_{21} is a circle, as shown in Fig. 26.13, and the complete performance of the polyphase induction motor may be derived from the circle diagram. This can best be illustrated by describing a common commercial test on induction motors. Two sets of readings are required.

1. At no load, corresponding to open secondary on a transformer.

2. Rotor locked, corresponding to short-circuit conditions for a transformer.

1. *At No Load.*—The rated voltage is applied to the terminals and readings taken of current and power input. The ammeter gives the exciting current ${}_mI$, and the wattmeter the core losses, windage and friction. The losses due to windage and friction are small; so the power indicated by the wattmeter may be considered as due to conductance in the exciting circuit as shown in Fig. 25.13. The exciting current has therefore components in time phase (power component) and time quadrature (magnetizing current) with the voltage, and their magnitudes are determined by the amperes, volts and watts taken at no load.

2. *At Standstill.*—For the second set of readings the rotor is clamped and sufficient voltage applied to give about twice the full-load current in the primary. Readings are taken to obtain the volts, amperes and watts. At standstill the rotor current is very nearly directly proportional to the impressed voltage and hence the value of the equivalent current in the primary at the rated voltage of the motor is found by multiplying the observed

value by the voltage ratio. Let ${}_{ss}I_{21}$ be the equivalent secondary current in the primary at standstill and at the rated voltage of the motor.

$$ {}_{ss}I_{21} = \frac{E_1}{x_1 + x_{21}} \text{ (approximately)} \tag{27.13} $$

The wattmeter reading represents the copper loss $(r_1 + r_{21})_{ss}I^2_{21}$ and hence is proportional to the square of the current.

From the two sets of readings, quantitative values are obtained for the power and reactive components of the currents at no load and at standstill, and from these data the constant and variable losses may be calculated.

At no load:

$$ {}_m\dot{I} = {}_mg\dot{E}_1 - j{}_mb\dot{E}_1. \tag{28.13} $$

${}_mI$ = exciting current.

${}_mgE_1$ = power component, due to core loss, windage and friction; the constant losses in the motor.

${}_mbE_1$ = reactive component or magnetizing current, providing the revolving field.

At standstill:

$$ {}_{ss}\dot{I}_1 = ({}_{ss}g + {}_mg)\dot{E}_1 - j({}_{ss}b + {}_mb)\dot{E}_1 \tag{29.13} $$

${}_{ss}I_1$ = current at standstill in the primary.

${}_{ss}gE_1$ = power component, due to copper losses in stator and rotor; the variable losses in the motor.

${}_{ss}bE_1$ = reactive components, due to leakage flux, or the stator and rotor reactance.

From these data the circle diagram may be constructed as in Fig. 26.13.

E_1 = constant impressed voltage, drawn from M along the Y-axis.

MN = ${}_mgE_1$, in time-phase with the voltage.

NO = $j{}_mbE_1$, magnetizing current in time-quadrature with the voltage.

Hence:

MO = ${}_mI$, the exciting current.

OS = $j\,{}_{ss}bE_1$, in time-quadrature with the voltage.

SQ = ${}_{ss}gE_1$, in time-phase with the voltage.

Hence:

$MQ = {}_{ss}I_1$, the primary current at standstill.
$OQ = {}_{ss}I_{21}$, the equivalent secondary current at standstill.

With the center on the line NO extended, draw the circle $OPQV$ passing through the points O and Q.

Divide the line QS at G so that QG and GS are in the same ratio as the secondary and primary copper losses, respectively. Connect O and G.

The complete performance of the motor may be determined from the diagram, Fig. 26.13. Since the circle is the locus of the

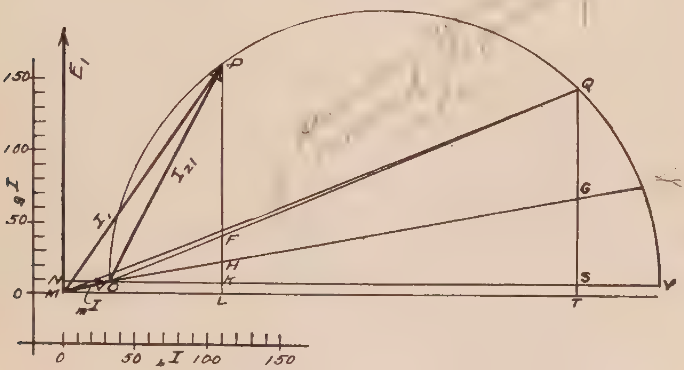


FIG. 26.13.—Circle diagram for polyphase induction motor.

secondary current, for any point P the following values are given directly from the diagram:

$PM = I_1$, the primary current.
$PO = I_{21}$, the secondary current.
$MO = {}_mI$, the exciting current.

That HK and FK are proportional to the primary and total copper losses for the current I_{21} is evident from the construction of Fig. 26.13. To show that HK and FK are directly proportional to I_{21}^2, connect V and P. From similar triangles, $OV:OP::OP:OK$. Hence $\overline{OP}^2 = OV \cdot OK$ or I_{21}^2 is proportional to OK, as OV is a constant. Under load, I_{21} is nearly equal to I_1.

If the quantities in the circle diagram represent values per phase, the corresponding quantities for the motor should be multiplied by n, the number of phases.

$PL = g_1 E_1$; hence $nE_1(PL)$ = input in watts (30.13)

$MN = {}_{m}g\, E_1$; hence $nE_1(MN)$ = core loss, windage and friction (31.13)

$HK = {}_{ss}g_1 E_1$; hence $nE_1(HK)$ = primary copper losses (32.13)

$FH = {}_{ss}g_2 E_1$; hence $nE_1(FH)$ = secondary copper loss (33.13)

$FP = g_3 E_1$; hence $nE_1(FP)$ = mechanical load, or output in watts (34.13)

$$\text{Power factor} = \cos\,(EMP) = \frac{PL}{PM} \tag{35.13}$$

$$\text{Efficiency} = \frac{PF}{PL} \tag{36.13}$$

$$\text{Slip} = \frac{\text{rotor loss}}{\text{rotor input}} = \frac{FH}{PH} \tag{37.13}$$

$$\text{Torque (in kg. at 1-meter radius)} = \frac{nE_1(PF)}{2\pi(9.806)(\text{r.p.m.})} \tag{38.13}$$

$$\text{Torque (in lb. at 1-ft. radius)} = \frac{nE_1(PF)33{,}000}{2\pi(746)(\text{r.p.m.})} \tag{39.13}$$

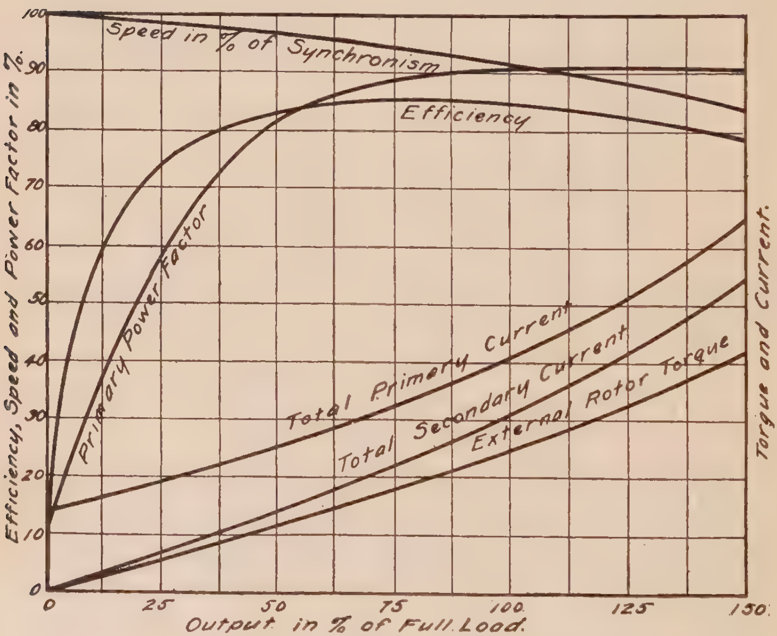


FIG. 27.13.—Performance curves for a three-phase induction motor.

A convenient way of expressing torque is in terms of *synchronous watts* or *synchronous horsepower*, which is the power that would be developed by the torque at synchronous speed.

$$\text{Torque (in synchronous watts)} = nE_1(PH) \qquad (40.13)$$

$$\text{Torque (in synchronous horsepower)} = \frac{nE_1(PH)}{746} \qquad (41.13)$$

$$\text{Torque (in ft.-lb.)} = nE_1(PH)\frac{7.04}{\text{syn. r.p.m.}} \qquad (42.13)$$

The maximum power output of the motor is found from the circle diagram by taking P at the point of tangency of a straight line tangent to the circle and parallel to OQ. The value of the losses, efficiency, power factor, torque, etc., at maximum output may be found for the given position of P in the same manner as above. Complete performance curves for the motor may be developed by plotting the values for a succession of positions of the point P on the circle diagram as in Fig. 27.13.

(*j*) **Starting Polyphase Motors.**—In general it has been standard practice for many years to start induction motors, larger than 5 hp., by means of a *starting compensator*, or autotransformer. This compensator reduced the voltage applied to the motor during starting to about 65 per cent of normal, thus limiting the heavy current drawn from the line during this period. As soon as the motor had reached full speed, a double-throw switch was thrown which placed the motor on full-line voltage and disconnected the autotransformer.

Recently there has been developed the so-called *across-the-line-start* type of motor which limits the starting current by its own reactance and does not require a starting compensator. Only very large motors are still provided with starting compensators. The rotor reactance is proportional to the slip, equation (4.13), and hence much greater when starting than at operating speed, and zero at synchronous speed. Moreover, the reactance, in the section of the rotor bars adjacent to the air gap, is much less than in the sections more deeply imbedded in the steel laminations.

In the across-the-line-start type of motors the outer part (radially) of the rotor bars has comparatively small cross-section and hence relatively high resistance. Moreover, these bars, due to their low reactance (low compared to the deeper part of the bars) during the starting period, carry most of the current. This crowding of the rotor current into the small conductor section holds the starting current within permissible limits.

As the motor approaches full speed the reactance of the deeper conductor sections becomes negligible and as a consequence the

current density becomes uniform in all parts of the rotor bars; *i.e.*, normal operation results.

When high starting torque is required from a squirrel-cage motor, two different sets of rotor bars are used: a set of high-resistance bars next to the air gap, and underneath a set of low-resistance bars. The rotor current during the starting period is forced through the high-resistance bars thereby producing the high starting torque. As the motor approaches full speed the current is gradually transferred to the inner set of low-resistance conductors. At normal speeds these so-called *high-starting-torque motors* have operating characteristics approximately like those of squirrel-cage induction motors.

(*k*) **The Revolving Field of a Single-phase Induction Motor.**— In order to produce a revolving field, two alternating-current

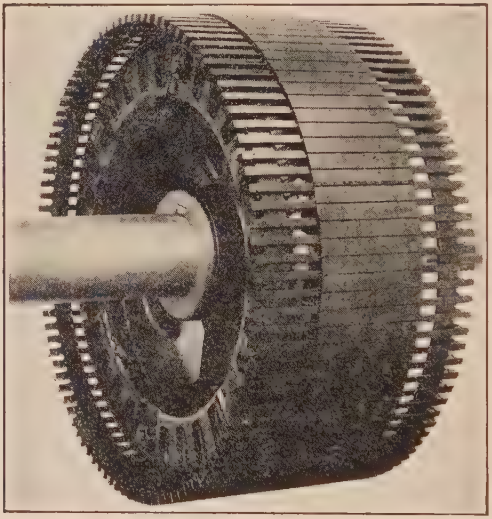


FIG. 28.13.—Squirrel-cage rotor, showing welded end rings. (*General Electric Company.*)

circuits differing in both time-phase and space-phase are required. In the single-phase motor only one of the component fields can be supplied directly from the mains. The single-phase current coming from the mains passes through the stator winding and produces a magnetic field at right angles to the plane of the coil. This flux, known as the *transformer field*, varies with the current from a maximum in the positive to an equal maximum in the negative direction. This is shown diagrammatically in Fig. 29.13.

Throughout the whole cycle the direction of the flux is always in the up or down direction in the figure. With the rotor at standstill this is the only flux in the motor. In Fig. 31.13 is shown the vector diagram with the instantaneous values as the projections

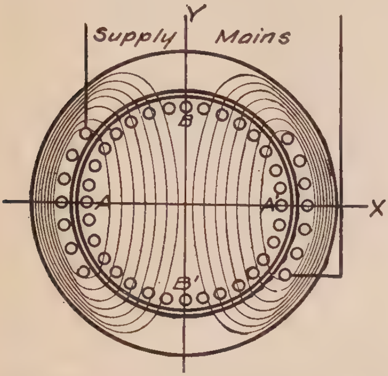


FIG. 29.13.—Transformer-field, single-phase induction motor.

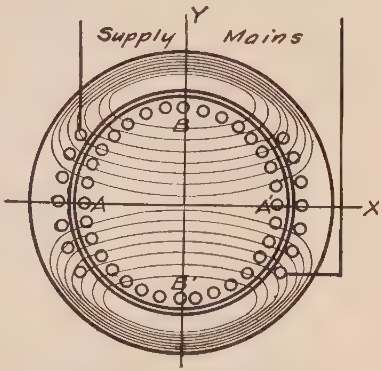


FIG. 30.13.—Speed-field, single-phase induction motor.

on the Y-axis. The pulsations of the transformer flux $^M_t\Phi$ generate a voltage and this, in turn, produces currents in the rotor conductors in the region marked AA' in Figs. 29.13 and 30.13. The maximum voltage is induced by transformer action in the conductors at AA' and no voltage at BB'.

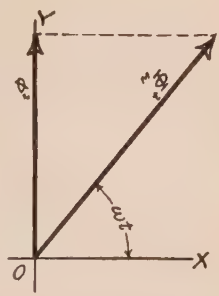


FIG. 31.13.—Magnetic flux, single-phase induction motor at standstill.

With the rotor in motion, the conductors at BB' cut the transformer flux and thereby generate a voltage in the conductors in the region BB'. The resulting current in BB' creates a field at right angles to BB', as shown in Fig. 30.13. This field, due to the rotation of the rotor, is known as the *speed-field.* From Figs. 29.13 and 30.13 it is seen that the *transformer-field* and *speed-field* are in *space-quadrature.* The voltage induced at AA' due to transformer action is in time quadrature with the flux in the transformer field. The maximum voltage generated in BB' due to rotation is necessarily coincident with the maximum strength of the transformer flux. As the maximum rate of cutting lines of force in the speed-field must be coincident with the maximum

voltage, the speed-field is in *time-quadrature* with the transformer field. Hence the two fields are in *space-quadrature* and *time-quadrature* and combined produce a revolving field. This is shown

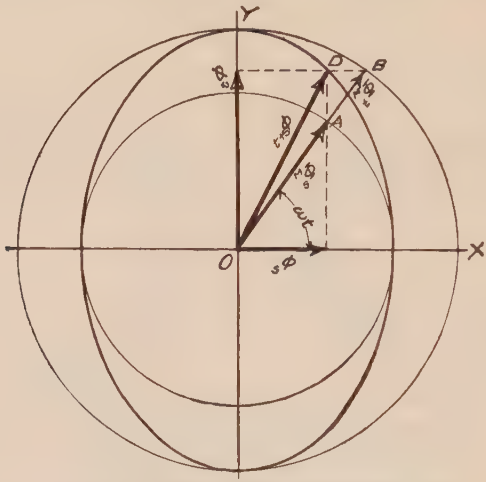


FIG. 32.13.—Revolving field, single-phase induction motor. Speed below synchronism.

graphically in Figs. 32.13, 33.13 and 34.13. By drawing the rotating vectors ${}^{M}_{t}\Phi$ and ${}^{M}_{s}\Phi$, representing the maximum values of the trans-

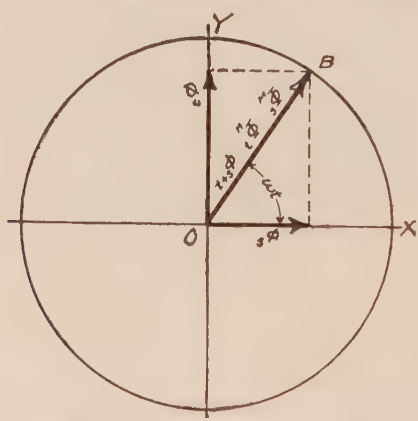


FIG. 33.13.—Revolving field, one-phase induction motor. Synchronous speed.

former and speed fields, along the same line, the diagram correctly expresses both the time-phase and space-phase relations if the projections of ${}^{M}_{t}\Phi$ along the Y-axis, and of ${}^{M}_{s}\Phi$ along the X-axis,

represent the instantaneous values of the two fields. If the speed is less than synchronous, the speed flux ${}^M_s\Phi$ is less than the transformer flux ${}^M_t\Phi$ and the resulting locus of the revolving field ${}_{t+s}\phi$ is an ellipse with the major axis as shown in Fig. 32.13. At synchronous speed the two fields are of equal magnitude and each equal to the resultant revolving field.

$$ {}^M_t\Phi = {}^M_s\Phi = {}_{t+s}\phi \text{ (at synchronous speed)} \qquad (43.13)$$

The revolving field is constant and its locus a circle as in Fig. 33.13. By increasing the speed of the rotor above synchronism the speed-field becomes stronger than the transformer-field and the locus of the revolving field again becomes an ellipse, with the major axis along the speed field as shown in Fig. 34.13.

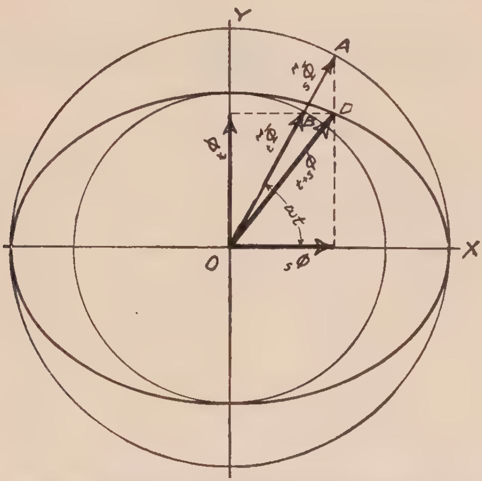


Fig. 34.13.—Revolving field, single-phase induction motor. Speed above synchronism.

Again referring to Figs. 29.13 and 30.13 it is evident that voltage is produced in the rotor conductors by two methods:

1. By transformer action; that is, the lines of force produced by the primary current cut the rotor conductors.

2. By the conductors cutting lines of force due to the rotation of the rotor.

In the conductors at AA' voltage is generated: first, by the pulsation of the transformer-field; and, second, by the cutting of the speed-flux by the rotation of the rotor. Likewise in the conductors at BB' voltage is generated: first, by the cutting of

the transformer-flux; and, second, by the pulsation of the speed-field.

${}_{pt}E_A$ = voltage in AA' due to pulsation of transformer-field.
${}_{rs}E_A$ = voltage in AA' due to rotor cutting speed-field.
${}_{rt}E_B$ = voltage in BB' due to rotor cutting transformer-field.
${}_{ps}E_B$ = voltage in BB' due to pulsation of speed-field.

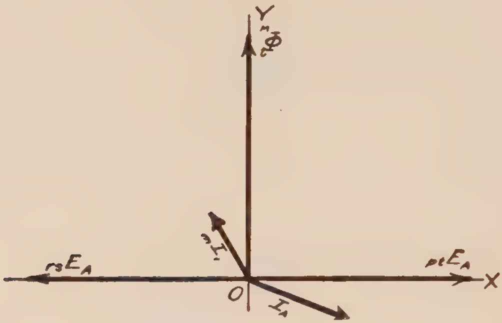


Fig. 35.13.

By noting the direction of the motion of the rotor conductors and also of the magnetic fluxes in the fields at any instant it is

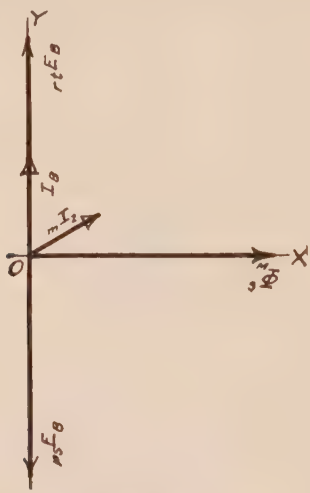


Fig. 36.13.

seen that the voltage induced at AA', and likewise at BB', by pulsation of one field is opposed by the voltage induced by rotation of the rotor. At synchronous speed the pairs of voltages are equal and a balanced condition exists. At any

speed below synchronism the difference in the opposing voltages causes a current to flow in the conductors AA' and BB'. The phase relations are shown in Fig. 35.13 for AA' and in Fig. 36.13 for BB'.

The exciting currents are shown as separate vectors to indicate the similarity of the action to that in the transformer and polyphase induction motor. Also the leakage or reactance in

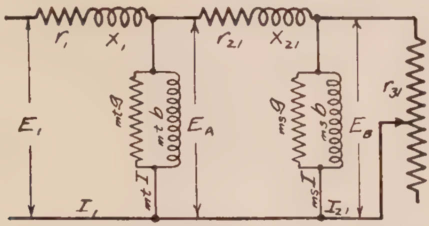


FIG. 37.13—Diagrammatic representation of circuits in a single-phase induction motor.

each circuit is omitted. By comparing Figs. 29.13, 30.13, 35.13 and 36.13 it is seen that I_A and ${}^M_s\Phi$, and likewise I_B and ${}^M_t\Phi$, are both in time-phase and space-phase. Hence there is a reaction or torque produced both at AA' and BB' when the rotor is in motion. At standstill ${}^M_s\Phi$ equals zero and hence I_B is also zero, and no torque is produced. Hence the single-phase induction motor has no starting torque and must be brought up to speed by means of special devices.

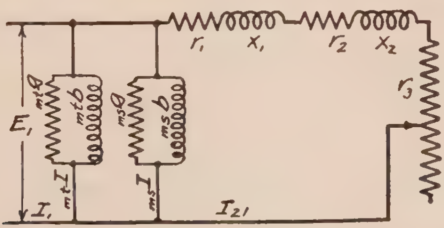


FIG. 38.13.—Representation of approximately equivalent circuits of a single-phase induction motor.

Circuit diagrams similar to the polyphase motor and transformer may be drawn for the single-phase induction motor. In Fig. 37.13 are shown the circuit diagrams representing the equivalent conditions of the single-phase motor. The transformer and speed fields require separate exciting currents as shown in the diagram. By modifying the diagram into the approximately

equivalent arrangement of Fig. 38.13 the solution of the problem is greatly simplified, while the errors introduced are relatively small.

${}_{mt}I$ is the exciting current for the transformer-field and has components ${}_{mt}gE_1$ in time-phase, power component, and ${}_{mt}bE_1$ in time-quadrature, magnetizing current, with the voltage.

Similarly, ${}_{ms}I$ is the exciting current for the speed-field and has components ${}_{ms}gE_1$ and ${}_{ms}bE_1$ in time-phase and in time-quadrature, respectively, with the voltage. With the same notation as used in the polyphase equivalent circuit diagrams the symbolic equations for the equivalent circuits in Fig. 38.13 are:

$$ {}_{mt}\dot{I} = {}_{mt}g\dot{E}_1 - j\,{}_{mt}b\dot{E}_1 \qquad (44.13)$$

$$ {}_{ms}\dot{I} = {}_{ms}g\dot{E}_1 - j\,{}_{ms}b\dot{E}_1 \qquad (45.13)$$

$$ \dot{I}_1 = {}_{mt}\dot{I} + {}_{ms}\dot{I} + \dot{I}_{21} \qquad (46.13)$$

$$ \dot{E}_1 = (r_1 + r_2 + r_3)\dot{I}_{21} + j(x_1 + x_2)\dot{I}_{21} \qquad (47.13)$$

(*l*) **The Circle Diagram for Single-phase Induction Motors.**—In the circuit in Fig. 38.13 the only variable is the load, which is

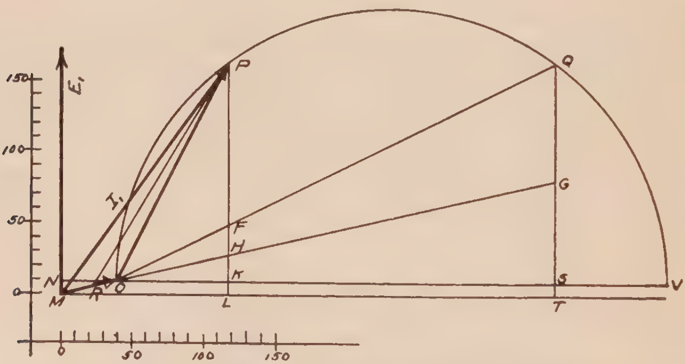


FIG. 39.13.—Circle diagram for single-phase induction motor.

represented by a variable resistance r_3. Hence the locus of the load current I_{21} is a circle and the performance curves of the single-phase motor may be derived from the circle diagram in Fig. 39.13.

With the motor running at synchronous speed the current in the primary is the vector sum of the exciting currents for the transformer and speed fields. In the diagram the power component at synchronous speed is represented by MN, and the reactive component by NO. The total exciting current is therefore MO, which is the vector resultant of the exciting currents for the transformer and speed fields. For convenience the two

component currents are shown as in phase and hence MO as the arithmetic sum. For actual machines this need not be the case. Let MO be divided at R so that MR represents the exciting current ${}_{mt}I$ for the transformer-field, and RO the similar current ${}_{ms}I$ for the speed-field. With the rotor locked, the power component is represented by QT and the reactive component (magnetizing current), by MT. The circle diagram is constructed by drawing a circle through O and Q with its center on NO extended, as in Fig. 39.13. It is often inconvenient to obtain the low voltage required for the locked rotor readings. As the center of the circle diagram must be in the line OV, the locus may be determined by finding the point P for any convenient load. A perpendicular from the middle point of the line joining O and P intersects the line OV at the center of the required circle OPV.

The complex equations are, at no load and synchronous speed:

$$ {}_{m}\dot{I} = {}_{mt}\dot{I} + {}_{ms}\dot{I} \tag{48.13} $$

$$ {}_{m}\dot{I} = ({}_{mt}g + {}_{ms}g)\dot{E}_1 - j({}_{mt}b + {}_{ms}b)\dot{E}_1 \tag{49.13} $$

$$ {}_{m}I = \text{total primary current, at no load} \tag{50.13} $$

$$ ({}_{mt}g + {}_{ms}g)E_1 = \text{power component due to core loss in both the transformer and speed fields} \tag{51.13} $$

$$ ({}_{mt}b + {}_{ms}b)E_1 = \text{reactive component, or magnetizing current, for both the transformer and speed fields} \tag{52.13} $$

Similarly at standstill:

$$ {}_{ss}\dot{I}_1 = ({}_{mt}g + {}_{ss}g_1 + {}_{ss}g_2)\dot{E}_1 - j({}_{mt}b + {}_{ss}b_1 + {}_{ss}b_2)\dot{E}_1 \tag{53.13} $$

$$ {}_{ss}g_1 E_1 = \text{power component due to copper losses in stator} \tag{54.13} $$

$$ {}_{ss}g_2 E_1 = \text{power component due to copper losses in rotor} \tag{55.13} $$

$$ ({}_{ss}b_1 + {}_{ss}b_2)E_1 = \text{reactive component due to leakage or stator and rotor reactance} \tag{56.13} $$

It should be noted that the current at standstill does not contain the exciting current for the speed field. In constructing the circle diagram this error is neglected and the construction made as if the power component of the current at standstill were due to the total losses in the motor while running.

After the circular locus of the current is found, complete performance curves may be determined directly from the circle diagram.

E_1 = impressed voltage

$MN = ({}_{mt}g + {}_{ms}g)\dot{E}_1$ = power component of exciting current (57.13)

$NO = j({}_{mt}b + {}_{ms}b)E_1$ = magnetizing current (58.13)

$MO = {}_{mt}\dot{I} + {}_{ms}\dot{I} = {}_{m}\dot{I}$ = exciting current (59.13)

$OS = j\,{}_{ss}bE_1$ (60.13)

$QS = {}_{ss}gE_1$, power component due to total copper losses at standstill (61.13)

QG = component due to rotor copper losses at standstill

SG = component due to stator copper losses at standstill.

From any point P on the circle draw PL, PM, PR and PO, and connect OQ and OG. Then from the diagram:

$PL = g_1E_1$; hence $E_1(PL)$ = input in watts (62.13)

$MN = ({}_{mt}g + {}_{ms}g)E_1$; hence $E_1(MN)$ = core loss windage and friction (63.13)

$HK = {}_{ss}g_1E_1$; hence $E_1(HK)$ = stator copper loss (64.13)

$FH = {}_{ss}g_2E_1$; hence $E_1(FH)$ = rotor copper loss (65.13)

$E_1 \times PH$ = total input to the rotor, not including speed field excitation loss (66.13)

$FP = g_3E_1$; hence $E_1(FP)$ = mechanical load or output in watts (67.13)

$E_1 \times FL = ({}_{mt}g + {}_{ms}g + {}_{ss}g_1 + {}_{ss}g_2)E_1^2$ = total losses in machine (68.13)

$$\text{Power factor} = \frac{PL}{PM} \tag{69.13}$$

$$\text{Efficiency} = \frac{PF}{PL} \tag{70.13}$$

In the polyphase motor the torque was taken directly from the diagram and when expressed in terms of synchronous watts was found to be equal to the rotor input. It must be noted that, while in the polyphase motor the fields produced by both phases are always equal at any rotor speed, in the single-phase motor the speed field varies directly with the speed as indicated by the elliptical locus of the revolving field, shown in Figs. 32.13 and 34.13. As the strength of the speed field is one of the torque factors, then torque must vary with the speed.

$$\text{Torque in synchronous watts} = \frac{\text{rotor speed}}{\text{synchronous speed}}(E_1)(PH) \tag{71.13}$$

Torque is also equal to the output divided by the speed. Hence torque in synchronous watts =

$$\frac{E_1(PF)\text{ synchronous speed}}{\text{rotor speed}} \tag{72.13}$$

From equations (71.13) and (72.13):

$$\frac{\text{Rotor speed}}{\text{Synchronous speed}} = \left(\frac{PF}{PH}\right)^{1/2} \tag{73.13}$$

Substituting this value in equation (71.13):

$$\text{Torque} = E_1\sqrt{(PF)(PH)},\text{ in synchronous watts} \tag{74.13}$$

(*m*) **Equivalent Single-phase Values.**—For convenience in solving numerical problems in polyphase systems, *equivalent single-phase values* for the current and the resistance are often used. By the *equivalent single-phase current* is understood that current which transmits the same power in a single-phase system *with the voltage between mains the same as in the corresponding polyphase systems.* Similarly, the *equivalent single-phase resistance* gives the same heat losses in the single-phase circuit with the *equivalent single-phase current* as is dissipated in the corresponding polyphase system.

Let

I_2 = current in one of the mains of a balanced, two-phase, four-wire system.
r_2 = resistance in either phase of the two-phase, four-wire system.
I_3 = current in one of the mains of a balanced three-phase, three-wire system.
r_3 = resistance, as measured between terminals or mains, of the three-phase, three-wire system.
I_1 = equivalent single-phase current.
r_1 = equivalent single-phase resistance.

For two-phase, four-wire systems:

$$I_1 = 2I_2 \tag{75.13}$$
$$r_1 = \tfrac{1}{2}r_2 \tag{76.13}$$

For three-phase, three-wire systems, either delta or Y:

$$I_1 = \sqrt{3}I_3 \tag{77.13}$$
$$r_1 = \tfrac{1}{2}r_3 \tag{78.13}$$

TABLE VII.—PERFORMANCE TEST. THREE-PHASE INDUCTION MOTOR

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Primary e.m.f., E	Primary watts, W_1	Primary watts, W_2	(2) + (3) Tot. pri. input $\sqrt{3}EI\cos\theta$	(2) − (3) $EI\sin\theta$	$\frac{(5)}{(4)}\sqrt{3}$ $\tan\theta$	$\tan^{-1}$(6) Primary angle of lag θ	cos (7) Pri. P.F. $\cos\theta$	sin (7) $\sin\theta$	$\frac{(5)}{(1)}$ $I\sin\theta$	$\frac{(10)}{(9)}$ Pri. amp. per lead	(11) $\sqrt{3}$ Tot. pri. amp., $\sqrt{3}I$	$\frac{(4)}{(1)}$ Power comp. pri. current	(10) $\sqrt{3}$ Quadrature comp. pri. current, $\sqrt{3}I\sin\theta$	(0.231) $(12)^2$ Pri. cu. loss, I^2R	(4) − (15) No-load data. Iron and friction fixed loss
103.5	425	−220	205	645	5.440	79° 35′	0.181	0.984	6.23	6.34	10.96	1.98	10.77	27.9	177
104.0	650	000	650	650	1.732	59° 58′	0.501	0.866	6.25	7.20	12.48	6.25	10.80	36.0	177
104.0	850	+160	1,010	690	1.182	49° 45′	0.646	0.763	6.63	8.20	15.02	9.70	11.48	46.7	177
103.7	940	240	1,180	700	1.028	45° 48′	0.697	0.717	6.75	9.40	16.28	11.40	11.69	61.2	177
104.0	1,050	330	1,380	720	0.903	42° 5′	0.742	0.670	6.92	10.30	17.85	13.25	11.98	73.5	177
103.0	1,140	440	1,580	700	0.766	37° 28′	0.794	0.608	6.80	11.20	19.35	15.32	11.78	85.7	177
103.5	1,275	540	1,815	735	0.700	35° 6′	0.820	0.573	7.10	12.40	21.50	17.60	12.29	107.0	177
103.8	1,375	630	2,005	745	0.645	32° 50′	0.840	0.542	7.18	13.20	22.90	19.30	12.42	120.8	177
102.7	1,525	790	2,315	735	0.552	28° 54′	0.876	0.483	7.16	14.80	25.60	22.55	12.40	152.0	177
102.0	1,730	960	2,690	770	0.494	26° 18′	0.897	0.443	7.55	17.10	29.40	25.40	13.08	202.5	177

Equivalent single-phase resistance: $R_1 = 0.23$ ohm. $R_2 = 0.30$ ohm. Synchronous speed 1,800 r.p.m.

17	18	19	20	21	22	23	24	25	26	27	28
(15) + (16) Total pri. losses with fric.	(4) − (17) Sec. input syn. watts, W_s	$\frac{(18)}{255.6}$ Rotor external torque, in lb.-ft.	(13) − 1.98 Power comp. sec. amp.	(14) − 10.77 Quadrature comp. sec. amp., $I_2 \sin \theta_2$	$\sqrt{(20)^2+(21)^2}$ Total sec. amp., I_2	(0.3) $(22)^2$ Sec. cu. loss, $I_2^2R_2$	$\frac{(23)}{(18)}$ Rotor slip, S	(18) − (23) Motor output, watts	$\frac{(25)}{746}$ Motor output, hp.	$\frac{(25)}{(4)}$ 100 Motor efficiency, per cent.	100[1 − (24)] Rotor speed, per cent. syn.
205	000	0.00	0.00	0.00	0.00	0.0	0.0000	000	0.00	0.0	100.0
213	437	1.71	4.27	0.03	4.28	5.5	0.0126	432	0.58	66.3	98.7
224	786	3.08	7.72	0.71	7.75	18.0	0.0230	768	1.03	76.0	97.7
238	942	3.69	9.40	0.92	9.45	26.8	0.0285	915	1.23	77.5	97.1
250	1,130	4.42	11.27	1.21	11.35	38.5	0.0340	1,092	1.46	79.1	96.6
263	1,317	5.15	13.32	1.01	13.40	53.7	0.0408	1,263	1.69	79.9	95.9
283	1,532	6.00	15.60	1.52	15.67	73.7	0.0480	1,458	1.96	80.4	95.2
298	1,707	6.68	17.30	1.65	17.40	90.5	0.0530	1,617	2.16	80.7	94.7
329	1,986	7.77	20.60	1.63	20.70	128.1	0.0650	1,858	2.49	80.3	93.5
380	2,310	9.05	23.40	2.31	23.50	165.7	0.0720	2,144	2.87	79.6	92.8

TABLE VIII.—PERFORMANCE TEST. SINGLE-PHASE INDUCTION MOTOR

1	2	3	4	5	6	7	8	9	10	11	12
Primary e.m.f., E	Primary amp., I	Primary watts, W	$\frac{(3)}{(1)\times(2)}$ $\cos\theta$	θ	$\sin\theta$	$(0.092)(2)^2$ Primary copper loss	(3) − (7) At no load iron and friction loss	(8) + (7) Total pri. loss with friction	(3) − (9) Secondary input syn. watts	$\frac{(10)}{255.6}$ Rotor external torque in lb.-ft.	(2) × (4) Power comp. pri. amp. $I\cos\theta$
108.0	19.3	690	0.331	70° 30′	0.943	34.3	656	690	000	0.00	6.4
107.7	20.6	980	0.442	63° 45′	0.896	39.0	656	695	285	1.12	9.1
106.9	24.3	1,600	0.616	52° 0′	0.788	54.3	656	710	890	3.48	15.0
105.5	31.0	2,500	0.763	40° 30′	0.646	88.4	656	744	1,756	6.86	23.7
105.1	35.4	3,000	0.805	36° 30′	0.595	115.1	656	771	2,229	8.74	28.7
104.5	39.6	3,400	0.822	34° 45′	0.570	144.1	656	800	2,600	10.18	32.6
103.2	49.0	4,300	0.848	32° 0′	0.530	222.0	656	878	3,422	13.40	41.6
103.0	51.0	4,650	0.885	27° 40′	0.463	239.5	656	895	3,755	14.70	45.2

$R_1 = 0.092$ ohm, $R_2 = 0.15$ ohm. Synchronous speed 1,800 r.p.m.

13	14	15	16	17	18	19	20	21	22	23
(2) × (6) Quadrature comp. pri. amp., $I \sin \theta$	(12) − 6.4 Power comp. sec. amp., $I_2 \cos \theta_2$	(13) − 18.2 Quadrature comp. sec. amp., $I_2 \sin \theta_2$	$\sqrt{(14)^2 + (15)^2}$ Total secondary current	(0.15) $(16)^2$ Secondary copper loss, $I_2^2 R_2$	$\frac{(17)}{(10)}$ Rotor slip, S	(10) − (17) Motor output watts, W_0	$\frac{(19)}{746}$ Motor output, hp.	$\frac{(19)}{(3)}$ 100 Motor efficiency, per cent.	100[1 − (18)] Rotor speed, per cent. syn.	Rotor resistance
18.2	0.0	0.0	0.00	0.0	0.000	000	0.00	0.0	100.0	0.15
18.5	2.7	0.3	2.72	1.1	0.004	284	0.38	23.0	99.6	0.15
19.2	8.6	1.0	8.66	11.3	0.013	879	1.18	53.0	93.7	0.15
20.2	17.3	2.0	17.40	45.5	0.026	1,710	2.29	63.4	97.4	0.15
21.1	22.3	3.0	22.50	76.0	0.034	2,153	2.88	71.6	96.6	0.15
22.6	26.2	4.4	26.55	106.0	0.041	2,494	3.35	73.3	95.9	0.15
26.0	35.2	7.6	36.00	198.0	0.058	3,224	4.34	75.0	94.2	0.15
23.9	38.8	5.7	39.63	236.0	0.063	3,520	4.72	75.7	93.7	0.15

(*n*) **Performance Curves by One Wattmeter[1] and Voltmeter.**— The performance curves of an induction motor may be calculated from data giving the primary and secondary resistances and the voltmeter and wattmeter input readings from no load to full load. The primary resistance is usually determined by direct-current methods and the secondary resistance from standstill data. These are obtained by clamping the rotor and impressing a sufficient voltage on the motor terminals to send approximately full-load current through the stator circuit. Under these conditions the input may be considered as equal to the primary and

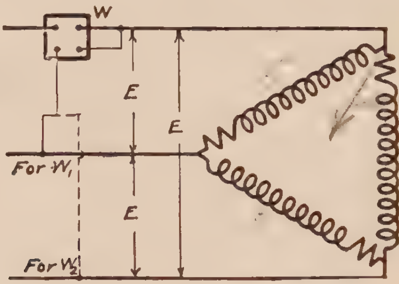


Fig. 40.13.

secondary copper losses, and the equivalent secondary resistance may be calculated. By taking voltmeter and wattmeter input readings for various loads, all the required points for determining the performance curves may be calculated. In three-phase circuits the readings of W_1 and W_2 may be obtained by means of one wattmeter by changing the voltage connection as shown in Fig. 40.13.

The method, as developed by Dr. McAllister, is best shown by an example with the calculations arranged in tabular form as in Table VII, which shows an analytical and numerical solution of the circle diagram.

The equivalent resistance $R_1 = 0.23$ ohm and $R_2 = 0.30$ ohm. The *observed input data* are given in columns 1, 2 and 3. The method for finding the calculated quantities is indicated at the top of each column. The corresponding performance curves are shown in Fig. 41.13.

The same method is applied to a single-phase motor in Table VIII. The *observed data* are given in columns 1, 2 and 3 and the calculated values are derived as indicated at the top of each

[1] McAllister, "Alternating-current Motors," p. 25.

column. The performance curves are plotted in rectangular coördinates, in Fig. 42.13. The simplicity of the experimental part of this method is apparent.

(*o*) **Starting Single-phase Induction Motors.**—The single-phase induction motor has no starting torque, and hence some other means must be provided to bring the machine up to speed. It is evident that for commercial purposes it is desirable to make the machine self-starting. Hence the starting device is

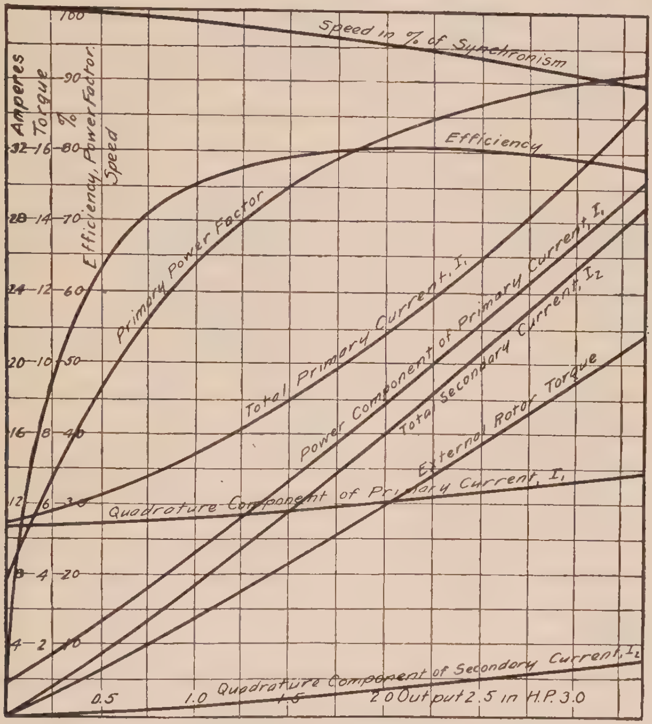


FIG. 41.13.—Performance curves of a 100-volt, three-phase, 60-cycle, 3-hp. induction motor. Table VII.

part of the machine and its action should be automatic. Some small motors are started by hand and have no self-starting arrangement. These are, however, unimportant exceptions. Of the automatic devices in commercial use, the repulsion motor, split-phase and shading coils are in general use.

Repulsion Motor Starting.—The repulsion motor is described in Chap. XVII. In order to make available the high starting torque of the repulsion motor for bringing the induction motor

up to speed, a wound armature with a commutator like that of a direct-current motor is required. The armature winding is short-circuited through a pair of brushes and the position of the brushes, relative to the field, determines the direction of rotation and the magnitude of the torque and starting current. When the motor reaches full speed the brushes are automatically removed from the commutator and simultaneously all the commutator bars are short-circuited, thus converting the repulsion motor to a squirrel-cage induction motor at full speed. Except

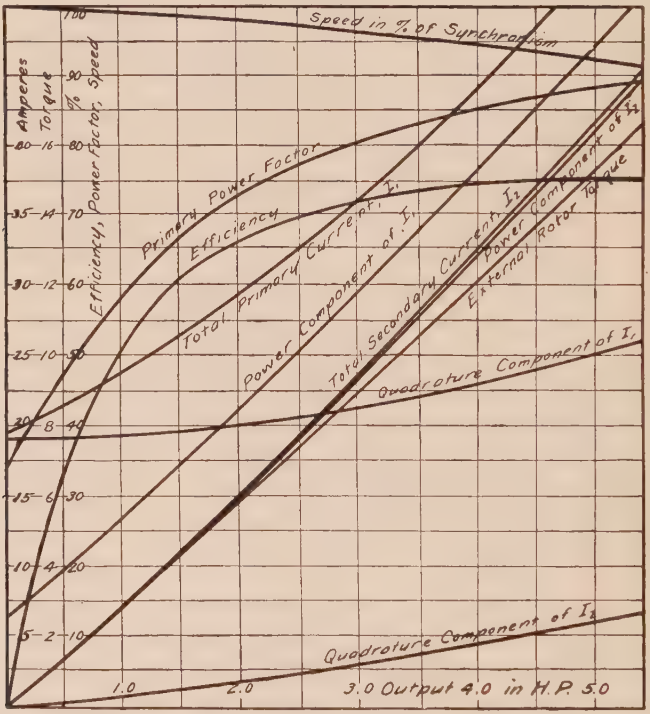


FIG. 42.13.—Performance curves of 104-volt, single-phase, 60-cycle, 5-hp. induction motor. Table VIII.

for small motors this method of starting is the best. The wound rotor and commutator with the automatic device for converting the winding at full speed to the squirrel-cage type greatly increase the cost of manufacture of single-phase induction motors as compared to polyphase motors of the same rating. The ideal simplicity of the polyphase induction motor is lost in providing a starting torque, and more care is required in operating single-phase motors.

Split Phase.—The requirement of two magnetic fields differing in both time-phase and space-phase for producing a revolving field may be provided from a single-phase circuit by means of the *split-phase* device. It consists essentially of two parallel circuits, as shown in Figs. 43.13 and 44.13, one of which, *B*, has resistance and the other, *A*, either inductive or condensive reactance. As the current is in phase with the voltage in circuit *B*, and either lags or leads in circuit *A*, the fields of magnetic flux are similarly out of phase. Hence by connecting the two circuits to windings differing in space phase the desired revolving field is obtained at standstill and consequently the required starting torque. The inductive reactance produces a low-power factor

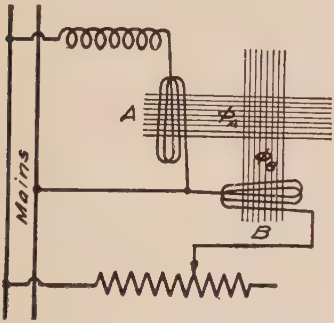


FIG. 43.13.—Diagram of inductor split-phase motor.

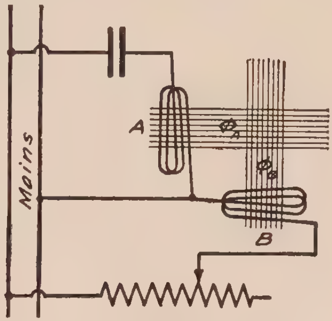


FIG. 44.13.—Diagram of capacitor split-phase motor.

on the line and gives only a weak starting torque, and hence is undesirable except for small motors. The split-phase device, using an inductive reactance, has a low first cost and small maintenance. It is used extensively for small motors requiring only a small starting torque. The condensive reactance or capacitor split-phase motor is rapidily increasing in commercial importance.

The difficulties found in the construction of capacitors for use in split-phase motors have been overcome. In present designs capacitor motors are rugged and entirely practical machines.

Shading Coil.—Another device frequently used on small fan motors and in single-phase induction-type ammeters, wattmeters and watt-hour meters is the *shading coil.* The motor field has salient poles like a direct-current motor, but made of laminated iron. Each pole is divided into two unequal parts as shown in Fig. 45.13. Around the small part is wound a short-circuited coil. In this coil induced currents flow, thus changing the time

phase of the flux inside the shading coil as compared to the flux in the main part of the pole. The combination of the two fluxes differing in both time and space phase produces a revolving field and consequently a starting torque. Necessarily, this torque is small and the device can be used only with small motors requiring small starting torques.

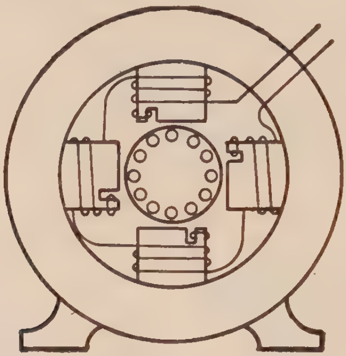


FIG. 45.13.

(*p*) **Starting Polyphase Induction Motors.**—The polyphase motor has inherently a torque at standstill and no special device is required. At standstill, however, the motor acts like a short-circuited transformer and necessarily takes comparatively large

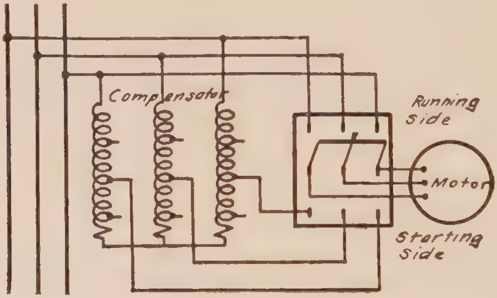


FIG. 46.13.

currents from the mains. With small motors the starting current is not sufficient to disturb the system from which the motor receives power and hence the small motor is started by throwing the supply switch directly on the mains. For overload protection the small motor generally has a double-throw switch; one side for starting directly on the line, and the other side a running position with the circuits properly fused to protect the motor under continuous operation. With large motors

the starting current at full voltage may be larger than can safely be supplied by the distributing system. To prevent excessive currents while starting, a *starting compensator* is placed between the mains and the motor. This apparatus is merely an autotransformer with circuit diagram as shown in Fig. 46.13. The number of steps in the compensator depends on the type of motor, the relative size of the motor and the carrying capacity of the line. In most cases the autotransformer has one position for starting and one for running in addition to the *off* position of the lever, when the circuit is open. In the running position the motor is connected directly across the mains.

(*q*) **Induction Meters.**—The fundamental principle upon which induction-type wattmeters are designed is that rotating fields can be produced by two alternating currents of the same frequency passing through circuits differing in space-phase. Essentially the instruments are small induction motors with cylindrical or disk rotors. The load for the integrating wattmeter consists of a metal disk rotating between the poles of a permanent magnet. In the indicating meter the torque produced by the rotating field is counterbalanced by spiral springs. The torque at any instant is proportional to the product of the voltage (current in the shunt coil) and the current (series coil), and hence for the complete cycle the average torque is proportional to the watts. In the integrating wattmeter the speed of the rotating element is proportional to the torque, and the instrument measures the energy delivered to the load.

PROBLEMS

1.13. In a test on a 5-hp., 60-cycle, six-pole, three-phase induction motor the following data were obtained:

Volts, E	Kilowatts		Ohms, r_1	Ohms, r_2
	W_1	W_2		
130	1.36	−0.90	0.265	0.18
130	1.45	−0.85	(Equiv. 1ϕ)	(Equiv. 1ϕ)
130	1.55	−0.75		
129	2.07	−0.20		
128	2.40	0.00		
129	3.05	0.55		
128	3.60	0.95		
128	4.25	1.42		

Calculate and plot performance curves as in Table VII and Fig. 41.13.

2.13. In a test on a 5-hp., four-pole, 60-cycle, single-phase motor the following data were obtained:

Volts, E	Amperes, I	Kilowatts, W	Ohms, r_1	Ohms, r_2
110	20.7	0.70	0.09	0.18
110	21.5	0.98		
110	25.6	1.67		
110	33.3	2.91		
110	37.5	3.46		
110	41.2	3.90		
110	48.6	4.69		
110	52.3	4.95		

Calculate data as in Table VIII and plot performance curves similar to Fig. 42.13.

3.13. A test on a 440-volt, three-phase, 60-cycle, six-pole induction motor gives the following data: no-load line current = 4.2 amp.; no-load power factor = 0.32; locked-rotor line current (reduced to rated voltage) = 55.2 amp.; locked-rotor power factor = 0.51; primary resistance = 2.28 ohms, between terminals.

Draw the circle diagram, Fig. 26.13, and plot the performance curves as in Fig. 27.13.

4.13. On test a 500-volt, three-phase, 50-cycle, ten-pole induction motor gives the following data: At no load the input is 53.5 kw. and the current is 14.6 amp. With blocked rotor and for 250 volts impressed, the motor takes 56.1 kw. and 802 amp. The motor is star-connected in both the stator and rotor. Stator resistance between mains = 0.03 ohm.

Draw the circle diagram and plot the performance curves.

5.13. Let the resistance of the rotor in problem 4.13 be increased until the starting torque is a maximum. Plot the corresponding performance curves for the motor.

6.13. Assume a simple balanced mine hoist with two cages, motor operated with motor direct connected to the drum. Depth of mine is 300 ft. Diameter of drum is 8 ft.

(*a*) How long will it take one cage to make the trip from bottom to top (300 ft.)? The running or full speed is 1,200 ft. per min. The system is accelerated to full speed, 4 sec., and brought to rest from full speed in 6 sec.

(*b*) It is desired to hoist 2,500 tons of coal from the mine in 8 hr., assuming:

Weight of coal per trip	2 tons
Delay between trips	5 sec.
Acceleration	4 sec.
Retardation	6 sec.

What should be the full speed of the motor, direct connected to the hoist drum, to secure this tonnage?

(c) How far will the cages travel during acceleration and during retardation?

7.13. A 10-hp. induction motor is furnished with external secondary resistance which reduces the output to 2.5 hp. at one-fourth speed, thus developing full-load torque. Find at what percentage speed the motor will develop 2 hp., assuming that for this value of secondary resistance the torque is directly proportional to slip.

CHAPTER XIV

ALTERNATORS

One of the notable features in the development of the electrical industry during the past 15 years has been the enormous increase in the size of the central stations. This has been due both to an increase in consumption of electric energy and to the combination of a large number of small plants into comparatively few large systems. Simultaneously with the growth of the central station has come the increase in the size of the generating units. At the Columbian Exposition in 1893 "Jumbo," the largest generator in the world, had a rating of 750 kv.a., while today single units deliver 150,000 kv.a. continuous load. Any discussion of the historical development of the modern alternator is outside the scope of this volume, and only a few of the more important features in the design are mentioned. The increase in speed, largely due to the invention of the steam turbine, is the chief factor in making possible economical designs of large units. By placing the armature winding in slots the reluctance of the field circuit is greatly reduced and also made to a large degree independent of the size of the armature conductor. With the field placed on the rotating spider, only the exciting current need pass through brushes and collector rings, while the generated power is transmitted through stationary and well-protected copper conductors. To provide adequate voltage regulation for rapid changes in load used to be a very difficult problem for the designers of alternators. The regulation is now secured, to a much more satisfactory degree than was possible in the self-regulating alternator, by means of regulating devices, outside the generator, such as the Tirrill regulator.

(*a*) **The Field.**—The number of poles is determined by the speed and the frequency. Except for small machines the field forms the rotating part. The poles are built up of punchings of sheet steel, about 0.025 in. thick, and riveted together between cast-steel end plates. The exciting current is led to the pole windings through brushes bearing on cast-iron or cast-brass

lip-rings. The exciting current comes from a separate exciter ntirely independent of the generator. The excitation voltage s usually 120 volts, and hence the fields have few turns and ake a comparatively large current. Except for small machines he winding is made from copper strips bent on edge, forming a hort rectangular-shaped coil.

(*b*) **The Armature.**—The armature conductors are placed on he inside of the stator surrounding the rotating field. In order o produce a magnetic circuit with low reluctance, the armature onductors are placed in slots, thus leaving only a small air-gap etween the iron cores of the field and armature. The armature ore is made from punchings of sheet steel of about half the hickness of the sheet used for the fields. Ducts are provided n the iron core through which the air circulates, keeping the nachine cool.

As already noted, the armature winding determines whether he machine is a single-phase, two-phase or three-phase alterator. The two-phase alternator has two single-phase windings paced 90 electrical degrees apart, and, similarly, the three-phase nachine has three single-phase windings spaced 120 electrical legrees. Very few alternators of the strictly two-phase and hree-phase design have ever been built. The so-called twohase and three-phase alternators are fundamentally designed as our-phase and six-phase machines with their windings spaced 0 and 60 electrical degrees, respectively. In the same manner s already explained for the induction motor, the four-phase nd six-phase designs are converted into the so-called twohase and three-phase alternators, respectively, by merely changng the connections inside the machine.

The three windings on the three-phase alternator may be onnected in star or in delta as explained in Chap. IX. Alterators are generally wound star-connected, as this requires fewer urns, eliminates the third harmonic and provides a convenient eutral point. The winding for any of the above alternators nay further differ by being placed in several slots for each pole nd for each phase, forming the so-called *distributed* winding. Moreover, the arrangement of the wires between each pair of oles may be of the wave or lap (chain) system, or a combination f the wave and lap form of winding. The windings may be in ingle or double layer, and the pitch may cover the full 180 lectrical degrees or only a fraction of that distance. All of

these factors influence the shape and magnitude of the generated voltage wave.

In Fig. 1.14 is illustrated a single-phase, wave-wound armature, having one conductor per pole. Figure 2.14, likewise, is a single-phase machine but has two partially distributed conductors per pole. A two-phase, full-pitch, distributed, lap winding is shown

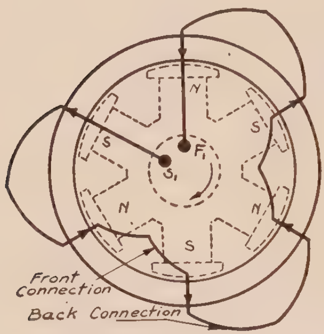


Fig. 1.14.—Single-phase, full-pitch wave winding.

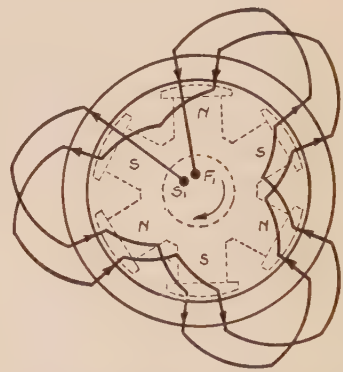


Fig. 2.14.—Single-phase, full-pitch partially distributed wave winding.

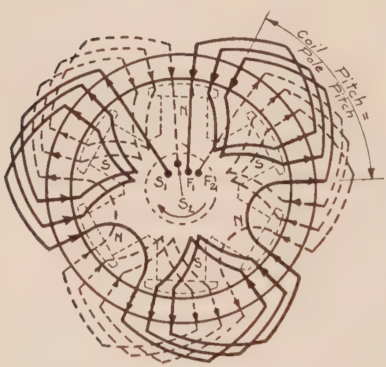


Fig. 3.14.—Two-phase, full-pitch, distributed lap winding.

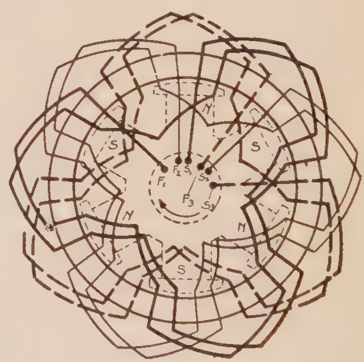


Fig. 4.14.—Three-phase, full-pitch, distributed wave winding.

in Fig. 3.14. Likewise, a three-phase, two-conductor, full-pitch wave winding is illustrated in Fig. 4.14.

Armature windings in commercial alternators differ in design, in order to meet satisfactorily various service requirements. Alternator armature windings may be classified in several ways,

as for example: wave, lap, chain or spiral windings; open-circuit or closed-circuit windings; bar or coil windings; whole-coiled or half-coiled windings; and single-phase or polyphase windings.

FIG. 5.14.

(c) **Wave Forms.**—The voltage wave produced by each armature conductor depends directly upon the distribution of the lines of force in the magnetic field. The distribution of the

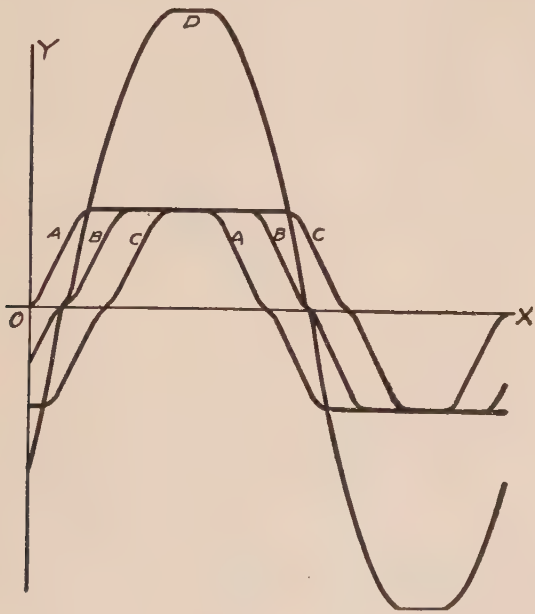


FIG. 6.14.

lines depends upon the reluctance of the magnetic paths, and hence is largely affected by the shape of the field pole. As a rule, the distribution is not such as to give a simple sine wave

of voltage but more of a rectangular shape, as in Fig. 5.14. If the winding is placed in three slots spaced γ electrical degrees, each one of the coils produces the same-shaped waves but differing in phase position by $\gamma°$. Hence the resultant wave at each point is the sum of the instantaneous values of the component waves, as shown in Fig. 6.14. In general, armature windings are distributed and placed in several slots per phase, per pole. The number of slots and the distribution of the field flux have a direct bearing on the harmonics in the voltage wave, as explained in Chap. XXIV. The magnitude of the total generated voltage per phase depends upon the number and spacing of the slots per phase per pole (Fig. 7.14). Assuming that the voltage wave for each conductor can be expressed by an equivalent sine wave, then the relation of the total voltage per phase to

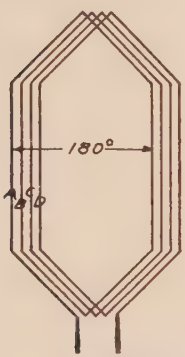


Fig. 7.14.

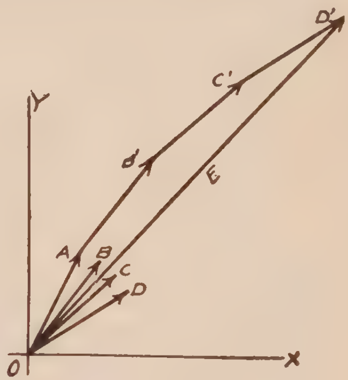


Fig. 8.14.

the e.m.f. generated in each conductor may be found by taking the vector sum of the component voltages as shown in Fig. 8.14. Let A, B, C and D be voltage vectors equal in magnitude and displaced in position to the same extent as the spacing of the slots on the armature. Then the resultant vector E represents the total voltage per phase both in magnitude and in phase position.

The ratio of the magnitude of the total voltage per phase to the product of the number of slots and the voltage generated in the conductor in each slot is known as the *distribution factor* and is represented by the letter k. The values of the distribution factor for several slots per pole are given in Table IX for two- and three-phase generators.

TABLE IX

Slots per pole	k, Two-phase	k, Three-phase
1	1.000	1.000
2	0.924	0.966
3	0.911	0.960
4	0.906	0.958
6	0.903	0.956

The value of the total voltage per phase is also reduced if the winding is *short-pitch;* that is, the two sides of the armature coil are less than 180 electrical degrees apart (Fig. 9.14) and hence the voltage generated under the north pole is not quite in phase with the voltage wave produced simultaneously by the corresponding south pole in the other side of the same coil. If the two sides of the coil are out of phase by δ electrical degrees the total voltage per phase is reduced by the factor $\cos \frac{\delta}{2}$, as may be seen from Fig. 10.14. Let the distributed winding be in four slots and the voltages on one side under a north pole be represented in phase and magnitude by the lines A, B, C and D, Fig. 10.14, and let the corresponding voltages on the other side, under a south pole, be represented in phase and magnitude by F, G, H and I. In each coil the two voltage waves are dis-

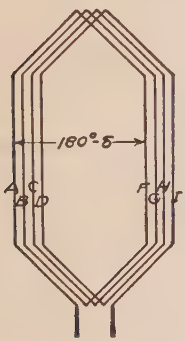


FIG. 9.14.

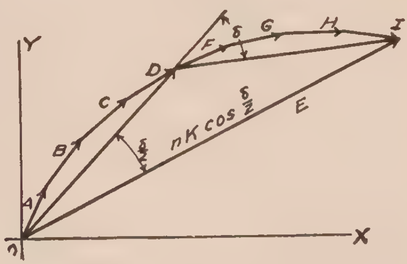


FIG. 10.14.

placed by δ electrical degrees and hence the resultant voltage is equal to the product of the voltage generated per conductor, the number of conductors, the distribution factor and $\cos \frac{\delta}{2}$. Hence the terminal voltage may be expressed in terms of the constants of the design, n the number of turns, k the distribution

factor, $\cos\frac{\delta}{2}$ the pitch factor and ${}_a\Phi$ the number of lines per pole cut by each conductor. Equation (18.2) gives the fundamental relation for the production of voltage by induction per turn. This is also the expression for a full-pitch concentrated winding; that is, all the turns placed in one slot per phase per pole.

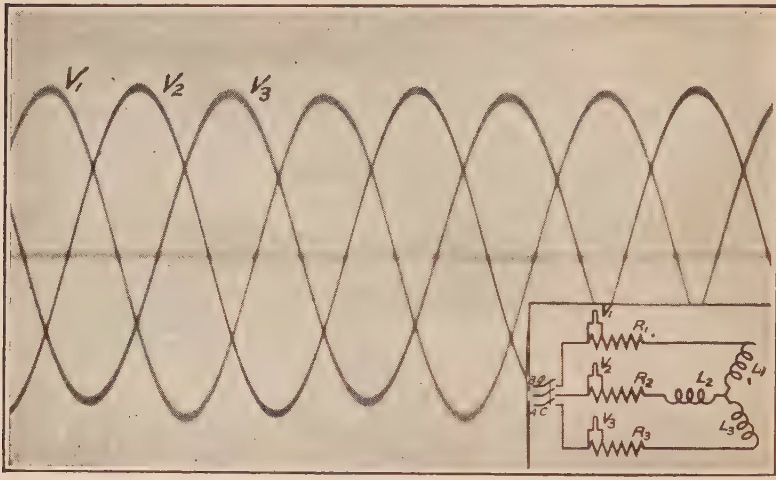


FIG. 11.14.—Oscillogram of three-phase alternating currents. $I = 8$ amp.; $E = 150$ volts; $f = 60$ cycles.

Let

n = the number of turns, or half the number of conductors.
f = the frequency.
${}_a\Phi$ = the total useful flux per pole.
k = the distribution factor.
$\cos\frac{\delta}{2}$ = the pitch factor.

The generated voltage per phase per pole therefore equals:

1. For concentrated winding, full-pitch,

$$4.44nf\ {}_a\Phi 10^{-8} \text{ volts} \qquad (1.14)$$

2. For distributed winding, full-pitch,

$$4.44knf\ {}_a\Phi 10^{-8} \text{ volts} \qquad (2.14)$$

3. For distributed winding, short-pitch,

$$4.44knf\ {}_a\Phi \cos\frac{\delta}{2} 10^{-8} \text{ volts} \qquad (3.14)$$

(*d*) **Armature Reaction; Reactance and Resistance.**—For a given machine running at constant speed the factors k, n, f

and $\cos \frac{\delta}{2}$ are constant. The only variable factor in equations (1.14), (2.14) and (3.14) for the generated voltage is the number of lines cut by the armature conductors. The m.m.f. producing useful flux depends, first, upon the *field excitation* and, second, upon the *armature reaction*. The field excitation is supplied from some independent source and is usually varied with the load requirements by means of an automatic regulator. The currents in the armature also produce magnetic fluxes that combine vectorially with the main field to form the useful flux $_u\Phi$ in equation (3.14). This factor is called the *armature reaction*, and it affects both the magnitude and time-phase of the generated voltage wave. Since the armature flux is in time-phase with the current, the armature reaction depends on both the magnitude of the load current and the load power factor. At no load the armature reaction is zero. For any load the armature flux is proportional to the current and also its effect upon the field flux depends upon the power factor.

In a three-phase generator carrying balanced, constant-load currents, the armature reaction is constant in magnitude but its space-phase relation to the field flux depends on the load power factor. The effects of the armature reaction on the field flux, and therefore on the generated voltage, for load currents, alike in magnitude but differing in power factor, are shown diagrammatically in Figs. 12.14 to 17.14, inclusive. In order to keep the drawings as simple as possible a two-pole field and a single turn per phase in the armature are used in the diagrams.

Let S_1, S_2, S_3 and F_1, F_2, F_3 denote the start and finish, respectively, of the three phases in the armature windings, and let the conventional notation in the diagram indicate the sequence of phases, the direction of rotation, field polarity and direction of flow of the armature current.

In order to readily compare the effects produced by the armature reaction under varying power factors in the load, all the diagrams in Figs. 12.14 to 17.14, inclusive, represent the same instant in the alternating-current cycle. The heavy cross and dot, in phase 1, indicate that the current flowing from S_1 to F_1 was at its maximum positive value. At this instant the currents in the two other phases are less in magnitude and reversed in direction as indicated by the lighter dots and crosses in S_2, F_2 and

S_3 and F_3. The armature flux ϕ_A is therefore the same in both magnitude and position in the six figures.

In Fig. 12.14 are shown the magnitude and phase relations of the field ϕ_F, and armature reaction ϕ_A, flux vectors for unity power factor load. The armature reaction ϕ_A is constant in magnitude and in phase position at right angles to the field flux

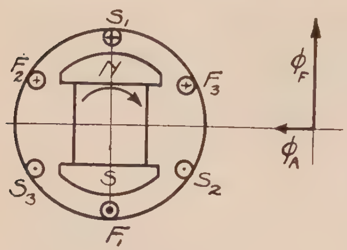


Fig. 12.14.

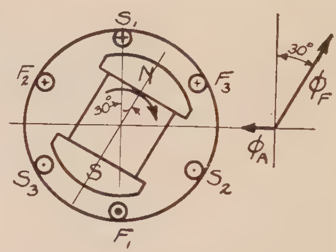


Fig. 13.14.

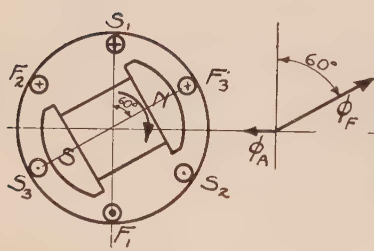


Fig. 14.14.

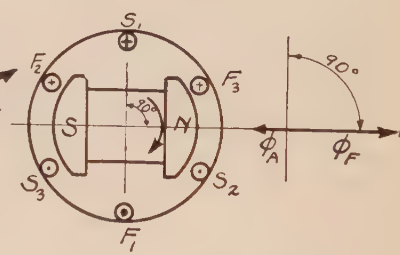


Fig. 15.14.

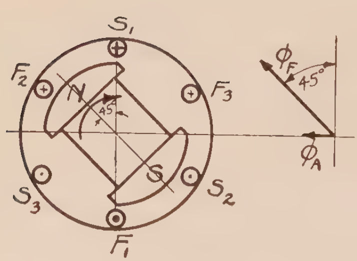


Fig. 16.14.

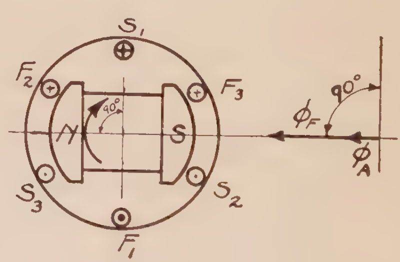


Fig. 17.14.

ϕ_F. The main effect of the armature reaction is a shift in the distribution of the resultant magnetic field, which causes a corresponding change in the generated wave form, as compared to the voltage wave under no-load conditions.

However, for load currents having power factors less than unity the effects on the field flux are markedly different. For current lagging 30° the field flux will be 30° in advance of its space-phase position for unity power factor, as shown in Fig. 13.14. The resultant total flux is less in magnitude than ϕ_F and shifted in time-phase. For 60° lagging current the demagnetizing effect of the armature reaction is larger and the phase shifting less. For load currents lagging 90° the armature reaction is in direct opposition to the field flux, as shown in Fig. 15.14; producing a corresponding reduction in the generated voltage.

For leading armature current essentially the reverse effect is produced as illustrated by the diagrams in Figs. 16.14 and 17.14. The resultant flux is greater than the field flux and, as a consequence, the voltage generated is increased in magnitude as compared to no-load voltage for the same field excitation.

For power factors less than unity but greater than zero the armature reaction may be considered as consisting of two components in phase and in quadrature, respectively, with the field flux. For lagging currents the in-phase component reduces the field magnetization and the quadrature component produces distortion as illustrated in Figs. 13.14 and 14.14. For leading currents the in-phase component adds to the field magnetization, and the quadrature component shifts the effective field flux as shown in Fig. 16.14.

In the preceding discussion it has been assumed that the armature reaction would be of the same magnitude for the same current in the armature circuits regardless of the power factor. For machines having salient poles this is not the case as the reluctance for the armature flux differs with the position of the field poles. This reluctance is least for low power factors and hence the armature reaction is greater for large values of θ than at unity power factor.

A small part of the armature flux, not affecting the field flux, surrounds the conductors in the same way as the leakage flux in transformers. Most of this leakage flux is on the ends of the armature, that is, around portions of the coils that do not pass under the fields. This portion, known as the *leakage flux*, appears in the circuit as an *armature leakage reactance*. Evidently the magnitude of the leakage flux depends directly upon the current and is not affected by the lag or lead of the load current. In modern alternators the effect of the armature reaction is

three or four times as large as the armature leakage reactance. The armature reactance like the armature resistance consumes voltage and hence the terminal voltage is less than the generated voltage in magnitude and differs in phase position. These relations are readily seen from a vector diagram. Referred

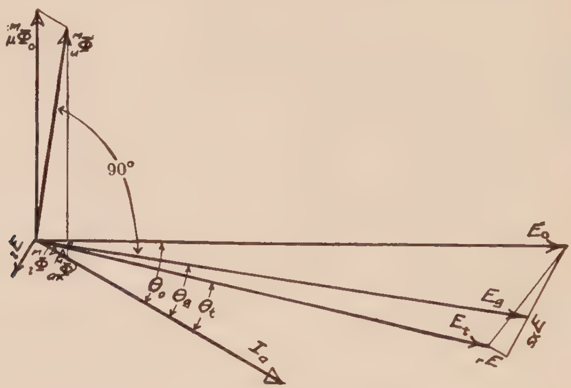


Fig. 18.14.—θ_0 = 30°, lagging current.

to the armature windings the field flux pulsates, although of constant value in space. Let the wave shape of the fluxes, voltages and currents be of the equivalent simple sine form in order that their phase relations may be represented by the crank vector diagram. Let the diagram in Fig. 18.14 be drawn for the flux, voltage and current relations in one phase of the armature winding.

${}^{M}_{u}\Phi_0$ = maximum flux due to the field excitation and passing through the armature windings; the useful flux at no load.
E_0 = voltage produced by the flux ${}^{M}_{u}\Phi_0$.
I_a = armature current (lagging in this case).
${}^{M}_{ax}\Phi$ = flux representing armature reaction.
${}^{M}_{l}\Phi$ = leakage flux, producing armature reactance.
${}^{M}_{u}\dot{\Phi} = {}^{M}_{u}\dot{\Phi}_0 + {}^{M}_{ax}\dot{\Phi}$ = useful flux.
E_g = voltage generated per phase.
${}_{x}E$ = voltage absorbed by armature reactance.
${}_{r}E$ = voltage absorbed by armature resistance.
E_t = terminal voltage.
θ_t = terminal phase angle.
$\cos\theta_t$ = load power factor.

With a lagging current the terminal voltage is less than the voltage generated in the alternator. In Fig. 19.14 the phase relations of the same quantities are shown for a load having a leading current. The terminal voltage is increased as the armature reaction adds to the field producing a larger useful flux than at no load.

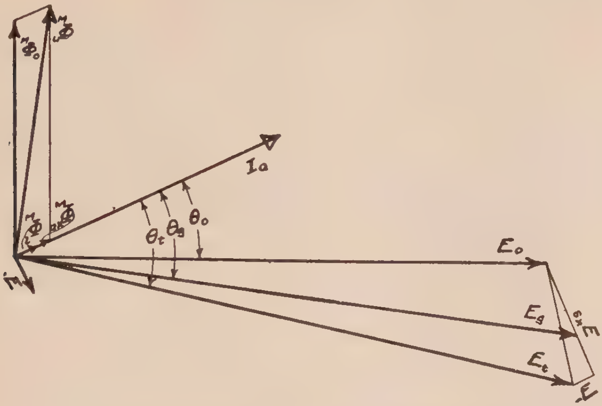


FIG. 19.14.—θ_0 = 25°, leading current.

In Fig. 20.14 is shown the corresponding vector diagram for $\theta_0 = 0°$, and in Figs. 21.14 and 22.14 the diagrams for currents of equal magnitude but lagging and leading the voltage, respectively, by 90°. For a given armature current the locus of the terminal voltage is a circle, as shown in Fig. 23.14.

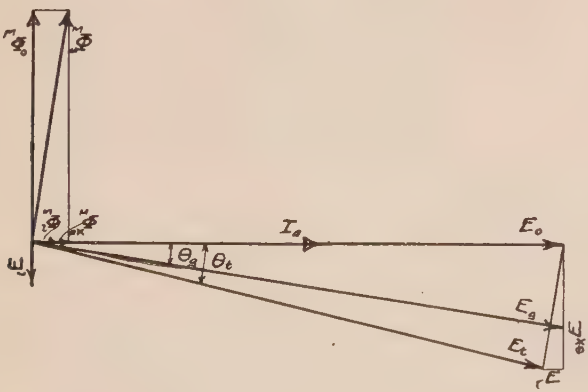


FIG. 20.14.—θ_0 = 0°, current in phase

The above vector diagrams refer to constant-current conditions. With constant load the current increases with the decrease of the power factor, and, as the drop due to armature reaction and

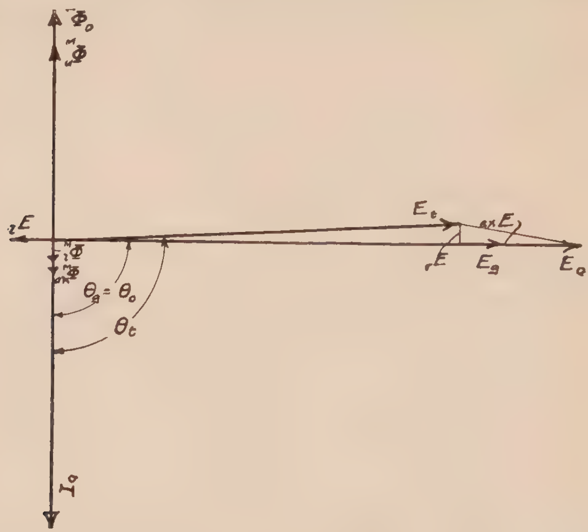


FIG. 21.14.—$\theta_0 = 90°$, lagging current.

reactance are proportional to the armature current, the effect at low power factors is greatly increased and the locus of the terminal voltage is no longer a circle. However, in commercial

FIG. 22.14.—$\theta_0 = 90°$, leading current.

systems the voltage is kept constant or even caused to rise at the terminals of the generator with increase of load by increasing the field excitation. To keep the terminal voltage constant, the field excitation must be increased so as to compensate for the

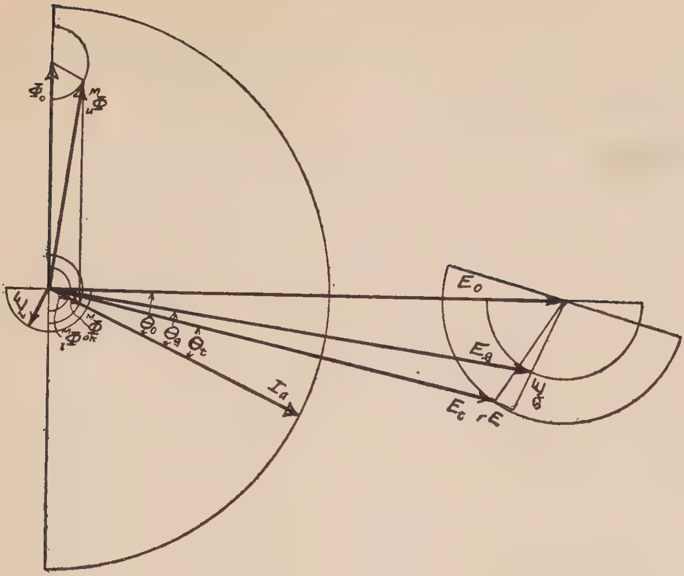


Fig. 23.14.—θ_0 variable. Flux, current and voltage loci.

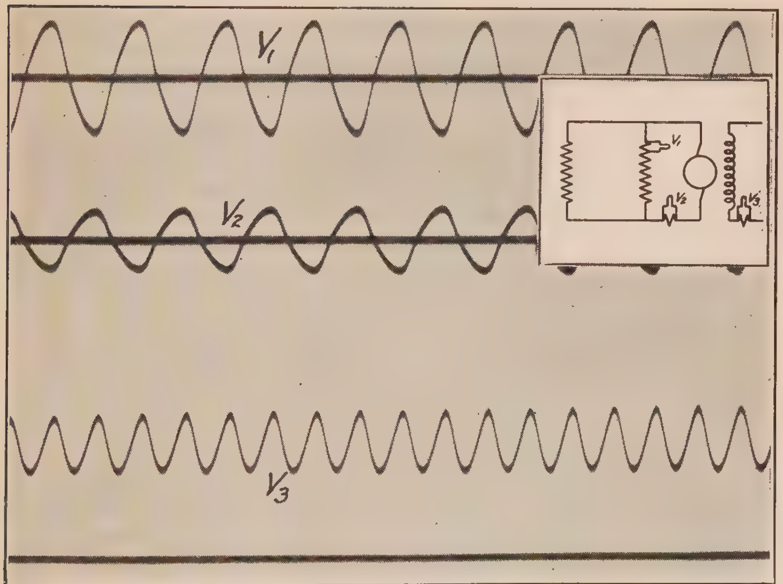


Fig. 24.14.—Oscillogram showing armature reaction on field circuit of single-phase generator. V_1, line voltage; V_2, armature current; V_3, field current.

effects of armature reaction, reactance and resistance. The number of ampere-turns required also depends upon the previous magnetization or the point on the magnetization curve at which the generator is operating.

The effects of the armature reaction in *single-phase generators* are basically the same as for three-phase machines with one important difference, namely, the armature reaction is not constant in magnitude but pulsates at double the frequency of the current flowing in the armature. This is illustrated by the oscillogram in Fig. 24.14. The three curves recorded by the oscillograph show the instantaneous values of the voltage V_1 at the terminals of the machine; the armature current V_2; and the field current V_3. It is evident that armature reaction must produce pulsations in the field current of double the frequency of the armature current, or of the impressed line voltage as the current in the armature reverses in direction for each 180° rotation of the field. Hence, the same relative reaction is produced by the armature current on *successive poles* not merely on alternate poles, that is, poles of like polarity. Hence, the pulsations produced by the armature reaction pass through a complete cycle for 180° rotation of the field, while the armature current requires 360° field rotation for each cycle.

(*e*) **Magnetization or Saturation Curve at No Load.**—As in direct-current generators, the magnetization curve is merely the relation between the terminal volts at no load and the field current. Usually the field currents are plotted as abscissæ and the corresponding volts as ordinates, as shown in Fig. 25.14. Experimentally, the data for the curve are found by running the alternator at the rated speed and taking a series of readings of field currents and the corresponding volts at the terminals of the machine. For predetermining the saturation curve, when designing an alternator, accurate values of the reluctance of the magnetic circuit for the increasing flux densities must be known.

(*f*) **Synchronous Reactance.**—Both the armature reaction and reactance drop are proportional in magnitude to the armature current, and the voltages consumed by both are in time-phase for all power factors as seen in Figs. 18.14 to 23.14. Both factors may therefore be represented by one equivalent reactance. This is called the *synchronous reactance* ${}_sx_a$, and is measured experimentally in the following manner: The armature is short-circuited through an ammeter and the generator is brought up to syn-

chronous speed without field excitation. Field current is then applied and a series of readings taken of the field currents

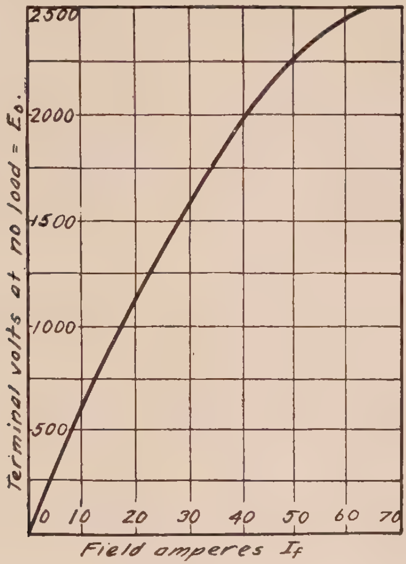


FIG. 25.14.

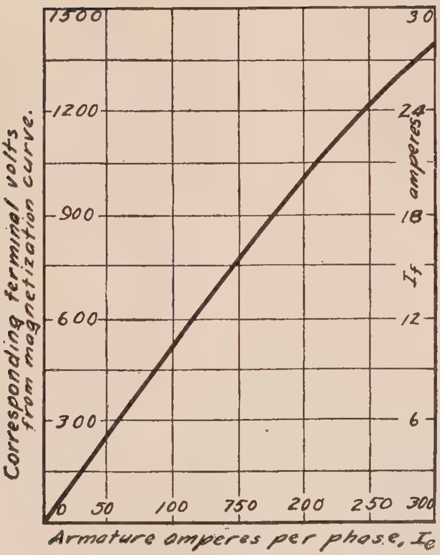


FIG. 26.14.

and the corresponding armature currents, until the armature current reaches about 50 per cent above normal full-load value.

From the saturation curve the voltages corresponding to the field-current readings are obtained. A curve is plotted as in Fig. 26.14 with armature currents as abscissæ, and the terminal volts corresponding to the field currents as ordinates. The curve is nearly a straight line, drooping slightly as the iron approaches saturation. Under short-circuit conditions the armature power factor is nearly zero and the vector relations are as shown in Fig. 21.14. The voltage consumed by the synchronous reactance is practically equal to the total voltage, as the part taken by the resistance is small and almost in time quadrature with the induced voltage.

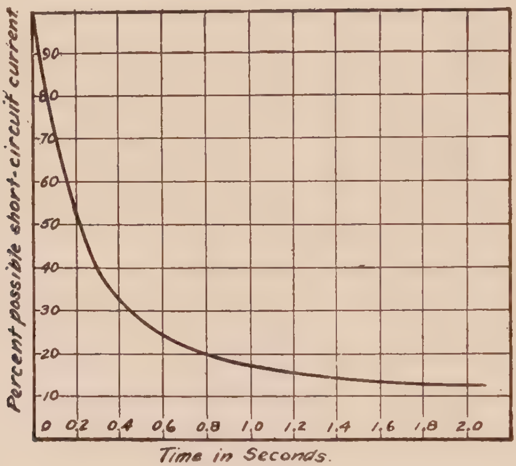


FIG. 27.14.

From the synchronous reactance curve:

$$E_0 = I_a\sqrt{r_a^2 + {}_sx_a^2} \tag{4.14}$$

or

$${}_sx_a = \frac{E_0}{I_a} \text{(approximately)} \tag{5.14}$$

The value obtained for the synchronous reactance by this method is, however, considerably larger than the actual synchronous reactance of the alternator when operating under full load. Moreover, the synchronous reactance, and hence the synchronous impedance, vary with the load. While the data gained by the saturation and short-circuit reactance tests are of importance when studying alternator characteristics, the cal-

culated regulation is only an approximation to the actual value. In modern alternators from 10 to 25 per cent of the total synchronous reactance is due to armature reactance, while the remainder is caused by armature reaction. This is illustrated by Fig. 27.14. The ordinates represent the short-circuit current of a large alternator and the abscissæ the time in seconds after the short-circuit began. The *instantaneous short-circuit current*, taken as 100 per cent, gives the ratio of the voltage to the armature impedance. After 2 sec., the current reached the *sustained short-circuit* value which represents the ratio between the voltage and the synchronous impedance. The quantitative values of the synchronous reactance and armature leakage reactance are usually given in terms of the voltage consumed by full-load current and expressed in per cent of the rated terminal voltage. A synchronous reactance of 40 per cent means that for full-load current the voltage consumed in the synchronous reactance of the armature is 40 per cent of the rated terminal voltage.

(*g*) **Field Excitation at Any Load and for Any Power Factor.**—From the saturation curve at no load, the synchronous reactance

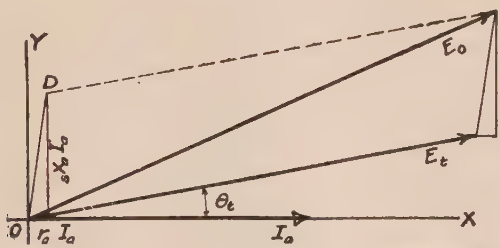


FIG. 28.14.

curve and the armature resistance, the approximate value of the required field excitation for any load and at any power factor may be determined. In the vector diagrams in Figs. 18.14 to 23.14, it is shown that the voltage drop caused by armature reaction, reactance and resistance may be represented by a triangle so drawn that the resistance drop is in phase, and synchronous reactance drop in quadrature, with the current. Let the conditions be a constant terminal voltage, constant load current and variable load power factor. In Figs. 28.14 and 29.14 the current I_a is drawn on the X-axis, and the terminal voltage E_t

is leading by θ_t°. The vector for the armature resistance drop is parallel to the current, and the corresponding vector for the armature synchronous reactance drop in quadrature with the current or perpendicular to the X-axis. The vector sum of E_t, $r_a I_a$ and ${}_s x_a I_a$ is the required voltage E_0. This is the voltage that appears at the terminals at no load, and hence is proportional to the main field flux. From the no-load saturation curve the field excitation corresponding to E_0 may be found.

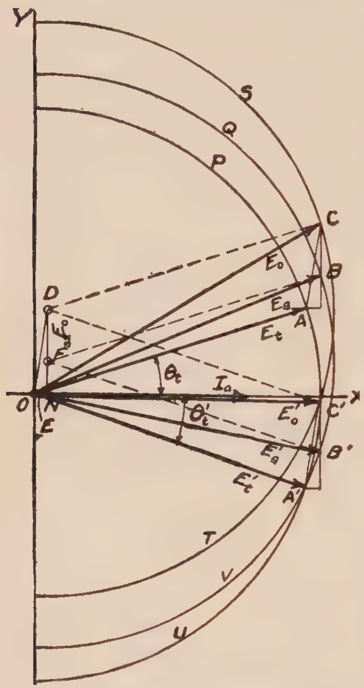


FIG. 29.14.

By varying the power factor but keeping the terminal voltage and armature current constant the loci of E_t and E_0 are circles, with E_t as radius and points O and D as centers, as shown in Fig. 29.14. The locus of the generated voltage may be found by separating the synchronous reactance drop into the parts proportional to the armature reaction and the armature reactance. Under ordinary operating conditions the load power factor, the terminal voltage and the armature current are all variable.

Assuming two factors as constant and the third variable, three combinations are possible. The corresponding loci are shown in Fig. 29.14

First Case.—E_t and I_a constant; θ_t variable.

I_a = armature current, assumed constant.
E_t = the terminal voltage, assumed constant.
r_a = armature resistance.
x_a = armature reactance.
${}_s x_a$ = synchronous reactance of armature, equivalent to both the armature reactance and reaction.
E_g = the generated voltage, proportional to the useful flux.

E_0 = the no-load voltage corresponding to the same field excitation, and proportional to the field flux.
θ_t = angle of lead or lag, variable.
$OA = E_t$
$ON = r_a I_a = {}_rE_a$
$NF = x_a I_a = {}_xE_a$
$ND = {}_sx_a I_a = {}_sE_a$
$OB = E_g$
$OC = E_0$

$$\dot{E}_g = \dot{E}_t + {}_r\dot{E}_a + {}_x\dot{E}_a = \dot{E}_t \cos \theta_t + {}_r\dot{E}_a + j(\dot{E}_t \sin \theta_t + {}_x\dot{E}_a) \quad (6.14)$$

$$\dot{E}_0 = \dot{E}_t + {}_r\dot{E}_a + {}_{sx}\dot{E}_a = \dot{E}_t \cos \theta_t + {}_r\dot{E}_a + j(\dot{E}_t \sin \theta_t + {}_{sx}\dot{E}_a) \quad (7.14)$$

Circle PAU = locus of E_t (O center, E_t radius).
Circle QBV = locus of E_g (F center, E_t radius).
Circle SCT = locus of E_0 (D center, E_t radius).
$\cos \theta_t$ = power factor of locus for perfect regulation.

Since the sides of the triangle OND are directly proportional to the armature current, similar constructions may be drawn for any load current and the value of the field excitation determined.

Second Case.—E_t and θ_t constant; I_a variable.

For constant power factor and terminal voltage the locus of the no-load voltage E_0 is along the line CA; for with a change in I_a the point D moves along the line OD, and as E_t and θ_t are assumed constant the point C must move along the line CA.

Third Case.—θ_t and I_a constant; E_t variable.

For constant power factor and current with variable terminal voltage, the triangle OND remains constant and the no-load voltage is the resultant of OD and the several values of E_t, and the locus of E_0 is along the line DC. The required field excitation may therefore be found from curves in Figs. 25.14, 26.14, and 29.14 for any load at any power factor and for any terminal voltage.

(*h*) **Regulation.**—The regulation of an alternator is defined as the ratio of the no-load minus the full-load terminal voltage to the full-load terminal voltage, the speed and field excitation remaining constant. The regulation is generally expressed in per cent of the full-load terminal voltage. Using the same

notation as in the preceding paragraph, the regulation is expressed by equation (8.14).

$$\text{Regulation (in per cent)} = \frac{E_0 - E_t}{E_t} \times 100 \qquad (8.14)$$

From the preceding discussion of field excitation for any load and at any power factor it is evident that the regulation depends very largely on the power factor of the load. This is shown by equation (9.14) which is obtained by combining equations (7.14) and (8.14).

Regulation (in per cent) =

$$\frac{\sqrt{(E_t \cos\theta_t + {}_rE_a)^2 + (E_t \sin\theta_t + {}_xE_a)^2} - E_t}{E_t} \times 100 \qquad (9.14)$$

In the above equation, the four main factors that affect the regulation of alternators, namely, armature reaction, armature reactance, armature resistance and the power factor of the load are included. A fifth factor, the changes in magnetic-flux leakage, particularly in relation to changes in the power factor of the load, cannot be measured directly, but enters to an approximate average extent under the test conditions generally used in determining the synchronous reactance. That is, the synchronous impedance is not constant in value for varying power factors. Thus flux leakage depends on the reluctance of the magnetic-flux paths, which in turn depends on the permeability of the iron and on the relative position of armature coils and slots with respect to the field poles at the instant the armature current is a maximum.

For determining the regulation of alternators four test methods have been developed.

1. Loading at specified output and power factor.
2. Synchronous-reactance method.
3. Magnetomotive-force method.
4. From test curves as recommended by the A.I.E.E.

Method I.—The regulation can be measured directly by loading the alternator at the specified output and power factor, and then reducing the load to zero and measuring the terminal voltage, without change in speed and field excitation. The regulation in per cent is then obtained by equation (8.14). This method is the most desirable if suitable load for test purposes is available which, however, is seldom the case.

Method II.—For this method the following test data are necessary:

a. Open-circuit saturation curve (Fig. 25.14).
b. Short-circuit synchronous reactance (Fig. 26.14).
c. Rated load current and armature resistance.

The regulation is obtained from equation (9.14).

Method III.—For this method the test requirements are the same as in method II. However, in place of combining the voltage drops under open-circuit and short-circuit conditions the resultant of the two field excitations is first determined and then the voltage E_0 is obtained from the saturation curve. For

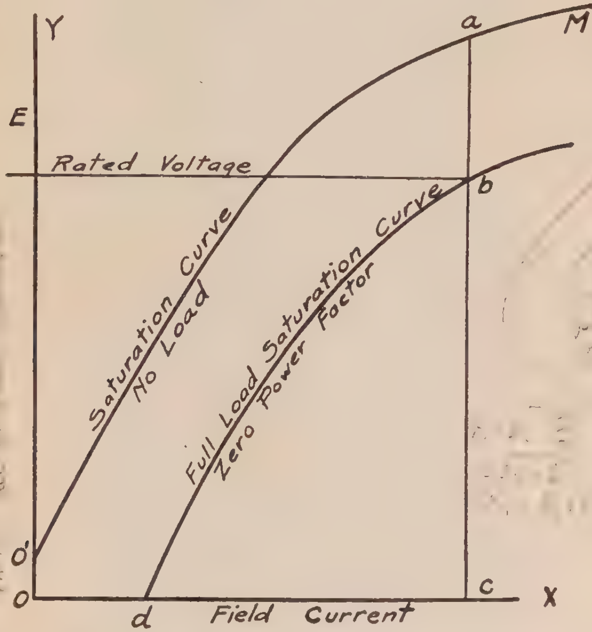


FIG. 30.14.—Open-circuit saturation curve and zero power-factor saturation curve of an alternator.

unity power-factor load the short-circuit field excitation is considered to be in quadrature with the open-circuit excitation. Hence, the resultant field excitation at full-load unity power factor is the square root of the sum of the squares of the component excitations. For other power factors the vector values must be used in obtaining the resultant field excitation.

The second method gives too high and the third too low values for E_0. The actual regulation is approximately the mean of the values obtained by the second and third methods.

Method IV.—This method consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero power-factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by overexciting the alternator while carrying a load of idle-running, underexcited synchronous motors. The power factor under these conditions is very low and the load saturation curve approximates very closely the zero power-factor saturation curve. From this curve (Fig. 30.14) and the open-circuit curve, points for the load saturation curve, for any specified power factor, can be obtained by means of vector diagrams. For specific details in applying this method see A.I.E.E. Standards for voltage regulation of alternators.

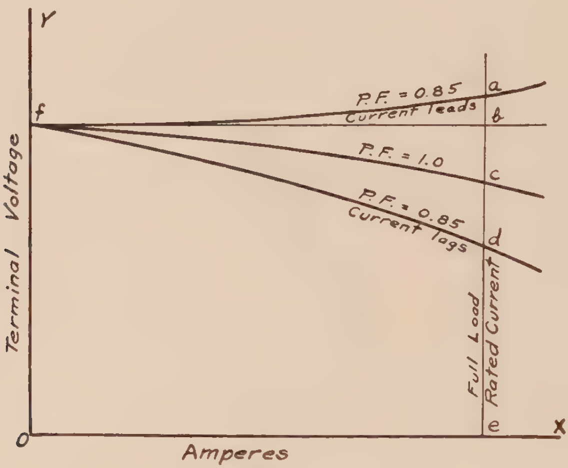


FIG. 31.14.—Regulation load curves.

In considering the voltage regulation of alternating-current circuits it should be kept in mind that alternators are not self-regulating as frequently is the case with direct-current generators. The voltage regulation in alternating-current circuits is accomplished by special regulating devices separate and apart from the alternators. The regulation of the alternators must, however, be known in order to properly adjust the auxiliary, automatic regulators.

The TA Regulator.—Since the invention of automatic regulators entirely separate from the alternator, all the earlier methods of

self-regulation have become obsolete. By proper adjustment the automatic regulator insures highly satisfactory regulation on the system, although the inherent regulations of the alternators may vary widely. With the use of an automatic regulator, like the TA type (Fig. 32.14), manufactured by the General Electric Company, inherent regulation of the alternator is of comparatively little

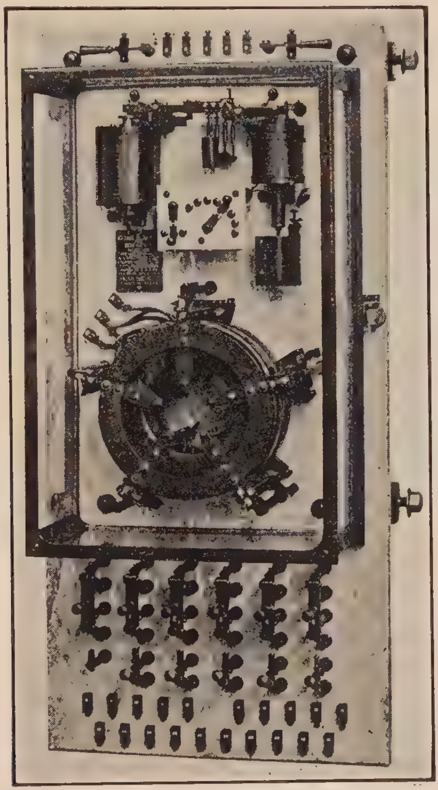


Fig. 32.14.—Type TA, form F-5, voltage regulator. (*General Electric Company.*)

importance, as the regulator so adjusts the excitation that constant voltage is obtained under all conditions of load.

An elementary diagram of the type TA, form A-2 regulator connections with an alternating current generator and exciter is shown in Fig. 33.14. The regulator has a direct-current control magnet, an alternating-current control magnet and a relay. The direct-current control magnet is connected to the exciter

busbars. This magnet has a fixed stop core in the bottom and a movable core in the top which is attached to a pivoted lever having at the opposite end a flexible contact pulled downward by four spiral springs. For clearness, however, only one spring is shown in the diagram. Opposite the direct-current control magnet is the alternating-current control magnet, which has a potential winding connected by means of a potential transformer

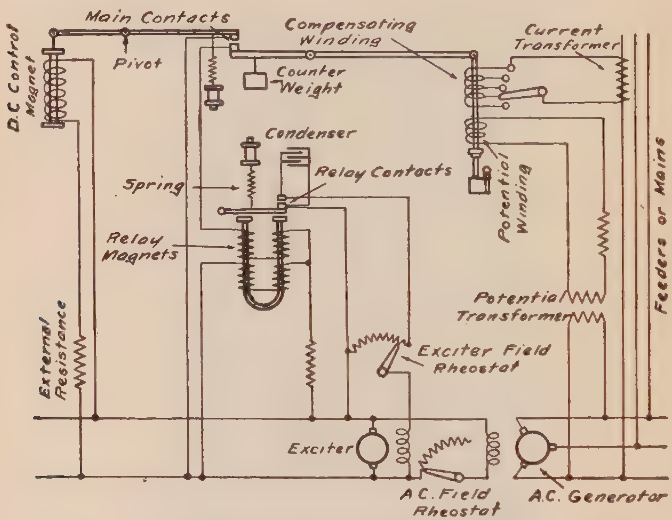


FIG. 33.14.—Circuit diagram of type TA, form A-2, regulator.

to the alternating-current generator or busbars. There is an adjustable compensating winding on the alternating-current magnet connected through a current transformer to the principal lighting feeder. The object of this winding is to raise the voltage of the alternating-current busbars as the load increases. The alternating-current control magnet has a movable core and a lever and contacts similar to those of the direct-current control magnet, and the two combined produce what is known as the *floating main contacts.* The relay consists of a U-shaped magnet core having a differential winding and a pivoted armature controlling the contacts which open and close the shunt circuit across the exciter field rheostat. One of the differential windings of the relay is permanently connected across the exciter busbars and tends to keep the contacts open; the other winding is connected to the exciter busbars through the floating main contacts, and when the latter are closed neutralizes the effect of the first

winding and allows the relay contacts to short-circuit the exciter field rheostat. Condensers are connected across the relay contacts to prevent severe arcing and possible injury. The circuit shunting the exciter field rheostat through the relay contacts is opened by means of a single-pole switch at the bottom of the regulator panel and the exciter rheostat turned in until the alternating-current voltage is reduced 65 per cent below normal. This

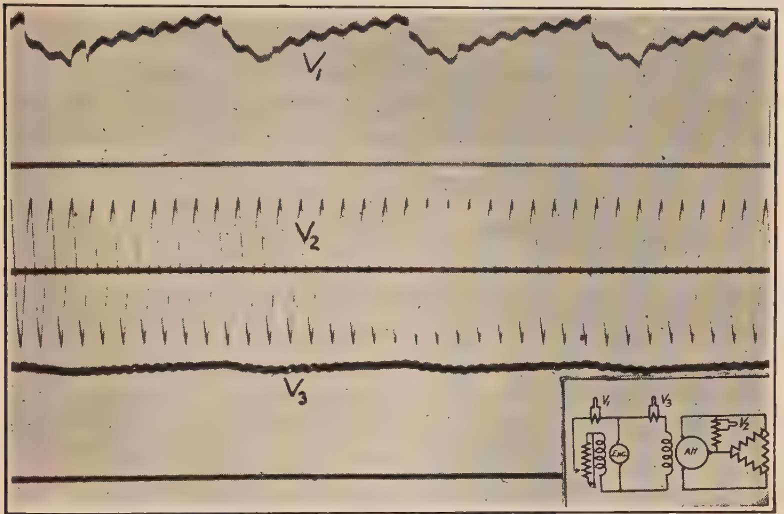


FIG. 34.14.—Oscillogram showing operation of TA regulator. V_1 = current in exciter field, 1.1 amp. V_2 = alternator voltage, 120 volts. V_3 = current in alternator field, 3.0 amp.

weakens both of the control magnets and the floating main contacts are closed. This closes the relay circuit and demagnetizes the relay magnet, releasing the relay armature, and the spring closes the relay contacts. The single-pole switch is then closed and as the exciter field rheostat is short-circuited the exciter voltage will at once rise and bring up the voltage of the alternator. This will strengthen the alternating-current and direct-current control magnets and at the voltage for which the counterweight has been previously adjusted the main contacts will open. The relay magnet will then attract its armature and, by opening the shunt circuit at the relay contacts, will throw the full resistance into the exciter field circuit, tending to lower the exciter and alternator voltage. The main contacts will then be again closed, the exciter field rheostat short-circuited through the relay contacts and the cycle repeated. This operation is continued

at a high rate of vibration due to the sensitiveness of the control magnets and maintains, not a constant, but a steady exciter voltage. The oscillogram in Fig. 34.14 shows the exciter and alternator field currents and the alternator voltage with the TA regulator in operation.

The Torque-motor Voltage Regulator (General Electric Company).—In order to meet the exacting regulation requirements of large central stations a motor-operated regulator has been developed. The type FA-4 regulator consists of a three-phase torque motor, which is balanced by means of a spring at normal voltage, to the armature of which are attached two pairs of contacts. One pair of contacts, in conjunction with a continuously rotating cam contact disk, controls the motor-operated exciter field rheostat to correct the excitation for small or gradual changes in system conditions. By virtue of the intermittent contact and the shape of the cam disk, this action varies from a single brief impulse to practically continuous operation of the exciter field rheostat, depending upon the correction required.

The other pair of contacts, in conjunction with a smooth contact disk, controls a pair of high-speed relays to meet comparatively violent alternating-current system disturbances by either short-circuiting the motor-operated exciter field rheostat or inserting a large block of resistance in the exciter field circuit and thus obtaining maximum rate of response of exciter voltage.

So long as the alternating voltage remains constant the torque motor armature is balanced by the spring with both pairs of contacts out of engagement. Should any condition arise to change the alternating voltage slightly, the torque motor balance is disturbed and one or the other of the front contacts swings over into engagement with the cam contact tips. As this disk is rotating, contact is made for a short time, energizing the contactors and operating the motor-operated exciter field rheostat in the proper direction to provide the required correction in excitation.

It should be noted that contacts are made with a sliding motion which cleans the contact surfaces and prevents sticking and consequent irregular operation. In addition, the regulator remains inactive unless excitation changes are required.

Westinghouse Voltage Regulators for Alternating-current Generators.—These regulators operate on the principle of controlling the alternating-current voltage indirectly, by varying the exciter

voltage by means of relays which open and close a shunt circuit across the exciter field rheostat.

The control element consists of a cast base on which are mounted all the control-element parts. The main control magnet as shown is of the solenoid type, having its core attracted upward and its core stem connected to the floating lever, which is pivoted to the bell crank lever of the vibrating magnet. A counterweight is used to assist the pull of the main control magnet, and to bring the lever and core to a balanced position at the normal voltage to be regulated. The vibrating magnet is also of the solenoid type, having its core attracted upward. Its core stem is connected to one end of the bell crank lever which is pivoted to the base, and its opposite end carries the floating lever of the main control magnet. The pull of this vibrating magnet is assisted by a single spring, as shown. These two magnets are energized from the same voltage transformer and actuate the movable main contact into and out of engagement with the fixed contact. These magnets are very sensitive and are provided with adjustable dashpots.

The relays are wound with two opposing windings, one of which is permanently energized while the other is energized intermittently through the main contacts. Springs on the relay armatures pull the relay contacts closed when the magnet is deenergized by equal currents in the opposing windings. When the main contacts are open, one circuit is interrupted and the other winding energizes the relay magnet and pulls the relay contacts open. Hence, the closing of the main contacts causes the immediate closure of all relay contacts. Opening of the main contacts causes the opening of all relay contacts.

An inspection of the schematic diagram, Fig. 36.14, shows one of the relays, called the vibration relay, connected so that the closure of its contacts shunts a small portion of the resistance in series with the vibrating magnet, thus increasing its pull and opening the main contacts. The opening of the main contacts opens all relay contacts and inserts the full resistance into the vibrating magnet circuit, weakening the pull and closing the main contacts again.

From the above it is evident that the system consisting of the main control magnet, vibrating magnet, levers, rheostat shunting and vibrating relay constitutes a vibrating system when the circuits are properly energized and the control element balanced.

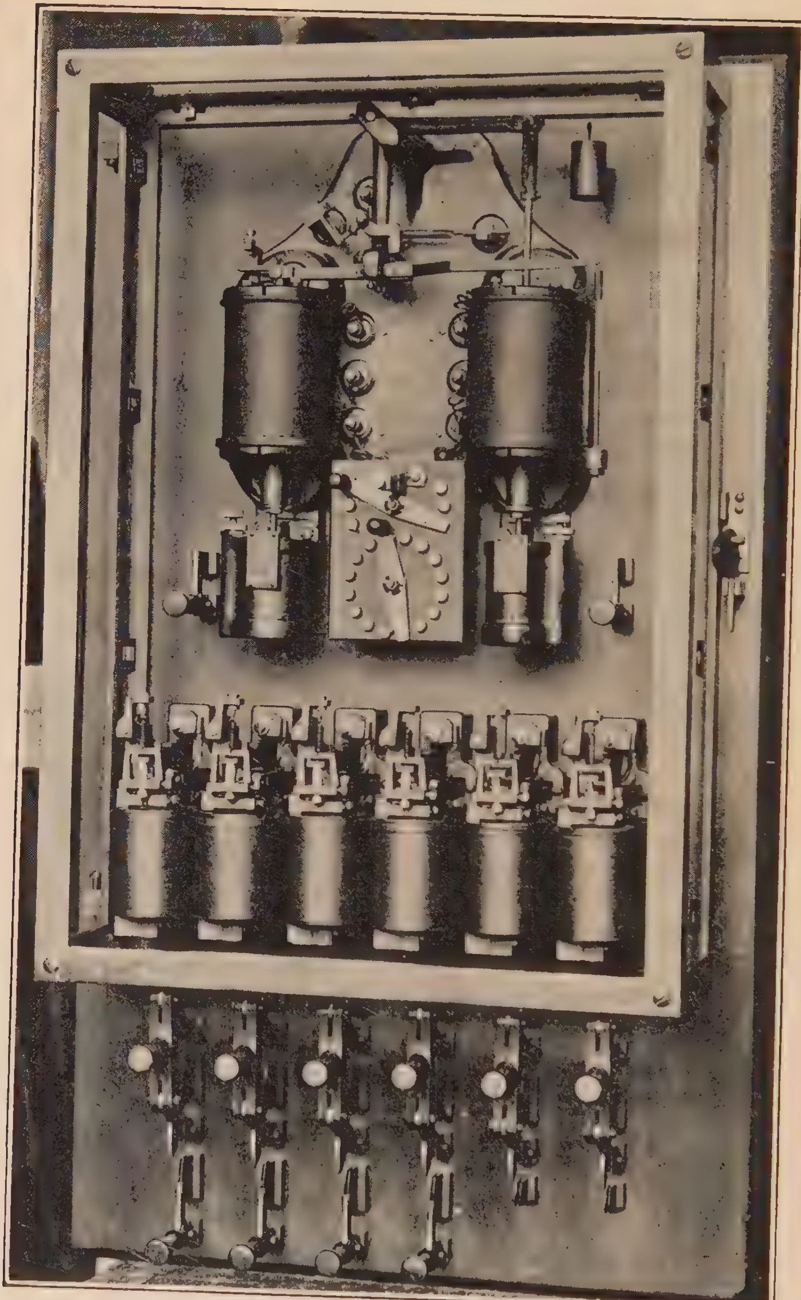


FIG. 35.14.—Westinghouse voltage regulator for alternating-current generators.

A necessary condition to the continuous vibration of the system is that the weight of the vibrating magnet core and the bell crank lever must be exactly balanced by the tension of the control spring plus the average pull of the magnet. This follows from the fact that when the system is vibrating, the main contacts merely play on each other without exerting or requiring any appreciable pressure, but they compel the lever system to assume a definite mean position. Since no force is transmitted through the main contacts, all the forces in the vibrating magnet

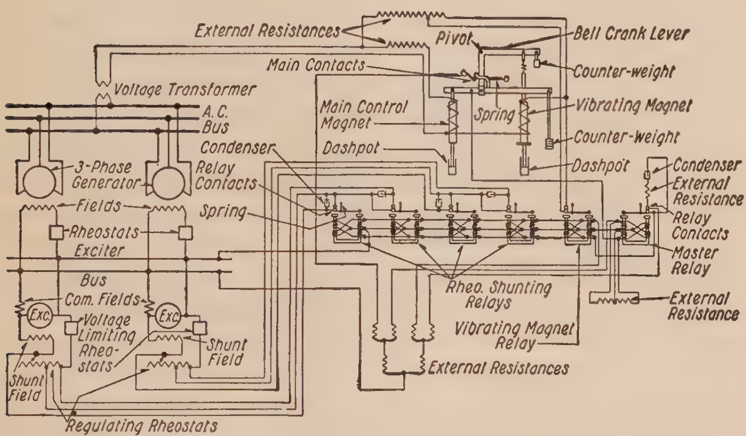


FIG. 36.14.—Schematic diagram of type AB-4 or AE-4 voltage regulator. (*Westinghouse Electric and Manufacturing Company.*)

system must be in equilibrium about the main pivotal point of the bell crank lever, that is, the spring tension and the magnet pull must balance the weight of the moving parts.

For a given line voltage, there is a definite upward pull on the vibrating magnet core when the vibrating relay contacts are open, and a definite pull of a greater value when the relay contacts are closed. When the vibrating magnet is in action, the upward pull of the magnet varies between the two values, contacts open and contacts closed. The average or effective upward pull depends upon the length of time the contacts are closed, as compared to the total time of opening and closing, that is, upon the time of contact engagement.

Assuming a condition of continuous vibration, an inspection of the lever system shows that for each position of the floating lever, as determined by the main control magnet core, there is a

definite position of the bell crank lever and a corresponding definite tension of the control spring. Since the spring tension added to the average magnet pull must always equal a constant force, namely the core weight, it follows that a change in spring tension compels a corresponding change in the average magnet pull and, therefore, of the time of contact engagement which produces this average pull. Hence, for every position of the floating lever there is a corresponding definite time of contact engagement.

When the regulator is connected to the system, the rheostat shunting relays open and close a circuit across the shunt field or regulating rheostat of the exciter and the effective resistance of this rheostat is determined by the time of contact engagement. For example, the longer the contacts remain closed, as compared with the time of opening and closing, the less will be the effective resistance of the exciter field rheostat, and, therefore, the higher the exciter voltage. An increase in time of contact engagement causes an increase in exciter voltage and a corresponding increase in excitation of the alternating-current generator. Since it has been shown that the position of the floating lever determines the time of contact engagement and the time of contact engagement determines the alternating-current generator excitation, it follows that the position of the floating lever determines the excitation of the alternating-current generator.

The main control and vibrating magnets are energized from the alternating-current bus. The main control magnet requires a certain alternating-current voltage to balance the weight of its core. The system voltage must come to equilibrium at this value, for if the alternating-current voltage is less, the pull of the main control magnet will be unable to sustain the weight of its core, which will move downward, thus increasing the time of contact engagement and therefore increasing the generator excitation. This excitation must continue to increase until sufficient alternating-current voltage is developed to reestablish the balance between the core weight and the magnet pull.

When in service, maintaining constant voltage under steady load, the regulator will have a definite rate of vibration and time of contact engagement which corresponds to a definite effective resistance of the exciter field rheostat. This effective resistance produces an exciter voltage sufficient to maintain the alternating-current voltage for which the regulator is balanced. If a load

is thrown on the system which lowers the alternating-current voltage, the main control magnet is weakened, thus upsetting the balance between the magnet pull and the core weight, allowing the core to descend. This closes the main contacts and stops vibration completely, or if vibration continues, it shifts the lower end of the bell crank lever to the right, thus weakening the tension of the control spring, which calls for an increased average magnet pull, resulting in an increase in time of contact engagement. Since the vibrating magnet is energized from the alternating-current bus, its pull (when vibrating relay contacts are opened, and when closed) is also weakened, and since the average magnet pull for any given spring tension must remain constant, the time of contact engagement must, of necessity, be much greater than if the main control magnet were acting alone. Since the damping of the vibrating magnet is very slight, the increase in time of engagement of a given drop in alternating-current voltage is almost instantaneous and proportional to the drop.

The effective resistance of the exciter field rheostat is rapidly reduced, thus causing the exciter to build up until the alternating-current voltage is sufficient to reestablish the balance between the magnet pulls and core weights. When this balance is reestablished the alternating-current voltage is normal and the regulator continues to vibrate at a new rate and with a longer time of contact engagement.

(*i*) **Rating.**—While the rating of an alternator includes a statement of the normal terminal voltage, whether wound for one, two or three phases, speed and frequency, the term essentially applies to the capacity or guaranteed output of the machine. With the other factors given, the output depends upon the temperature rise on continuous operation at full load. This increase of temperature is usually specified, in degrees above room temperature, for continuous full load and short period of overload. The temperature rise depends directly upon the rate of heat generation by the losses in the alternator, and the rate of heat dissipation from the surface of the machine. The temperature rises until the dissipation equals the heat production. Since the copper losses vary as the square of the current, and for a given load the current varies inversely as the power factor, the temperature rise, and hence the rating, cannot be given for any specific load in kilowatts without stating the power

factor. Therefore alternators are rated in *kilovolt-amperes* (*kv.a.*). This is necessary, since the manufacturer can have no control over the power factor of the load that the machine will be required to carry in service. As the iron losses also change somewhat for different power factors, the rating in kv.a. is often supplemented by a statement of temperature rise for two power factors, usually unity and 85 per cent lagging current.

(*j*) **Losses.**—The losses in alternators are similar to those of direct-current generators[1] and may be grouped under *no-load or open-circuit losses* and *load losses.* The open-circuit losses are essentially constant while the load losses vary with the load.

a. Open-circuit Losses:
1. Friction.
2. Windage.
3. Hysteresis and eddy-current losses.
4. Field-excitation losses.

b. Load Losses:
5. Armature copper losses.
6. Stray-power load losses.

1. *Friction.*—In alternators the brush friction caused by the brushes for the field excitation is very small and may be neglected. The friction in the bearings depends on many factors and is either computed from empirical formulas or measured in combination with windage and no-load iron losses. Bearing friction is proportional to the length and to the diameter of the bearing. It is also proportional to the three-halves power of the linear velocity of the shaft, as expressed by the following empirical formula:

$$\text{Bearing friction loss} = 0.81dl\left(\frac{v}{100}\right)^{3/2} \text{ watts} \qquad (10.14)$$

Where

d = diameter of bearing in inches.
l = length of bearing in inches.
v = rubbing velocity of the bearing in feet per minute.

[1] MAGNUSSON, C. E., "Direct Currents," Chap. XVIII, McGraw-Hill Book Company, Inc.

2. *Windage.*—The windage loss is small, except for high-speed machines as turbo-alternators, and cannot be directly calculated. It is either neglected (low-speed machines) or measured by using a calibrated motor, or an empirical amount is assumed, based on values obtained in previous tests on similar machines.

3. *No-load Hysteresis and Eddy-current Losses.*—All hysteresis and eddy-current losses not directly or indirectly due to the armature current are included in the no-load iron losses. The losses vary greatly with the design, and particularly with the speed factor. Eddy-current and hysteresis losses in end plates, bolts and pole faces may in some designs add considerable amount to the ordinary iron losses.

4. *Field-excitation Loss.*—The excitation loss is the RI_f^2 copper losses in the field windings including the field rheostat. The excitation loss is not constant, but varies with the load and with the power factor.

5. *Armature Copper Loss.*—The main load loss comes from the RI_a^2 loss in the armature conductors. As the resistance of the armature is small it cannot readily be measured with accuracy, but is usually computed from the dimensions and specific resistance of the armature conductors.

6. *Stray Load Losses.*—All hysteresis and eddy-current losses due to the armature current are included in the stray load losses. Losses due to eddy currents in the armature conductors, caused by variations in the field flux as a consequence of the armature slots and teeth moving in front of the poles, are included in the stray load losses. The stray load losses are sometimes combined with the armature copper loss by using an effective resistance, R_l, in place of the ohmic resistance R, and thus letting the load losses $P_l = R_l I_a^2$. Stray load losses are determined experimentally by operating the machine on short-circuit and at rated load current. This, after deducting the windage and friction and RI_a^2 losses, gives the stray load loss for single-phase or polyphase generators or motors.

For methods of measuring losses under a variety of specific conditions the reader is referred to the current editions of the A.I.E.E. Standards, adopted by the American Institute of Electrical Engineering, the N.E.M.A. Apparatus Standards, sponsored by the National Electrical Manufacturers' Association and the A.E.S.C. Standards, approved by the American Engineering Standards Committee.

(*k*) **Efficiency.**—The efficiency of an alternator is the ratio of the power output to the power input, and is usually expressed in per cent of the input.

$$\text{Efficiency in per cent} = \frac{\text{power output} \times 100}{\text{power input}} \qquad (11.14)$$

For three-phase alternators the efficiency is expressed by equation (12.14).

$$\text{Efficiency in per cent} = \frac{\sqrt{3}EI \cos \theta \times 100}{\sqrt{3}EI \cos \theta + P_w + P_f + P_i} \qquad (12.14)$$

E = terminal voltage.
I = line current.
$\cos \theta$ = power factor.
P_w = friction, windage, hysteresis and eddy-current losses.
P_f = field-excitation loss.
P_i = copper and stray power-load losses.

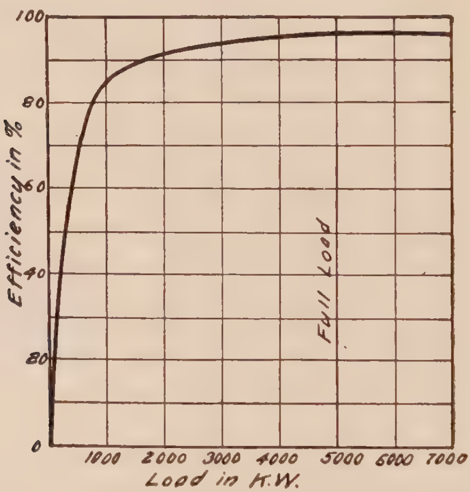


FIG. 37.14.—Efficiency-load curve for an alternator.

The necessary data for determining the efficiency of alternators are obtained by measuring the output and the losses. For special rules and recommendations see the A.I.E.E. Standards referred to in the preceding paragraph.

For Table X the losses are given separately for several alternators. The relative amounts of energy changed into heat

in the armature and field vary considerably for different designs. The ratios between the field amperes for no-load and normal terminal voltage (taken from the no-load saturation curve) to the field amperes for full-load current with the terminals short-circuited (taken from short-circuit saturation curve —straight line) are given in the S.C. ratio column. Armature resistance is between terminals at 25°C. Synchronous reactances are given in per cent of terminal voltage at full-load current.

TABLE X

Full load, kv.a.	E_t Volts	f Cycles	Speed, r.p.m.	Phase	S.C. ratio	Iron loss, kw.	Friction and windage, kw.	Field RI^2 loss, kw.	Armature resistance across terminals, ohms	x_a Per cent
500	2,300	60	360	2	1.4	18.7	6.1	5.0	0.250	45
500	2,300	25	300	3	1.8	8.1	3.2	5.4	0.273	44
750	2,300	60	400	3	2.2	17.0	7.1	6.8	0.111	40
1,000	2,300	25	500	3	1.6	12.5	9.5	7.3	0.084	42
1,500	2,300	60	200	3	2.2	24.6	10.0	7.8	0.037	44
1,500	2,300	25	750	3	1.5	31.4	15.3	7.5	0.043	45
1,800	2,300	60	360	3	1.6	28.1	11.8	7.8	0.023	47

(*l*) **Parallel Operation of Alternators.**—In most power plants it is necessary to connect several generators to the same distribution system, as this arrangement gives the best operating economy. In constant-potential systems the alternators must therefore be operated in parallel. The alternating nature of the voltage wave and the constant speed required for constant frequency make the operation of alternators fundamentally different from similar arrangements of direct-current generators. Consider the two alternators in Fig. 38.14, operating in parallel and delivering energy to the same busbars. If both machines produce simple sine voltage waves, it is possible to adjust the field excitation and the phase position of the two machines so that the voltages from the two alternators shall at every instant be equal. At the busbars the two voltages arrive in phase, are of equal magnitude and may be represented by one vector, as in Fig. 39.14. The load current is the sum of the currents coming from the two machines. With respect to the busbars the two machines are therefore in parallel and the voltages and current waves are in phase. However, between the machines, the two armature windings and the connections to the busbars form a series circuit, and therefore relatively to each other the

two voltage waves are in opposition, and may be represented by the diagram in Fig. 40.14. The two diagrams in Figs. 39.14 and 40.14 represent the same voltages; the difference in phase position merely depends on whether the parallel circuits for the load or the series circuit through the two armatures are under discussion. The conditions for parallel operation, with no cross-currents flowing between the alternators in the series circuit and for simple sine voltages, are:

1. The machines must be in synchronism, that is, generate voltage at the same frequency.
2. The voltages must be equal in magnitude.
3. The voltages must be in phase (referred to the parallel circuit) or the machines must be *in step*.

4 Same phase sequence

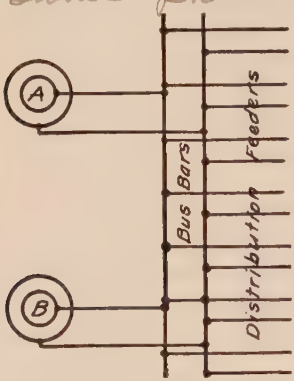


Fig. 38.14.

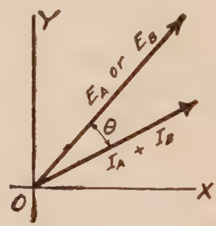


Fig. 39.14.

(*m*) **Synchronizing Current.**—Given the two alternators operating in parallel and the voltages represented by vectors 180° apart as in Fig. 40.14. Let the power supplied by the prime-mover driving machine *A* be increased by a small amount. This tends to increase the speed of *A* and bring the voltage vectors less than 180° apart, as indicated in Fig. 41.14. The voltages in this phase relation will not neutralize completely but form a resultant voltage $_sE$, causing a current $_sI$ to flow in the series circuit through the armatures of both alternators. The synchronizing current lags behind the resultant voltage $_sE$, by an angle $_s\theta$, depending upon the total resistance and reactance of the series circuit, therefore including the resistance and reactance of both armatures.

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r = total resistance in the series circuit.
${}_sx$ = total synchronous reactance in the series circuit.
E_{gA} = open-circuit voltage in alternator A.
E_{gB} = open-circuit voltage in alternator B.
${}_sE$ = resultant voltage causing the current ${}_sI$ to flow in the series circuit.
${}_sI$ = synchronizing current.

$$\cos {}_s\theta = \frac{r}{{}_sz} = \frac{r}{\sqrt{r^2 + {}_sx^2}} \tag{13.14}$$

With the two voltages E_{gA} and E_{gB} $180° - \gamma$ apart, as in Fig. 41.14, the synchronizing current leads E_{gA} by $90° - ({}_s\theta + \frac{1}{2}\gamma)$ and lags behind E_{gB} by $90° + ({}_s\theta - \frac{1}{2}\gamma)$. With ${}_s\theta$ large compared to γ, the angle of lag of the synchronizing current with respect to B is greater than 90° and hence the cosine negative,

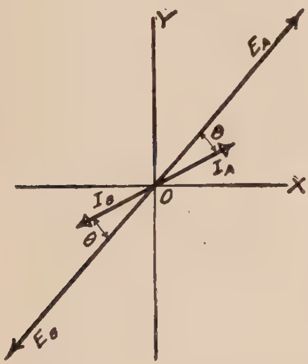

FIG. 40.14.

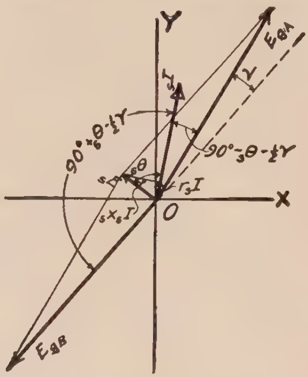

FIG. 41.14.

while for generator A the angle of lead is less than 90° and hence the cosine is positive. This means that alternator A gives out power, that is, acts as a generator, while machine B receives power from the series circuit. Hence the synchronizing current transmits power from A to B and therefore tends to retard A and accelerate B and thus bring the two machines back into phase opposition, or to keep them in synchronism. In the series circuit between the two alternators:

$$\begin{aligned} &\text{The power supplied by } A = E_{gA}\,{}_sI \cos (90° - {}_s\theta - \tfrac{1}{2}\gamma) \\ &\text{The power received by } B = E_{gB}\,{}_sI \cos (90° + {}_s\theta - \tfrac{1}{2}\gamma) \end{aligned} \tag{14.14}$$

or

$$\text{Using the negative sign} \quad = -E_{gB\,s}I \cos (90° - {}_s\theta + \tfrac{1}{2}\gamma) \tag{15.14}$$

The difference in the power supplied by A and the power received by B is equal to the power lost in the series circuit, or ${}_sE_sI \cos {}_s\theta$.

$$E_{gA\,s}I \cos (90° - {}_s\theta - \tfrac{1}{2}\gamma) + E_{gB\,s}I \cos (90° + {}_s\theta - \tfrac{1}{2}\gamma) = {}_sE_sI \cos {}_s\theta$$

or

$$E_{gA\,s}I \cos (90° - {}_s\theta - \tfrac{1}{2}\gamma) - E_{gB\,s}I \cos (90° - {}_s\theta + \tfrac{1}{2}\gamma) = {}_sE_sI \cos {}_s\theta \tag{16.14}$$

$$E_{gA} = E_{gB};\ {}_sE = 2E_{gB} \cos (90° - \tfrac{1}{2}\gamma) \tag{17.14}$$

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If several alternators are connected to one set of busbars the synchronizing current tends to keep all the machines in step. If the prime mover of any one machine tends to increase its speed, a resultant voltage is generated, causing a synchronizing current to flow. The machine that leads delivers power, is thus retarded and thereby tends to hold the machine in synchronism and in step. If, on the other hand, the prime mover of one of the alternators gives less than its share of the load the machine lags behind and receives power by the synchronizing current and therefore is in phase relation similar to machine B in Fig. 41.14. The power supplied by the synchronizing current accelerates the lagging alternator, thus tending to keep the machine in synchronism. The maximum value of the synchronizing torque is proportional to the maximum power supplied or received through the series circuit. For the alternator tending to run too fast, the expression for power is given in equation (14.14), and for the lagging machine in equation (15.14).

From (14.14)

$${}_sP_A = E_{gA\,s}I \cos (90° - {}_s\theta - \tfrac{1}{2}\gamma) \tag{18.14}$$

$$= 2E_{gA}^2 \frac{({}_sx \cos \tfrac{1}{2}\gamma + r \sin \tfrac{1}{2}\gamma)(\sin \tfrac{1}{2}\gamma)}{r^2 + {}_sx^2} \tag{19.14}$$

Considering r and γ constant, differentiate with respect to ${}_sx$ and equate to zero, to find the value of the synchronous reactance in terms of the resistance and phase angle for maximum torque.

$$\frac{d_sP_A}{d_sx} = 0. \tag{20.14}$$

Hence,

$$(r^2 + {}_sx^2) \cos \tfrac{1}{2}\gamma - ({}_sx \cos \tfrac{1}{2}\gamma + r \sin \tfrac{1}{2}\gamma)2{}_sx = 0 \quad (21.14)$$

or

$${}_sx \doteq \frac{(1 - \sin \tfrac{1}{2}\gamma)r}{(\cos \tfrac{1}{2}\gamma)} \quad (22.14)$$

For small values of γ, that is, with the machines nearly in the desired 180° phase position, $\cos \tfrac{1}{2}\gamma$ is very nearly unity and $\sin \tfrac{1}{2}\gamma$ very small, and therefore the condition for maximum torque with the machines in step is:

$${}_sx = r \quad (23.14)$$

For the machine tending to lag, the value of the synchronous reactance:

$${}_sx = \frac{(1 + \sin \tfrac{1}{2}\gamma)}{(\cos \tfrac{1}{2}\gamma)}r \quad (24.14)$$

Hence, the condition for maximum torque is the same when the value of γ is very small whether the machine tends to go faster or slower than synchronous speed. With the alternators in step the maximum synchronizing force is obtained when the synchronous reactance and resistance are equal, or ${}_s\theta = 45°$. For larger values of γ, the value of ${}_sx$ is less than r, to give maximum synchronizing torque. It is, however, neither necessary nor desirable to design the machines for a maximum synchronous torque, as a considerably smaller value is ample for practical operation. Other factors entering into the design of alternators make it necessary to let the synchronous reactance be several times as large as the armature resistance.

(n) **Synchronizing[1] and synchroscopes.**—In order to connect an alternator to a system already in operation, the conditions for parallel operation must be met, or the alternator must be synchronized before it is connected to the busbars. The incoming machine must be adjusted to comply with the three required conditions:

1. The machine and busbar voltages must be approximately equal in magnitude.
2. The voltages must have the same frequency.
3. The voltage waves must be in phase.

The first condition is met by adjusting the field excitation, using

[1] For an extended discussion on Synchronizing see article, "Various Methods of Synchronizing," *Gen. Elec. Rev.*, Vol. 28, p.692.

the switchboard voltmeter as indicator. The voltage is usually kept a little higher on the incoming machine. Conditions 2 and 3 are of much greater importance than a careful adjustment of the voltages. To determine the speed and phase relations for conditions 2 and 3 special indicators or synchronizing devices are used. These may be grouped as indicating and automatic. The indicating synchronizing device, or synchroscope, should show three things:

1. Whether the speed of the incoming machine is too fast or too slow.

2. The amount of the difference in speed from synchronism.

3. The time of coincidence of phase relations of the incoming machine and the busbars.

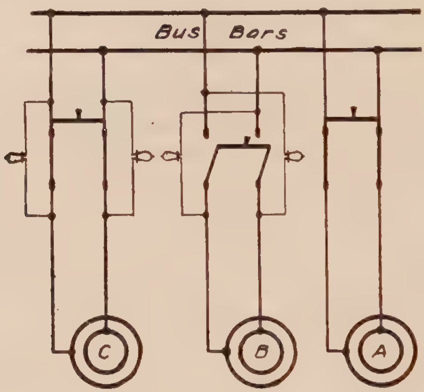


FIG. 42.14.

The simplest of all synchroscopes is the incandescent lamp; when used for this purpose it is called a *synchronizing lamp.* In Fig. 42.14 let machines *A* and *C* be in operation while *B* is being synchronized. The synchronizing lamps, connected as indicated in the diagram, will pulsate in brightness as the machine *B* approaches synchronous speed. The rapidity of the pulsations indicates the difference in speed. When the machine runs in synchronism with the busbar voltage the pulsations cease; and when the lamps are at their greatest brilliancy, the phase relations are correct and the machine may be thrown into circuit. By connecting the lamps as on machine *C*, Fig. 42.14, the lamps are dark when the machine and busbars are in phase. For machines having a higher voltage, transformers are used and the

connections made so as to have the lamps either *dark* or *bright* when the machines are in phase, as shown in Figs. 43.14 and 44.14. For three-phase machines, as in Fig. 45.14, the lamps become bright in a definite order while the speed of the incoming machine

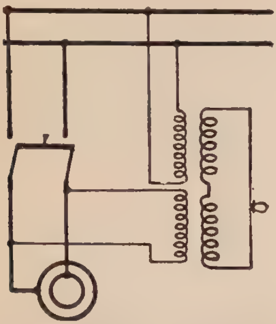


FIG. 43.14.—Synchronizing *bright*, using two transformers with the secondaries in series.

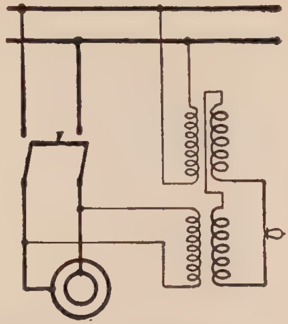


FIG. 44.14.—Synchronizing *dark* using two transformers with the secondaries in opposition.

is too low, and in the reverse order when the speed is above synchronism. At synchronism the right-hand lamp will be dark and the other two of equal brightness. For small machines the synchronizing lamps serve the purpose but for large alternators more accurate indicators are necessary.

The Lincoln Synchroscope.—A pointer moving in front of a dial, like the hands of a clock, indicates the relative speeds and phase positions of the machines to be synchronized. The direction of rotation shows whether the speed of the incoming machine is too fast or too slow. The speed of the pointer indicates the relative speeds of the alternators. When the machines are running at the same speed and the voltage waves are in phase, the pointer remains stationary in an upright position. The circuit diagram of the instrument is shown in Fig. 46.14. A stationary laminated iron core MM' is magnetized by a current from the busbars flowing through coil A and resistance r_2. The rotating part carrying the pointer consists of two coils B and D, rigidly attached to the axle, in space quadrature to each other. Both coils are connected to the terminals of the incoming

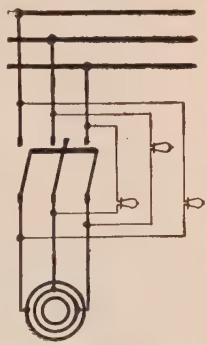


FIG. 45.14.

machine; B through a resistance r_1, and D through an inductive reactance x_1. The currents flowing in B and D therefore differ

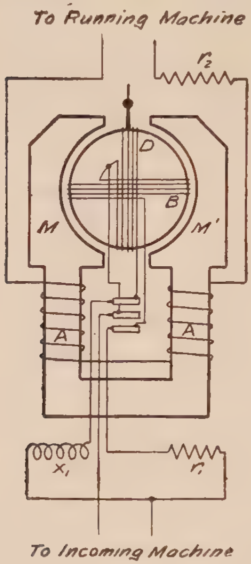


FIG. 46.14.—Lincoln synchroscope.

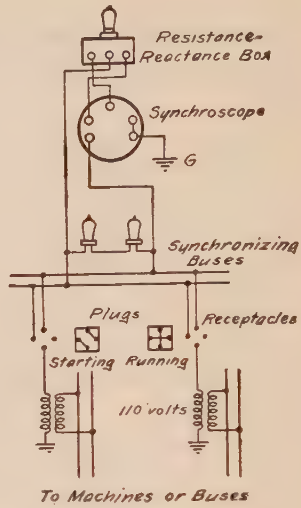


FIG. 47.14.

in time phase by nearly 90°. Hence the resultant magnetic field produced by the currents in B and D rotates in synchronism with the frequency of the voltage from the incoming alternator. The magnetic flux in MM' pulsates in synchronism with the busbar voltage. The reaction between the stationary pulsating flux produced by the current in coil A and the constant rotating flux produced by the currents in coils B and D give the required torque for turning the pointer. When the voltage from the incoming machine has the same frequency and is in time-phase with the busbar voltage the torque during one-half cycle is balanced by an equal torque in the opposite direction during the next half wave. The pointer is fastened to the

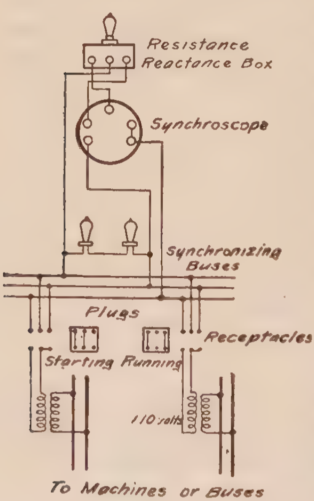


FIG. 48.14.

movable coils so as to take an upright position when the machines are in synchronism. Circuit diagrams showing the connections to the synchroscope in grounded and ungrounded systems are shown in Figs. 47.14 and 48.14.

Induction Synchroscope.—The mechanism consists of three stationary coils, A, B and D, Fig. 49.14, with a central movable iron core, to which is attached a pointer. Coil A is connected to the busbars through a resistance r_2. The current in A produces a pulsating flux in the iron armature. Coils B and D, in space quadrature, are connected to the incoming machine; coil B through a resistance, r_1, and coil D through an inductive reactance

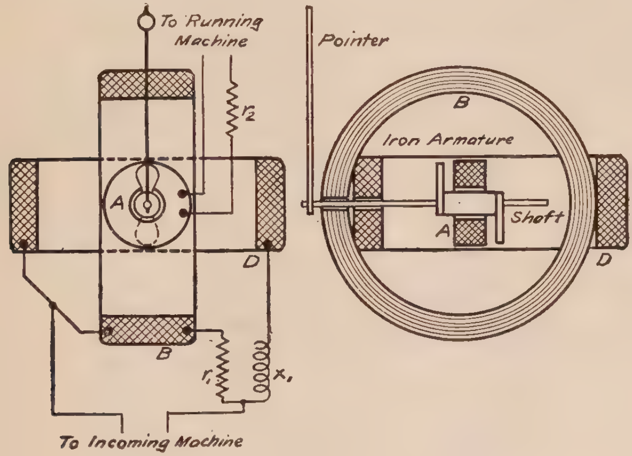


Fig. 49.14.—Induction synchroscope.

x_1. The currents in coils B and D produce a constant rotating field. The torque is produced, in the same manner as in the Lincoln synchroscope, by the reaction between the pulsating and rotating fields. If the speed of the incoming machine is too slow, the pointer rotates in a counterclockwise direction; if too fast, the direction of rotation is reversed. When the machines are in synchronism the pointer remains stationary in an upright position.

(*o*) **Division of Load, in Parallel Operation.**—The division of the load depends fundamentally upon the governors of the prime movers. Without entering into an extended discussion of the problem the essential principle involved may be shown by Fig. 50.14, assuming similar machines.

Let the diagram represent the current and voltage relations for one phase.

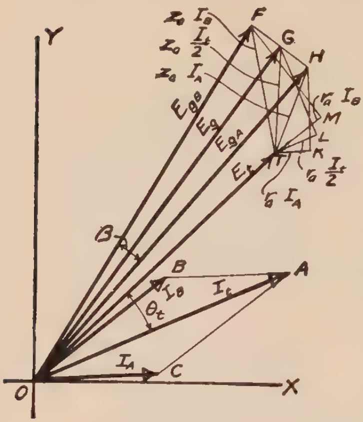


FIG. 50.14.

$OT = E_t$, constant busbar or terminal voltage.

$OA = I_t$, constant-load current.

θ_t = angle of lag for I_t and E_t.

r_a = armature resistance.

x_a = armature reactance.

First Condition.—The two machines are in phase. The voltage drop in each armature per phase is:

$$z_a\frac{\dot{I}_t}{2} = r_a\frac{\dot{I}_t}{2} + jx_a\frac{\dot{I}_t}{2} \text{ (each alternator supplies half the busbar current)} \tag{25.14}$$

$$\dot{E}_0 = z_a\frac{\dot{I}_t}{2} + \dot{E}_t \tag{26.14}$$

$OG = E_0$, generated voltage in each machine.

Angle $GTL = {}_s\theta$ = armature angle of lag.

The two machines are dividing the load equally.

$$\text{Power generated in } A = E_{0A}\frac{I_t}{2}\cos GOA \tag{27.14}$$

$$\text{Power output from } A = E_t\frac{I_t}{2}\cos\theta_t \tag{28.14}$$

$$\text{Power generated in } B = E_{0B}\frac{I_t}{2}\cos GOA \tag{29.14}$$

$$\text{Power output from } B = E_t\frac{I_t}{2}\cos\theta_t \tag{30.14}$$

Second Condition.—Let the governor for the prime mover of alternator B be adjusted slightly to increase the amount of power supplied. This tends to increase the speed of B and hence to make the voltage differ from A by an angle β.

Keeping the same position for the terminal voltage OT, the generated voltage of B is $\frac{1}{2}\beta$ ahead and of A $\frac{1}{2}\beta$ behind OG. It is also assumed that

$OF = OH$.

$OF = E_{0B}$ generated voltage in B.

$$OH = E_{gA} \text{ generated voltage in } A.$$
$$TF = z_a I_B \text{ armature drop in } B.$$
$$TH = \dot{z}_a I_A \text{ armature drop in } A.$$

Upon TF and TH construct right triangles similar to TGL.

$$TM = r_a I_B;\ FM = x_a I_B \tag{31.14}$$
$$TK = r_a I_A;\ HK = x_a I_A \tag{32.14}$$

From O draw OB parallel to TM, and OC parallel to TK. In length, let

$$OB:\frac{1}{2}OA::TM:TL, \text{ or } OB:\frac{1}{2}I_t::r_a I_B:r_a\frac{I_t}{2} \tag{33.14}$$

Hence $OB = I_B$, the current from machine B, in phase and magnitude.
Similarly, $OC = I_A$, the current from machine A, in phase and magnitude.

$$\dot{I}_t = \dot{I}_B + \dot{I}_A \tag{34.14}$$
$$\text{Power generated in } B = E_{gB} I_B \cos FOB \tag{35.14}$$
$$\text{Power output from } B = E_t I_B \cos TOB \tag{36.14}$$
$$\text{Power generated in } A = E_{gA} I_A \cos HOC \tag{37.14}$$
$$\text{Power output from } A = E_t I_A \cos TOC \tag{38.14}$$

Under the conditions assumed for the diagram it is evident that B delivers more power than A.

In the above discussion it is assumed that the generated voltages in the two machines are equal in magnitude. This need not be the case, but the division of the load will in any event depend upon the power supplied by the prime mover to each alternator; and since the alternators must run in synchronism the amount of power supplied by the prime movers and therefore the division of the load depends upon the adjustment of the governors. In direct currents the generated voltage depends upon the product of the speed and the field excitation. For constant voltage an increase in one requires a corresponding decrease in the other. For alternators running in parallel both the speed and the terminal voltage are constant, while the field is separately excited and hence variable. Referring to Figs. 18.14 and 23.14 it will be recalled that the useful flux has two com-

ponents, one from the field excitation and another from the armature reaction. The product of the useful flux and the speed determines the generated voltage and for constant speed and constant voltage the useful flux must also be constant.

Hence, if the field excitation of one alternator is changed, while the speed remains constant, the armature reaction must make a corresponding counterchange in the useful flux in order that both alternators may produce the same terminal voltage. This is done automatically by the circulating current produced by the difference in generated voltages and a *very slight* phase displacement caused by the tendency of the machine of increased field excitation to increase its load and as a consequence to retard in speed.

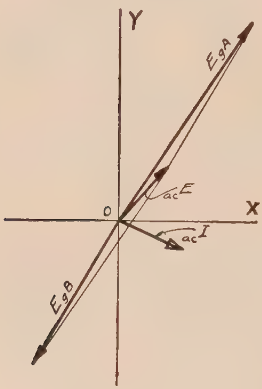


FIG. 51.14.

Let two alternators operating in parallel have equal field excitation and each taking half the load. If the excitation of A is increased, the voltage E_{gA} increases (Fig. 51.14), which causes a *slight* decrease in speed due to the tendency, with increasing voltage, to take a greater part of the load. The increase in E_{gA} combined with the very slight change in phase position produces a resultant voltage, $_{ac}E$, which causes a circulatory current, $_{ac}I$, to flow in the series circuit of the two armatures. Since the armature reactance is inductive, the circulatory current $_{ac}I$ lags with respect to E_{gA} and leads E_{gB}. The resulting armature reaction decreases the field flux in A and correspondingly increases the useful flux in B. Hence, the armature reaction produced by the circulatory current $_{ac}I$ automatically equalizes the resultant useful fluxes of the alternators so as to produce equal terminal voltages.

It should be noted that the field excitations of all the alternators operating in parallel are pooled, so that any increase or decrease in the excitation of one of the alternators produces a corresponding change in the total field excitation available for the group. As a consequence any increase or decrease in the field excitation of any one of the alternators produces a corresponding change in the common terminal voltage. The circulatory current, produced by the inequality of field excitations,

automatically distributes the change in the field excitation made on any one unit over all the alternators operating in parallel, so that the useful fluxes in the several machines remain in the same relative proportion. Hence, any increase or decrease in the total field excitation of alternators operating in parallel causes a corresponding change in the terminal voltage common to all the units in the group.

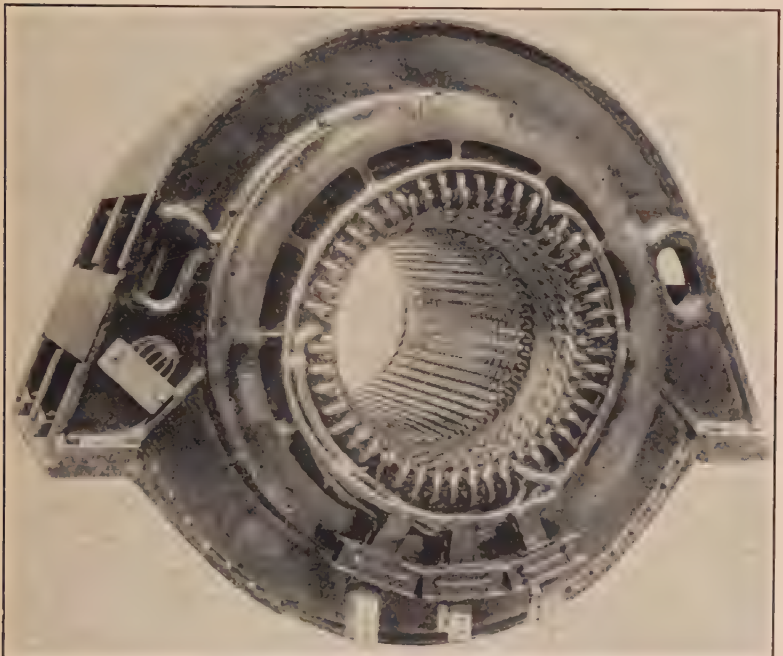


Fig. 52.14.—Armature or stator of a 6,000-kv.a. generator. (*General Electric Company.*)

(*p*) **Hunting of Alternators Operating in Parallel.**—Consider two alternators, *A* and *B*, in parallel operation, driven by two prime movers, as internal-combustion or steam-reciprocating engines. The average speed required for synchronous operation can readily be made the same for the two units, but since the driving torque of reciprocating engines varies from zero at dead centers to a maximum at intermediate points in the piston cycle the instantaneous speeds of the two machines may well differ, so that during the cycle, machine *A* first goes slower and later faster than machine *B*. At the period in the cycle when the

speed of A is slower than B, the synchronizing current flowing in the series circuit of the two armatures tends to speed up A and slow down B. The synchronizing power, combined with an increase in the driving torque from the prime mover for A and a decrease for B, increases the speed of A until it becomes greater than that of B. The process then reverses until the speed of B becomes greater than that of A. Oscillation in the relative speed of machines operating in parallel, or cyclic variations in the speed of single alternators, is termed *hunting*, an entirely undesirable feature which must be held within narrow limits for satisfactory operation. In steam and water-turbine-driven alternators there is little if any hunting tendency as the driving torque is constant. On the other hand, special means for the damping of hunting, as heavy flywheels and amortisseur windings must be used for units driven by internal-combustion engines. A more extended discussion of hunting by synchronous machines is found under Synchronous Motors in Chapter XV.

(*q*) **Voltage Wave Forms of Alternators Operating in Parallel.** In present-day alternators of good commercial design the terminal voltages are very nearly simple sine waves. It is evident that to avoid cross-currents in the series-armature circuits it is necessary that not merely the *effective terminal voltages* of alternators operating in parallel be equal, but that the *instantaneous values* at every point in the voltage cycle be essentially equal. Assume that two generators having voltage waves, respectively, like those shown in Figs. 1.24 and 2.24 in Chapter XXIV, are in parallel operation. It is evident that even if the effective terminal voltages of the two machines were equal there would be large differences in the instantaneous values during the voltage cycle. As a consequence, large currents would flow in the series-armatures circuit wasting electric energy and raising the temperature of the two armatures. It seems quite likely that the cross-currents produced by so large differences in wave form, as shown in the above-referred-to figures, might cause cross-currents to flow that would raise the temperature of the machines to the maximum permissible value, although neither alternator carried any outside load. Cross-currents caused by differences in the wave forms of the terminal voltages are wholly undesirable and should be kept at a minimum. This is one reason for having the simple sine wave as standard for voltage waves in alternating-current circuits.

PROBLEMS

1.14. From the data given in Table X find the efficiencies of the several alternators at 25, 50, 75, 100, 125 and 150 per cent full load. Assume that the load losses vary with I^2 and at full load are equal to 50 per cent of the iron losses. Neglect field-rheostat losses.

(a) For $\cos \theta_t = 1.0$. Load loss = 532

(b) For $\cos \theta_t = 0.85$.

2.14. With E_t constant and I_a constant and equal to the values for 50 per cent full load, and $\cos \theta_t$ varying from 70 per cent lagging current to 70 per cent leading current, draw the loci for E_0 and E_g for alternators Nos. 2 and 7 in Table X. Let X_a be 12.5 per cent of ${}_sX_a$.

3.14. In problem 2.14 let E_t be constant and $\cos \theta_t$ = 88 per cent (lagging current) and let I_a vary from no load to full load.

Draw the loci for E_0 and E_g.

4.14. Given two alternators operating in parallel. The two machines have equal excitation, but the prime mover for *B* tends to make it rotate faster than *A*.

(a) Draw vector diagram for the synchronizing circuit.

(b) With the same notation as in the diagram write the equations for the power delivered by *A* and *B*.

(c) Prove that for the synchronizing circuit alone machine *B* is a generator and machine *A* a motor.

5.14. Let *A* and *B* be alternators of the same type and rating as Nos. 3 and 5 respectively, in Table X. The prime mover of *A* tends to run too fast, so that the voltage wave from *A* leads that from *B* by 3°.

(a) Find the synchronizing current.

(b) Find the power in the synchronizing circuit flowing from *A* to *B*.

6.14. In problem 4.14 let the load current lag 32° behind the busbar voltage.

(a) Draw the vector diagram for the load circuit.

(b) With the same notation as in the vector diagram, write the power equations for the output in the load circuit for *A* and *B*.

(c) From the equations show that *B* takes more than half the load.

7.14. Two alternators like No. 6 in Table X operate in parallel. The voltage wave from *A* leads that from *B* by 2°. Let X_a = 12.5 per cent of ${}_sX_a$. Assume that the prime mover regulators are adjusted so that machine *A* takes half of the load plus the required synchronizing power, while machine *B* takes half the load minus the synchronizing power. The load power factor at the busbars is 87 per cent, current lagging. Total load 1,700 kw.

(a) What is the load, in kw., for *A*? in kv.a.?

(b) What is the load, in kw., for *B*? in kv.a.?

(c) If *B* delivers full load, what is the load on *A*? Assume 70 per cent power factor for machine *B*.

(d) What power is passing from *A* to *B* in the synchronizing circuit?

8.14. The following data apply to the test for core loss of a three-phase, 100-kv.a., 2,300-volt generator when driven by a separately excited motor at normal speed:

Condition	Motor		Generator		
	E	I	E_L	I_f	I_L
Belt off	523.5	4.0			
Belt on	525.5	6.4	0.0	0.0	0.0
Open circuit	527.5	8.5	2,000.0	7.0	0.0
Open circuit	528.0	9.5	2,500.0	9.5	0.0
Open circuit	529.0	10.6	2,900.0	12.2	0.0
Open circuit	530.0	11.7	3,200.0	14.6	0.0
Short circuit	528.5	10.0	0.0	4.8	20.0
Short circuit	530.5	12.1	0.0	6.0	25.0
Short circuit	532.5	13.7	0.0	6.8	28.2

Star-connected ammeters used during short-circuit test.

Resistance of direct-current motor armature, 0.87 ohm.

Resistance of generator between terminals, 0.862 ohm.

Calculate the efficiency and regulation of the above machine at full load on 85 per cent power factor (lagging).

9.14. Alternators like Nos. 3 and 5 in Table X are run in parallel. Before No. 3 is connected, No. 5 supplies 1,500 kw. to a 2,300-volt load at 85 per cent power factor lagging. Alternator No. 3 is now connected and the governors of each driving turbine so adjusted that each machine carries its share of the load (500 kw. for No. 3 and 1,000 kw. for No. 5). The field of No. 5 is not changed and that of No. 3 is adjusted to bring the line voltage to 2,300 volts. What are the load currents and power factors of each machine? Give a complete vector diagram.

10.14. Two similar 1,670-kv.a., Y-connected, 2,300-volt, 25-cycle, three-phase generators, designated as A and B, are installed in a generating station and may be connected to the same set of busses. A is running, and it is desired to synchronize B with it. When synchronizing, the operator carelessly throws the switch of machine B, when it is 15° ahead of exact-phase opposition to A. (*a*) Which machine supplies power to bring the two machines into step? (*b*) How many kw. are supplied? (*c*) What are the value and phase position of the resultant voltage in the circuit of the two machines? (*d*) What is the value of the circulating current?

The synchronous reactance of each machine is 40 per cent, and the resistance 1.5 per cent.

11.14. The machines in problem 10.14 are driven by prime movers such that the speed regulation of machine A is 4 per cent and that of machine B is 5.5 per cent at full load of 1,670 kv.a. and 85 per cent power factor, and normal frequency. The rise or drop in speed is a constant amount per kw. change of load. (*a*) What is the combined kv.a. output of the two machines when A is delivering 125 per cent of its rated kv.a output? (*b*) What is the frequency at this load?

12.14. In problem 11.14 the combined load of the two machines is reduced to 2,400 kv.a., load power factor remaining constant. (*a*) What is the frequency? (*b*) How many kw. does A deliver? (*c*) How many kw. does B deliver?

CHAPTER XV

SYNCHRONOUS MOTORS AND SYNCHRONOUS CONDENSERS

Consider two similar alternators, connected in parallel to the same busbars as in Fig. 1.15, with equal field excitations, and running in synchronism carrying equal loads. In the series circuit between the alternators no cross-current is flowing and the voltage vectors are 180° apart. If the power supply of machine M is reduced, the voltage vector E_M lags behind the first position, relative to the voltage E_0 by an angle γ, as indicated in Fig. 2.15. The synchronizing current flowing in the series circuit transmits power from G to M as explained in Chap. XIV. If all power supply is cut off from the prime mover, machine M still continues to run, receiving the necessary power from alternator G. If, in addition, a mechanical load is connected to M, additional power is transmitted from G by an increased current in the series circuit. The difference in phase position of the voltage vectors for the two machines is larger with M carrying a mechanical load than before, but the two machines continue to run in synchronism. Alternator M is thus changed into a motor running in synchronism with an alternator G by varying the phase position of the voltage waves. A synchronous motor is, therefore, in principle, simply a reversed alternator. Instead of receiving mechanical energy and generating electric energy, it receives electric energy at synchronous speed and delivers mechanical energy. If the excitation of the motor is less than the generator, the resultant voltage and the synchronizing current change somewhat in phase position and magnitude but the motor stays in synchronism and power is transmitted from the generator G to the synchronous motor M under wide differences in field excitation and for a considerable range of load. For 80

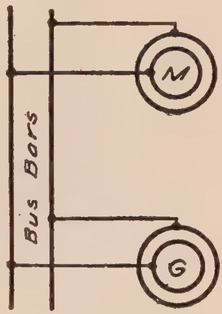


Fig. 1.15.

per cent field excitation of the motor the current lags behind the generator voltage, as shown in Fig. 3.15, and power is still transmitted from G to M; or G is a generator and M a motor. For changes in load the generator and motor voltages vary in phase position, and for any given excitation of the motor the locus of the motor voltage is a circle as shown in Fig. 4.15. For changes in the field excitation of the motor and for phase positions of the motor voltage E_M with respect to the generator voltage E_g the

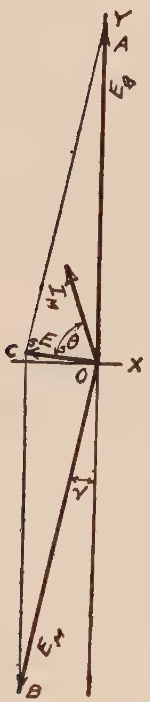


FIG. 2.15.

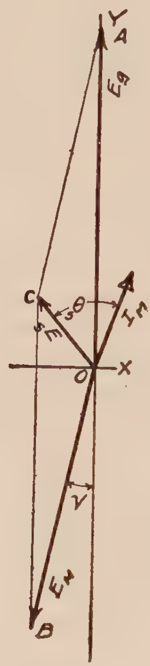


FIG. 3.15.

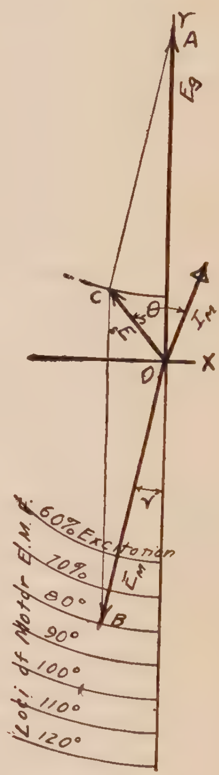


FIG. 4.15.

resulting voltage $_sE$ changes in both magnitude and phase position. Hence the current flowing in the motor likewise changes in both magnitude and phase.

(*a*) **Current Loci. Power Input.**—The interrelation of the motor voltage, current and power is best shown graphically by vector diagrams. The loci of the voltages and motor current for

different field excitations and loads are shown in Figs. 4.15 and 5.15.

E_g = generator terminal or busbar voltage, assumed constant.
E_M = motor-generated counter e.m.f., depending upon the motor field excitation, and given in per cent of generator voltage E_g.
${}_sE$ = resultant voltage causing current to flow in the motor armature.
I_M = current in motor armature.
${}_sr$ = effective resistance of motor armature and leads at synchronous speed.
${}_sx$ = synchronous reactance of motor.
${}_sz = {}_sr + j\,{}_sx$ = synchronous impedance of motor.
${}_s\theta = \cos^{-1}\frac{{}_sr}{{}_sz}$ (assumed constant).

$$ {}_s\dot{E} = \dot{E}_g + \dot{E}_M \tag{1.15} $$

$$ \dot{I}_M = \frac{{}_s\dot{E}}{{}_sz} \tag{2.15} $$

The line OF is drawn at an angle ${}_s\theta$ from E_g, and the distance OF is equal to the quotient of E_g and ${}_sz$.

$$ OF = \frac{E_g}{{}_sz} = {}_oI_M \tag{3.15} $$

The current vector I_M in Fig. 5.15 is drawn for 80 per cent motor field excitation. The locus of the motor voltage E_M (Fig. 4.15) is a circle, and hence the locus of the resultant voltage ${}_sE$ is also a circle with A as center and a radius equal to E_M. Therefore, the locus of the motor current I_M is a circle (Fig. 5.15), with F as center and a radius equal to the motor voltage divided by the synchronous impedance. For 80 per cent excitation the radius for the current locus is 80 per cent of FO, Fig. 5.15. With F as a center a series of circles may be drawn with radii directly proportional to the motor excitation, expressed in per cent of E_g. These circles are the current loci for different motor excitations and for varying load. The component of current in phase with E_g represents power and hence the power component and the corresponding power input are measured directly by the projection of the current along the Y-axis. Likewise the quadrature component

of the current and the corresponding reactive power are measured by the projection of the current along the X-axis. Thus the diagram shows directly the power input for any motor excitation and also the corresponding power passing to and fro between the motor and the generator.

(*b*) **Losses. Current Loci for Mechanical Power.**—Since the synchronous motor is not self-exciting the energy consumed in the field need not be taken into consideration when discussing

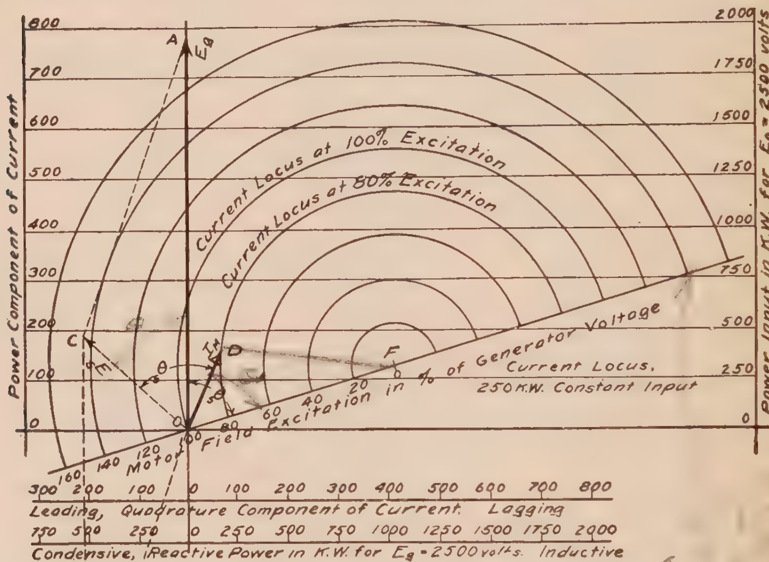


FIG. 5.15.

the characteristics of the machine, except for the total efficiency. The energy delivered to the rotor is therefore consumed by the rotor copper and iron losses, windage, friction and the outside load. Since the power absorbed by the iron losses, windage and friction is practically constant for all loads, it is convenient to combine it with the outside load into one term in the equations. *Mechanical power* (as used in this chapter) = outside load + (windage, friction, rotor iron losses), or

$$\text{Mechanical power} = \text{outside load} + \text{a constant} \qquad (4.15)$$

When discussing the synchronous-motor characteristics it may therefore be assumed that:

$$\text{Power input} = {}_{e}rI_M^2 \text{ losses} + \text{mechanical power} \qquad (5.15)$$

In Fig. 6.15, E_o, the generator busbar voltage, the line OF and the broken lines for the current loci are the same as in Fig. 5.15. For zero motor field OF represents the motor current in both magnitude and phase and the projection on the Y-axis, OB, the power input. As no mechanical power is produced, the input is consumed by the copper losses.

$$OB = {}_sr_oI_M^2 \qquad (6.15)$$

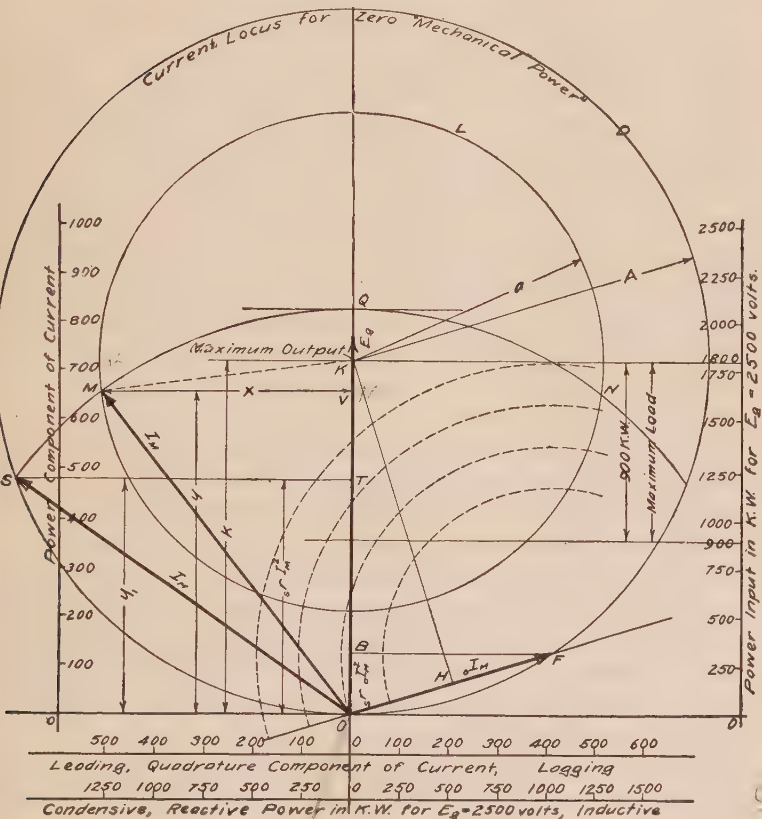


FIG. 6.15.

With its center along the E_o vector, a circle is drawn through O and F. This is the locus of currents for zero mechanical power. From similar triangles,

$$OB:OF::OH:OK, \text{ or } OB:{}_oI_M::\tfrac{1}{2}\,{}_oI_M:OK \qquad (7.15)$$

Hence:

$$OB = \frac{{}_o I_M^2}{2OK} \tag{8.15}$$

Similarly, for any point S on the circle OFD the projection on E_g represents the power input from equations (6.15) and (8.15),

$$OT = \frac{\overline{OS}^2}{2OK} = \frac{I_M^2}{2OK} = {}_s r I_M^2 \tag{9.15}$$

and hence zero mechanical power. Therefore, in order to produce any mechanical power the current vector must fall inside the circle $OFDS$. As the mechanical load at the shaft includes all but the ${}_s r I_M^2$ losses, it is evident that for any power input the losses will be least when E_g and I_M are in phase, or for unity power factor. The power input for a current in phase with the voltage, as OQ, is given directly from the diagram. The copper loss may be found by drawing a circle with OQ as radius and O as center. The intersection with the locus for no mechanical load determines the ${}_s r I_M^2$ loss equal to the projection OT. The mechanical load is therefore given by the difference of OQ and OT and is equal to TQ.

The *maximum mechanical power* that the motor theoretically can carry may be shown to be one-half of the power represented by OK, the radius of the current locus for zero mechanical power The term *maximum mechanical power* in this discussion refers to conditions which cannot actually be met by the motor. It is based on the assumption that the resistance and the counter e.m.f. are the only factors opposing the flow of the current, as in a direct-current motor. The synchronous impedance limits the armature current to values much smaller than indicated in the diagram. Hence the *maximum power possible* is fictitious, as it cannot be produced by the machine.

That the current loci for any constant mechanical power output of the motor are circles concentric with the locus for zero mechanical power may be proved as follows:

In Fig. 6.15 draw any circle LMN with K as center and of radius a less than A, the radius of the circle OFD. Let the circular arc SMQ intersect the circle LMN at M. Draw MV perpendicular to OK. Then TV represents the mechanical power delivered by the motor since OV represents the total input and OT the losses. Hence, if the distance TV is proved constant

for point M on the circle LMN corresponding to any point S on OFD, then the circle LMN must be the current locus for the constant mechanical power TV.

Let

$$OT = y_1; OV = y; MV = x; OS = OM = I_M$$

From the equation of the circle OFD, or from equation (9.15)

$$I_M = \sqrt{2Ay_1} \qquad (10.15)$$

$I_M = \sqrt{KM^2 + MO^2}$

From the triangle OMV,

$$I_M = \sqrt{x^2 + y^2} \qquad (11.15)$$

From the triangle MVK,

$$a^2 = x^2 + (A - y)^2 \qquad (12.15)$$

Eliminating I_M and x and solving for TV,

$$y - y_1 = TV = \frac{A^2 - a^2}{2A}, \text{ a constant} \qquad (13.15)$$

$a^2 = RM^2 + A^2 - 2A(OM)$

For maximum mechanical load a equals zero, and the corresponding current locus is the center of the circle OFD.

$a^2 = A^2 - 2A\,OM$

The efficiency at maximum load

$$= \frac{y - y_1}{y} = \frac{\frac{A}{2}}{A} = 50 \text{ per cent} \qquad (14.15)$$

$\frac{A^2 - a^2}{2A} = OM - OV = MV$

when (a) is

The larger the load, the smaller is the radius of the current locus. For the maximum load the circle of the current locus has contracted into a point, at the center of the set of current loci for the loads between zero mechanical output and the largest load the motor can carry. A convenient way of graphically determining the locus for any given load is shown in Fig. 7.15, in which the generator voltage, the current locus for zero mechanical power and the scales for current and power are the same as in Fig. 6.15. The maximum power is represented by half the radius, and the corresponding current by the radius of the locus for zero mechanical power. With O as center and OK as radius draw the arc LKM. The line LM bisects OK and hence the length NK represents the maximum output or maximum mechanical power of the motor.

A scale for the output may be drawn as in Fig. 7.15. The current locus for any load, as for 500 kw., may be determined by

drawing a line parallel to the X-axis from the output scale and intersecting the arc LKM at P. The circle drawn with K as center and KP as radius is the required current locus for an output of 500 kw. mechanical power. Similarly, the locus for any other output may be found as shown in Fig. 7.15 for 350, 800 and 150 kw. In practical operation the current vector must be within

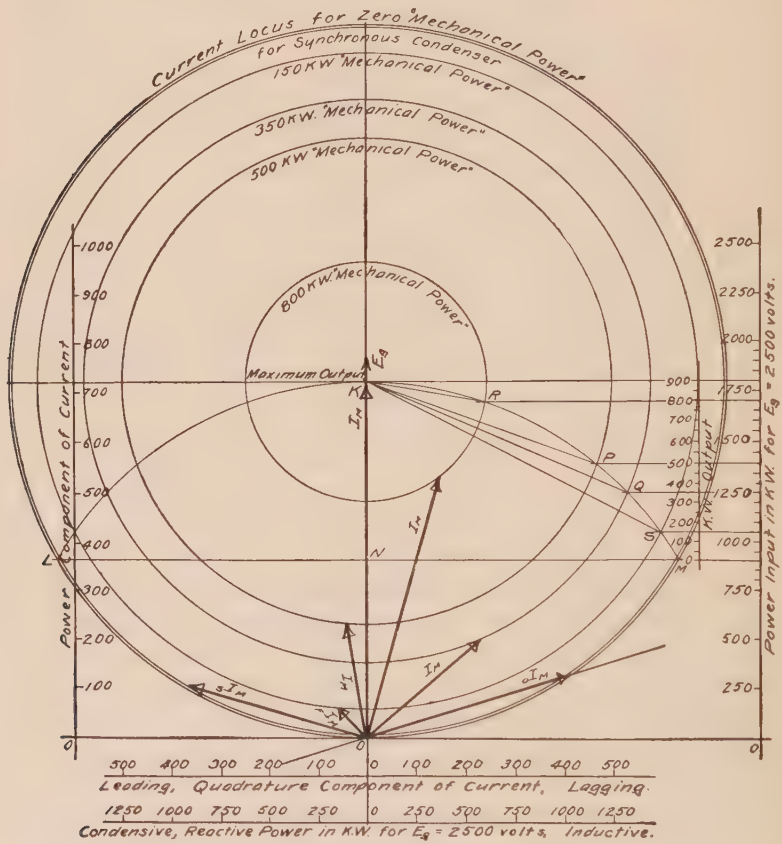


FIG. 7.15.

the locus for zero mechanical power, since even with no load some power is required to overcome friction, windage and iron losses. The current locus for the machine operating as a synchronous condenser (no mechanical load) is shown in Fig. 7.15.

The copper losses are excessive unless the motor operates near unity power factor and in the lower part of the diagram.

At maximum load the copper loss at unity power factor is 50 per cent. Hence for an efficiency greater than 50 per cent the current must be within the arc *LKM*. The ordinary operation of the synchronous motor for the conversion of electric to mechanical power is within much narrower limits. This may be seen from the diagram, as 150 kw. is full load for this motor. To obtain the best efficiency for any load the power factor must be near unity and hence the current vector lies near the E_g vector and varies with the load from zero to the value corresponding to the maximum load. In Figs. 5.15, 6.15 and 7.15, the power component is represented by the projections of the current vector on the Y-axis in phase with E_g. The product of the power component of the current and the voltage gives the power input to the motor.

$$E_g I_M \cos \theta_t = \text{motor input, or generator power output} \quad (15.15)$$

Similarly, the quadrature current and the reactive power are represented along the X-axis, in quadrature with E_g.

$$E_g I_M \sin \theta_t = \text{reactive power} \quad (16.15)$$

From Fig. 7.15 it is seen that the quadrature component of the current may be either lagging or leading with respect to the impressed voltage E_g; and hence the reactive power may be either inductive or condensive, depending upon the excitation of the motor field. The fact that the current taken by a synchronous motor may be made either leading or lagging by merely adjusting the field excitation, and hence under control of the operator, is of great commercial importance. With a leading current the excitation of the motor field is under most conditions, as may be seen from Fig. 7.15, over 100 per cent of the generator voltage. This has led to the statement that an "overexcited synchronous motor acts like a condenser." With a leading current the corresponding reactance is condensive and in that respect, *and that only*, the overexcited synchronous motor acts like a condenser. However, the ability to receive a leading current, and hence to store power in time-phase opposition to magnetic inductance, is the important commercial feature of a condenser, and for the same reason the chief advantage of the overexcited synchronous motor. The commercial importance of an adjustable condensive reactance may be seen in the design of power distribution networks and especially in long-distance

transmission systems. A simple illustration of improving the power factor of the load by operating a synchronous motor in parallel with an induction motor will be given.

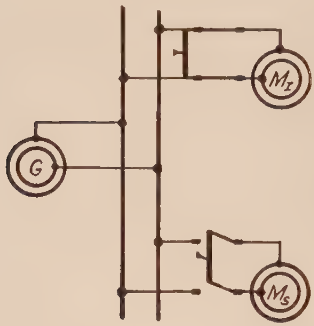


Fig. 8.15.

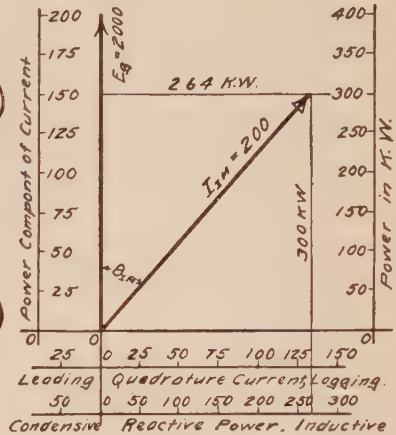


Fig. 9.15.

In Fig. 8.15 let a generator G supply power to an induction motor M_I. Let the rated capacity of the generator be 400 kv.a.,

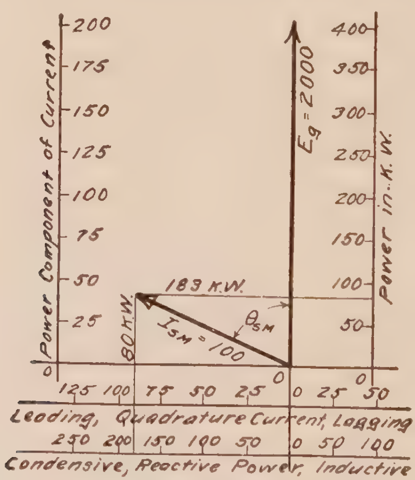


Fig. 10.15.

the motor load 300 kw. at 75 per cent power factor. The vector diagram of the voltages and currents with scales showing the real and reactive power is shown in Fig. 9.15. It is required to carry

an additional load of 80 kw. by the same generator. This evidently cannot be done by using an induction motor as the 400 kv.a. taken by the first motor is equal to the rated capacity of the generator. It is, however, possible to improve the power factor by using an overexcited synchronous motor for the additional load, thereby reducing the kv.a. so that the larger load takes less current from the generator. In Fig. 10.15 is shown the vector diagram of voltage and current for a 200-kv.a. synchronous motor taking 80 kw. at 40 per cent power factor, leading current. Combining the vector diagrams for the induction and synchronous motors in Fig. 11.15 gives the relative magnitudes

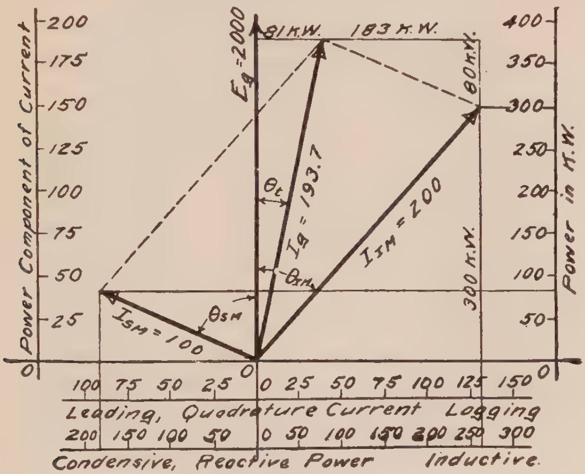


FIG. 11.15.

and phase relations of the currents in each of the motors and in the generator.

Power taken by the induction motor:

$$E_g I_{IM} \cos \theta_{IM} = 2{,}000 \times 200 \times 0.75 = 300 \text{ kw.}$$

Power taken by the synchronous motor:

$$E_g I_{SM} \cos \theta_{SM} = 2{,}000 \times 100 \times 0.40 = 80 \text{ kw.}$$

Power delivered by the generator to both motors:

$$E_g I_t \cos \theta_t = E_g I_{SM} \cos \theta_{IM} + E_g I_{SM} \cos \theta_{SM} = 380 \text{ kw.}$$

Reactive power in the induction motor:

$$E_g I_{IM} \sin \theta_{IM} = 2{,}000 \times 200 \times 0.66 = 264 \text{ kw. lagging current.}$$

Reactive power in synchronous motor:

$$E_g I_{SM} \sin \theta_{SM} = 2{,}000 \times 100 \times 0.915 = 183 \text{ kw., leading current.}$$

Reactive power in generator:

$$E_g I_{IM} \sin \theta_{IM} - E_g I_{SM} \sin \theta_{SM} = 264 - 183 = 81 \text{ kw., lagging current.}$$

Current in generator:

$$\dot{I}_o = \dot{I}_{IM} + \dot{I}_{SM} \qquad (17.15)$$
$$= I_{IM}(\cos \theta_{IM} - j \sin \theta_{IM}) + I_{SM}(\cos \theta_{SM} + j \sin \theta_{SM}) \qquad (18.15)$$
$$= \dot{I}_{IM} \cos \theta_{IM} + I_{SM} \cos \theta_{SM} - j(I_{IM} \sin \theta_{IM} - I_{SM} \sin \theta_{SM}) \qquad (19.15)$$
$$= 190 - j40, \text{ or } \dot{I}_g = 193.7 \text{ amp.}$$
$$\cos \theta_t = 0.98 \text{ and } \theta_t = 11^\circ\ 28'.$$

Hence, although 80 kw. additional load is supplied by the generator with both motors in operation, the current in the generator is reduced from 200 to 193.7 amp. As the loss in the generator copper varies as the square of the current, the heat is reduced in the proportion of $\overline{200}^2:\overline{193.7}^2$. From the diagram in Fig. 11.15 it is evident that by means of a synchronous motor the power factor of the load at the generator may be changed at will. Aside from the economy effected by operating at or near unity power factor, as illustrated above, the control of the power factor is a useful method for regulating the voltage in long-distance transmission systems, as explained in Chap. XXIII.

(*c*) **Synchronous and Static Condensers.**—In large power systems, and particularly in connection with long-distance transmission lines, overexcited synchronous motors are kept *floating on the line*, that is, running at synchronous speed but not carrying any load, for the purpose of supplying the necessary leading or lagging current to control the power factor and thereby regulate the voltage. Under such conditions the synchronous machine is called a *synchronous condenser*. The derivation of the term is plain, for the synchronous condenser produces the same effect in causing a leading current as a static condenser of equivalent condensance. The mechanical power is expended in overcoming friction and windage and hence is constant in value. The current locus is therefore a circle for mechanical power equal to the losses caused by friction, windage and losses in the iron, as shown in Fig. 7.15.

(*d*) **Power Equations.**—From the vector diagrams used in the above graphical analysis of the synchronous motor the algebraic

expressions for power are readily obtained. Let the diagram in Fig. 13.15 represent the general case.

E_g = the impressed voltage on the motor terminals.
E_M = the voltage corresponding to the motor counter e.m.f.
I_M = the current in motor armature.
${}_sx$ = synchronous reactance of motor.
${}_sr$ = effective resistance of motor armature at synchronous speed.

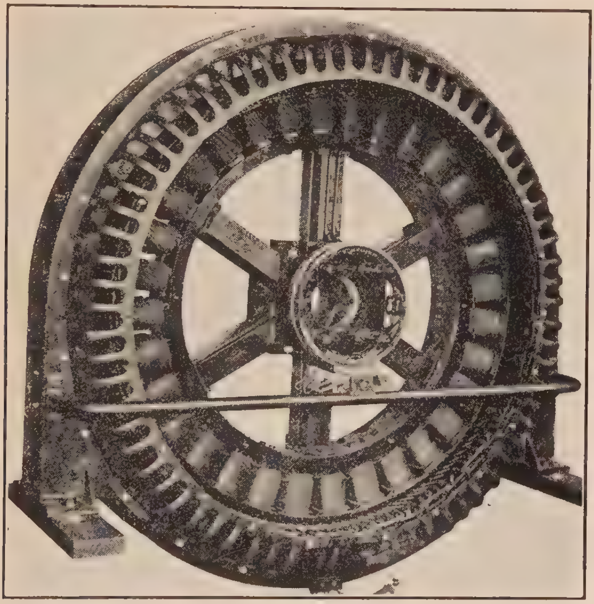


FIG. 12.15.—Synchronous motor showing welded frame construction. Type HR. (*Westinghouse Electric and Manufacturing Company.*)

$$\cos {}_s\theta = \frac{{}_sr}{\sqrt{{}_sr^2 + {}_sx^2}} \tag{20.51}$$

$\cos \theta_t$ = terminal power factor.
γ = time-phase angle between E_g and E_M.

Power input $= E_g I_M \cos \theta_t$ (21.15)
Mechanical power $= E_M I_M \cos (\theta_t - \gamma)$ (22.15)
Copper losses $= {}_srI_M^2$ (23.15)
Power output = mechanical power − friction and windage and rotor iron loss (24.15)

It is sometimes desirable to have the power input and the mechanical power expressed in terms of E_g, E_M, $_s\theta$, γ, $_sr$ and $_sx$. These may be derived directly from equations (21.15) and (22.15) by substituting for I_M and θ_t.

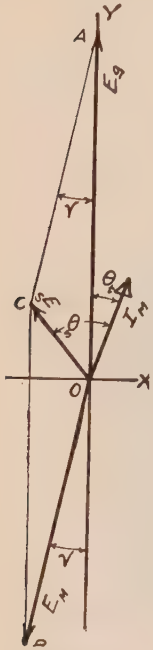


FIG. 13.15.

From Fig. 13.15:

$$I_M = \frac{_sE}{\sqrt{_sr^2 + {_sx^2}}} \tag{25.15}$$

$$_sE \cos ({_s\theta} - \theta_t) = E_g - E_M \cos \gamma \tag{26.15}$$

and

$$_sE \sin ({_s\theta} - \theta_t) = E_M \sin \gamma \tag{27.15}$$

Substituting in equation (21.15):

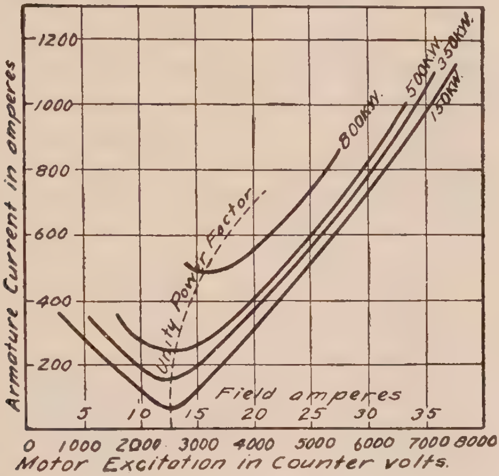
$$\text{Power input} = \frac{-E_g E_M}{\sqrt{_sr^2 + {_sx^2}}} \cos (\gamma + {_s\theta}) + \frac{E_g^2}{\sqrt{_sr^2 + {_sx^2}}} \cos {_s\theta} \tag{28.15}$$

$$\text{Mechanical power} = \frac{E_g E_M}{\sqrt{_sr^2 + {_sx^2}}} \cos (\gamma - {_s\theta}) - \frac{E_M^2}{\sqrt{_sr^2 + {_sx^2}}} \cos {_s\theta} \tag{29.15}$$

(*e*) **Test Data. Rating. Efficiency.**—The saturation and synchronous reactance curves are obtained by running the synchronous motor as a generator and taking the data as explained for alternators. From the synchronous reactance and armature resistance, $_s\theta$ may be found. The value of E_M is determined from the current in the field and the saturation curve. E_g, I_M and θ_t may be taken directly by voltmeter, ammeter and wattmeter readings for any given load and field excitation. The fixed losses (windage, friction and iron losses) may be found by measuring the input with the motor running without load. Adjusting for unity power factor keeps the copper losses at a minimum. These must be subtracted from the input to obtain the fixed losses. The rating of a synchronous motor is based on the same principle as already discussed for alternators. The capacity is given in hp. or kv.a. with a statement of temperature rise at unity power factor. The efficiency is the ratio of output to input, usually expressed in per cent. As with alternators, the efficiency may be found by measuring input and output or

by measuring either the input or the output and the losses. For large machines the latter method is preferable.

(*f*) **The V- and O-curves.**—In a synchronous motor giving constant mechanical power the armature current is a minimum if the power factor is unity. By varying the field excitation of the motor the power factor may be varied with corresponding change in the armature current. Curves plotted with



IG. 14.15.

the field excitation as abscissæ and the armature current as ordinates for a constant load, assume a form like the letter *V*, as seen in Fig. 14.15, and are known as the *V-curves* of a synchronous motor. A series of curves may be obtained for a number of loads, as in Fig. 14.15.

The *V*-curves may be found graphically from the current loci diagram in Fig. 7.15 in a simple manner, as shown in Fig. 15.15. The voltage or field excitation of the motor is laid off on a line from the origin to the intersection of the current locus for a constant load with the loci for different field excitations and this distance used as ordinate in obtaining a point on the *V*-curve. By plotting the values of the current for all points on the circular load loci, the *V*-curves are converted into closed curves, sometimes called the *O-curves* of the synchronous motor. In Fig. 15.15 the parts that may be found experimentally are drawn in heavy lines.

The broken line in Fig. 15.15 indicates the field excitations and corresponding minimum currents for unity power factor. In attempting to extend the experimental observations further by either increasing or decreasing the motor field excitation the machine is thrown out of synchronism. Curves drawn as in Fig. 15.15 are necessarily only approximations to the actual characteristic curves of commercial machines, since several assumptions were made that are only partly fulfilled in practice.

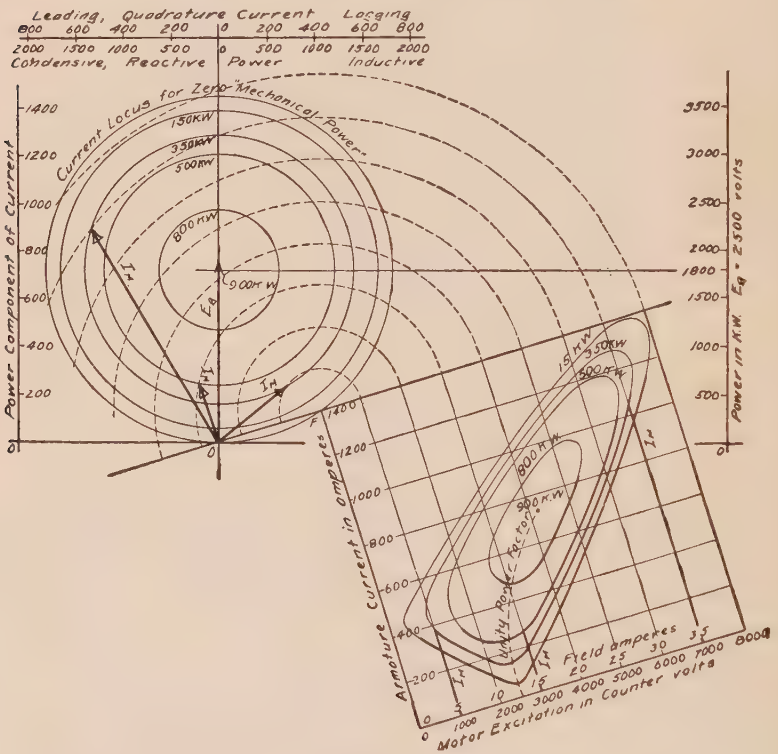


Fig. 15.15.

1. It was assumed that the motor voltage E_M could be varied indefinitely, while in the actual motor the field flux is limited by magnetic saturation. Although sufficient field excitation might be applied so as to obtain readings, during a short test, as high as indicated by the heavy lines in the diagram, the limits are far beyond the voltage that may be applied continuously to the motor.

2. The synchronous reactance and impedance were assumed as constant, while these vary in the commercial motor with both the saturation of the iron and the position of the armature with regard to the field. These variations in the reactance and impedance are most pronounced at lower power factors.

3. The characteristic curves in Figs. 14.15 and 15.15 represent conditions for constant electric power, including all but the copper losses. The loss due to friction and windage is practically constant, but the iron losses depend upon the field excitation, and increase with the magnetic induction. Hence for constant mechanical output the whole system of curves is shifted somewhat toward the lower field excitation.

(*g*) **Load Curves.**—The curves for current, power factor and efficiency, as a function of the output at constant excitation, are often called *load curves.* These may be derived directly from Figs. 7.15 and 15.15, if the same assumptions be made, that the core loss, friction, windage and synchronous reactance remain constant. By plotting the curves in rectangular coördinates, with power output as abscissæ, the characteristics of the synchronous motor may readily be compared to those of the induction motor. For the latter, one set of curves determines the performance of the machine for any given impressed voltage. In the synchronous motor the field excitation can be varied, and hence an infinite series of load curves may be obtained for any given impressed voltage. The power-factor curves are of special interest. By inspection of Fig. 5.15, it is readily seen that unity power factor is obtained only at points of intersection of the current locus for the given field excitation with the voltage vector along the *Y*-axis. The load curves for a synchronous motor at three field excitations are illustrated in Figs. 16.15, 17.15, and 18.15. For a field excitation of 80 per cent, Fig. 16.15, the current locus does not intersect the voltage vector and the power factor never reaches unity. Under these conditions the load curves are similar to those of the induction motor. At an excitation of 99 per cent the power factor is unity at two loads, as shown in Fig. 17.15.

By referring to Figs. 5.15 and 7.15 it is evident that the limits within which the power factor will be unity for two loads, as illustrated in Fig. 17.15, are approximately 98 per cent and 100 per cent field excitation. For excitations above 100 per

cent the power factor becomes unity for one load only, as illustrated in Fig. 18.15.

It should be noted that for a field excitation slightly below 100 per cent, the power factor is near unity from zero to full load. As the losses are a minimum at unity power factor the most desirable field excitation for economical operation is about 99 per cent when running simply as a motor. In most cases the synchronous machine is required to supply a leading current;

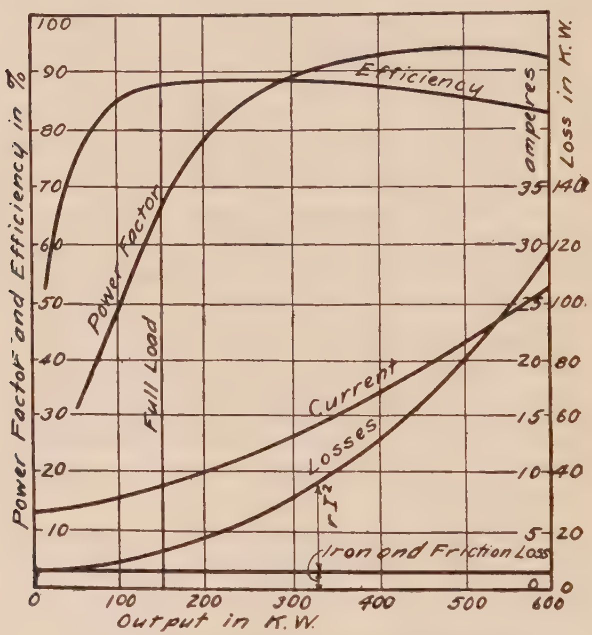


FIG. 16.15.—Curves for 80 per cent excitation.

that is, to act partly as synchronous condenser, in order to bring the power factor of the system nearer unity. In general, the economical operation of the whole system determines the percentage of field excitation of the synchronous machines.

(*h*) **Polyphase Synchronous Motors. Equivalent Single-phase.**—The above discussion is based on single-phase circuits and single-phase apparatus. The same treatment may be applied without modification to polyphase apparatus, provided *equivalent single-phase values* are used for the current, resistance, synchronous current, synchronous reactance and synchronous impedance of the armature and supply circuits.

For two-phase motors the *equivalent single-phase* resistance, synchronous reactance and synchronous impedance equal one-half the respective values as measured in one of the phases; and the *equivalent single-phase* current is twice the current in one phase.

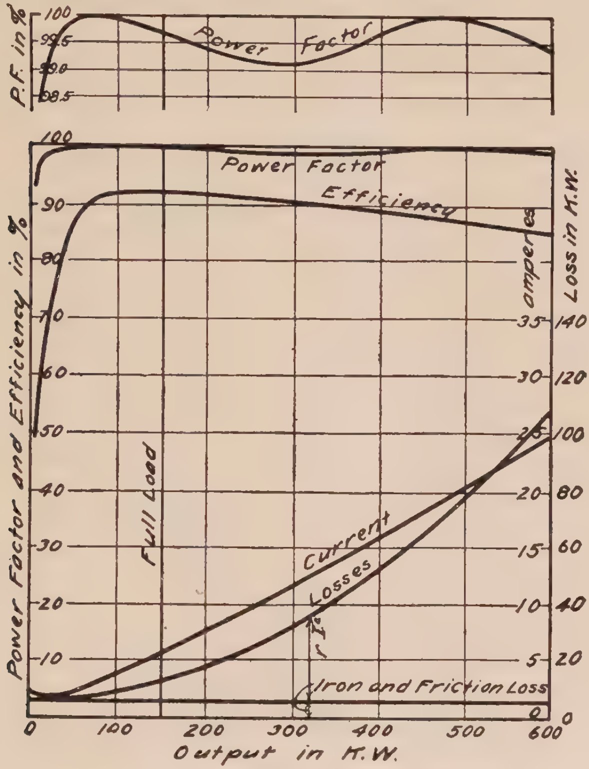


FIG. 17.15.—Curves for 99 per cent excitation.

For three-phase motors (whether star- or delta-connected) the *equivalent single-phase* resistance, synchronous reactance and synchronous impedance equal one-half the respective values as measured between two terminals; and the equivalent single-phase current equals $\sqrt{3}$ times the current in one of the mains. The equivalent single-phase voltage is the same as measured between the mains for two-phase or three-phase.

It should also be noted that, while the synchronous motors discussed above operate on a constant-potential system, it is seldom that the impressed voltage is constant at the terminals of the motor. If the busbars are kept at constant voltage, the

resistance and reactance of the leads from the busbars to the motor should be combined with the resistance and synchronous reactance of the armature to obtain the data from which the performance of the motor may be calculated. In some cases the synchronous motor may be located at a considerable distance from the busbars or the generator where the voltage is kept constant. The resistance and reactance of the line from the point of constant voltage must in all cases be included with the

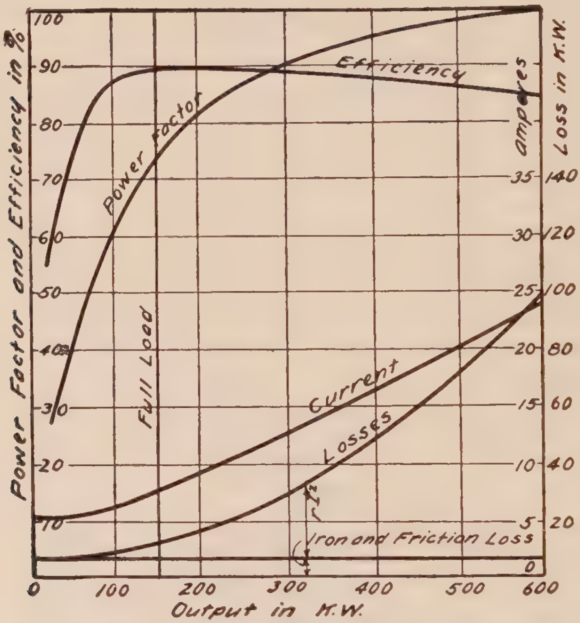


FIG. 18.15.—Curves for 110 per cent excitation.

resistance and synchronous reactance of the armature in preparing the data from which the performance curves of the synchronous motor may be calculated. Necessarily, if the constants of the leads to the busbars or a considerable length of transmission line are included, the performance curves differ from the set obtained from the same motor operating under the condition of constant potential at the terminals of the motor. Hence, in order to determine the operating characteristics of a synchronous motor the constants of the line from the point of constant potential to the terminals of the motor must be known, as well as the resistance and synchronous reactance of the machine.

(*i*) **Surging or Hunting of a Synchronous Motor.**—In Chap. XIII it is shown that the torque of an induction motor is a function of the speed; that a decrease of speed increases the torque. Hence, if the load is suddenly increased on an induction motor the speed decreases until the torque becomes sufficient for the new load. Conversely, if the load is decreased the motor reacts by an increase in speed. This change in torque and speed is gradual and without any oscillation.

In the synchronous motor the torque is *not a function of the speed* but of the *relative phase positions of the rotor and the impressed voltage.* Hence, for a change of load the rotor is required to change its relative phase position without change of speed. Strictly, the two requirements are incompatible and the change is effected in a series of oscillations which may produce serious disturbances in the system. Suppose the load is decreased. The torque is then in excess of the amount required by the load and thus causes an acceleration of the armature until the phase position is reached at which the torque is just sufficient for the load. However, upon reaching this position the speed of the armature is above synchronism and hence moves past the desired position. This further change of phase position also decreases the torque, which becomes less than the load, and the difference must be supplied by a deceleration of the armature. When the armature has again reached synchronous speed its phase position is such as to give less torque than required by the load. The deceleration of the armature, therefore, continues and the phase position is retraced until the second position, where the torque balances the load, is again reached. However, the armature rotates by this time below synchronous speed and hence the armature again passes the desired position. The change in the kinetic energy in the rotating mass therefore causes the armature to oscillate around the desired position, where the torque is just sufficient for the load.

Under certain conditions[1] the action is cumulative, the successive oscillations increase in magnitude, and as a result the machine, in a short time, drops out of synchronism. In most cases the synchronous motor is designed so that the successive oscillations decrease in magnitude and rapidly become negligible. The most common and useful means for rapidly reducing hunting

[1] Steinmetz, "Instability of Electric Circuits," *Trans. Am. Inst. Elec. Eng.*, Vol. 32, p. 2007.

oscillations is the *amortisseur winding* or *damper*. This consists simply of a short-circuited grid, similar to the squirrel-cage rotor of an induction motor, placed in slots at the surface of the field iron core, Fig. 19.15. The oscillations of the armature cause a

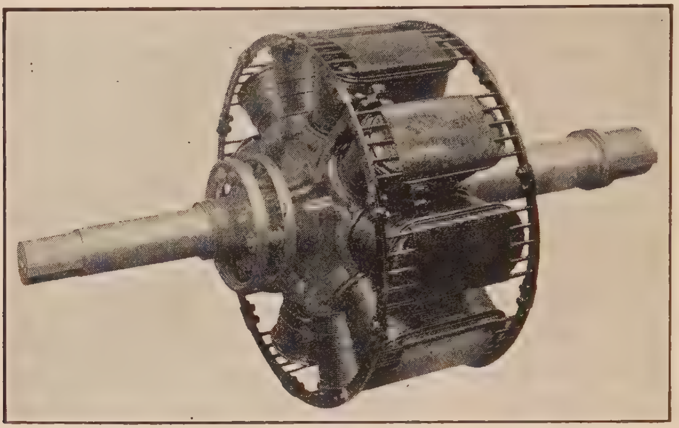


Fig. 19.15.—Amortisseur winding, damper or squirrel-cage winding. (*General Electric Company.*)

corresponding motion in the magnetic flux, which induces a voltage causing currents to flow in the amortisseur winding. The rI^2 losses from these currents act as a brake on the armature oscillations, and the hunting of the synchronous motor is thereby rapidly decreased. The amortisseur winding is sometimes used as a squirrel-cage rotor when starting the machine as an induction motor. When running in synchronism no lines of force cut the amortisseur winding and hence it does not affect the operation of the motor under normal conditions. Its chief purpose is to provide the means for a gradual adjustment of the motor-phase positions for changes in load, and thus to eliminate troublesome surging or hunting in the synchronous machine.

(*k*) **Torques of Synchronous Motors.**—In the practical operation of motors the type of load carried imposes definite torque requirements, which to a large extent determine important features in the design. The following torques are of special importance in the application of synchronous motors to commercial loads.

1. The *initial or starting torque*, which determines the standstill friction the motor can overcome in starting its load. This torque and the corresponding starting current can be varied over

wide limits by design as illustrated in the torque-speed curves in Fig. 20.15. In recent years starting torques have been greatly improved so that, in general, synchronous motors, unless specially designed for low or no-load starting, can start any load they can carry at synchronous speed. General-purpose, high-speed, synchronous motors have starting torques well within the range of standard squirrel-cage induction motors.

2. The *accelerating torque*, which brings the load up to the induction-motor slip speed with more or less rapidity. The rate of acceleration and the corresponding line current are, within a wide range, subject to design.

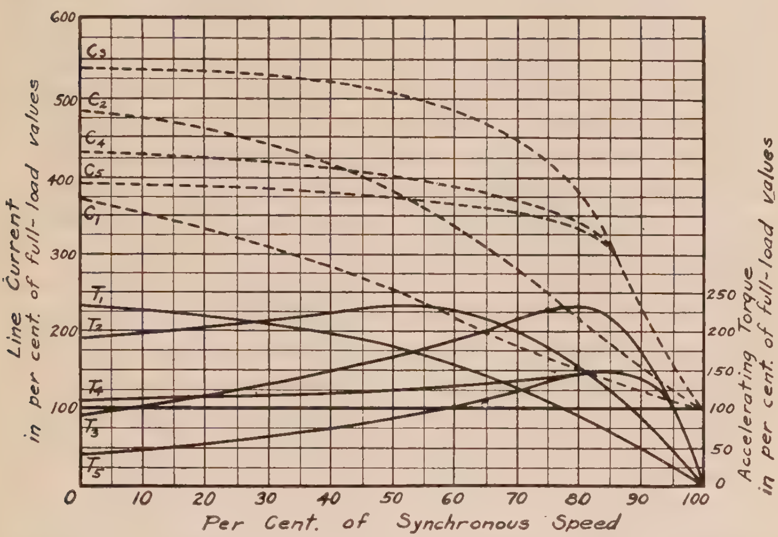


FIG. 20.15.—Speed-torque curves of five synchronous motors of different designs. (*Electric Machinery Manufacturing Company.*)

3. The *pull-in torque*, which determines the motor's ability to pull its load up from the induction-motor slip speed to synchronous speed. In the design of the motor this is often the most difficult requirement to meet. However, if the load requires a maximum pull-in torque a satisfactory motor can be designed for this service.

4. The *full-load torque*, which combined with the speed determines the rated horsepower of the motor.

5. The *pull-out torque*, which is a measure of the motor's ability to carry overload; that is, up to the point where the motor falls

out of step and stops. Both the full-load and the pull-out torques are subject to design and may be greatly varied.

It is, of course, evident that while the design of the motor determines the magnitude of the torques, as listed above, it is not possible to secure maximum torques at all speed points in the same motor. For example, the requirements for high starting torque and high pull-in torque are, in fact, incompatible. The gain in one is obtained at a sacrifice in the other. The initial starting torque can be increased by using higher resistance in the amortisseur or cage winding but this necessarily produces a lower slip speed. That is, for the greatest initial starting torque the resistance of the cage winding should be high, but for the best pull-in torque the resistance should be low, so as to have the slip speed nearly the same as the synchronous speed. For certain loads, like line shafts, the starting torque is high while the pull-in torque is comparatively small. For this type of load the cage winding should have high resistance. For other loads, as fans or centrifugal pumps, the starting torque is low and the pull-in torque high. For loads of this type motors having low-resistance cage windings should be used.

The super-synchronous motor developed by the General Electric Company is an interesting example of special design to meet exacting torque and line current requirements. The construction is unique in that the stator is mounted, so that during the initial starting period it revolves in the reverse direction while the rotor and load are at rest. The stator is brought up to speed and phased in with the stationary rotor field. A large band brake on the outer surface of the stator frame is then applied gradually. The rotor and load start to rotate in the proper direction by slowing down the stator; the sum of the two speeds remaining constant and equal to the synchronous speed. After 10 to 20 sec., as determined by the load torque and line current requirements, the rotor has reached full speed, the stator stops, the brake is locked and the motor continues to operate at synchronous speed. In this design the full pull-out torque is available for the starting, accelerating and pull-in operations in bringing the load from standstill to synchronous speed.

(*l*) **Starting Synchronous Motors.**—It is evident that the synchronous motor as such is not self-starting, since it would be impossible to change the rotating element from standstill to synchronous speed instantly. However, the synchronous motor

as a machine is self-starting but operates (without field excitation) as an induction motor during the starting period. The amortisseur winding (Fig. 19.15) becomes the squirrel-cage rotor of the induction motor. When the rotor reaches the rated slip speed, the machine operating as an induction motor is converted into a synchronous motor by applying full excitation to the field. The starting of synchronous motors therefore involves one more switching operation, namely the excitation of the field, than for induction motors.

In order to keep the starting current within the limits prescribed by the power supply system, the voltage applied to the terminals of large synchronous motors is generally reduced below the line voltage by means of starting compensators (autotransformers). However, larger synchronous motors than induction motors are connected directly to line voltage. Many factors, as size and speed of motor, size of generating station, feeders and mains, enter into the problem of whether or not to use starting compensators in the practical operation of either induction or synchronous motors. Since the starting torques for any given motor varies approximately as the square of the impressed voltage the reduction of voltage is often limited by the type of load to be carried.

In starting synchronous motors three forms of sequence in the switching operations are used, the choice depending on the design of the motor and service conditions.

First Method.

1. Line voltage impressed on armature.
2. Excitation applied to field.

Second Method.

1. Reduced voltage impressed on armature.
2. Line voltage impressed on armature
3. Excitation applied to field.

Third Method.

1. Reduced voltage impressed on armature.
2. Excitation applied to field.
3. Line voltage impressed on armature.

The first method is the simplest, does not require starting compensators and requires the least time for bringing the motor to synchronous speed. Automatic starters operated by means of

push buttons are coming into general use for starting synchronous motors. A button is pressed to start the motor and another button to stop the motor. The automatic switch completes both operations in the predetermined sequence.

For loads requiring both high initial starting and pull-in torques it is sometimes desirable to use clutch starting with synchronous motors. The motor is started running idle and after synchronous speed has been reached the load is started gradually by means of a suitable clutch. By this means the full pull-out torque of the synchronous motor is available throughout the load-starting period. Magnetic as well as mechanical clutches have been developed for this service.

Other methods used in early installations, as starting by means of an auxiliary induction motor or by using the exciter as a motor for starting the synchronous motor are by now obsolete.

(*m*) **Field Excitation.**—The synchronous motor requires direct-current field excitation and, hence, in most cases must be provided with a direct-current exciter. This may appear as a troublesome handicap in comparison with the induction motor, but is, in fact, in many cases a distinct advantage.

It should be clearly kept in mind that all electric motors and generators require field excitation and that the number of ampere-turns necessary under specified voltage and load conditions must be provided at some point or points in the system. If several generators operate in parallel and provide power for a motor load the total field excitation for the system is essentially the same whether induction or synchronous motors are used, or a combination of synchronous and induction motors. If only induction motors are installed, the field excitation of the generators must be sufficient to supply the total excitation required by the induction motors as well as the alternators. If part of the induction motors be replaced by synchronous motors, the alternator field excitation is reduced by an amount equal to the excitation provided by the synchronous motors. By overexciting the synchronous motors the alternators may be entirely relieved of the excitation required for the induction motors as the necessary ampere-turns are provided by the fields of the synchronous motors. Under such conditions the alternators would provide their own field excitation and carry load having unity power factor. The synchronous motor is therefore not an extra expense but merely the means of producing the necessary field excitation where it is needed, instead

of forcing the alternators to provide the field excitation for all the machines, motors as well as alternators, in the system.

(*n*) **Synchronous vs. Induction Motors.**—In selecting motors for the bulk of industrial power loads the choice usually lies between synchronous and induction motors. Therefore a summary[1] of the principal features of synchronous motors in comparison to squirrel-cage induction motors should be of interest.

Advantages.

1. Unity power factor at normal excitation.
2. Adjustable for leading power factor to compensate for the lagging power factor of induction-motor loads.
3. Constant speed at all loads.
4. Unsurpassed efficiency.
5. Mechanical ruggedness.
6. Can be wound for high voltage.
7. Rotor and slip rings carry only small direct current at low voltage.
8. Pull-out torque less affected by voltage fluctuations.
9. Inherent tendency to maintain constant voltage on system.
10. Cost is less in large sizes, or in low-speed ratings.

Disadvantages.

1. Limitation in pull-in torque.
2. Requires direct-current excitation.
3. Higher cost in high speed ratings.
4. Pull-out torque (unity power-factor excitation), in general, somewhat lower.
5. Constant-speed feature limits usefulness of flywheels.
6. May have to be restarted if dip in line voltage causes it to drop out of synchronism.

During the past few years many important improvements have been made in the design of synchronous motors, so that the range in torques and speed and, as a consequence, the types of load that can be advantageously carried by synchronous motors have been greatly increased.

PROBLEMS

1.15. From the data in Table XI find the efficiency of the several motors for 25, 50, 75, 100, 125 and 150 per cent full load. Assume that the load

[1] DuBois, A. Dawes, The Power Factor Book, p. 50, Electric Machinery Manufacturing Co.

losses vary with I^2 and at full load are equal to 50 per cent of the iron losses. Neglect losses in rheostats.

(a) For unity power factor.

(b) For 85 per cent power factor.

TABLE XI

	Full load, hp.	E_t, volts	f, cycles	Speed, r.p.m.	Phase	S.C. ratio	Iron loss, kw.	Friction and windage, kw.	Armature resistance across terminals, ohms	x_s per cent.
1	400	2,300	25	150	3	1.6	5.4	4.2	0.573	40
2	750	2,400	60	720	3	1.1	10.1	7.2	0.166	44
3	1,000	2,300	60	514	2	1.2	15.0	4.9	0.092	44
4	1,000	2,200	25	500	3	1.0	16.0	6.5	0.061	45
5	1,500	2,200	25	500	3	1.5	16.7	13.8	0.061	42
6	1,500	2,300	60	514	3	1.0	17.0	16.0	0.068	37

2.15. For one of the motors in Table XI draw current loci as in Fig. 7.15 for zero mechanical power and for full load.

3.15. An induction motor and a synchronous motor are connected to the same 2,200-volt mains. The induction motor takes 137 kw. at 84 per cent power factor. The synchronous motor takes 85 kw. The power factor of the total load in the mains is 98 per cent, lagging current.

(a) Find the power factor of the synchronous motor.

(b) Draw the vector diagram showing the currents in the motors and in the mains and their time-phase relations to the voltage.

4.15. A synchronous motor is installed in a substation having the following load: incandescent lamps 76 kw.; induction motor 85 kw. at 82 per cent power factor; induction motor 120 kw. at 87 per cent power factor. The load for the synchronous motor is 92 kw. The power factor at the busbars is 99 per cent, current lagging. $E = 2{,}200$ volts.

(a) Find the power factor of the synchronous motor.

(b) Find the rating of the synchronous motor if its load is 75 per cent of full load.

(c) Draw the vector diagram showing the magnitude of the currents in the lamps and motors and their time-phase relation to the busbar voltage.

5.15. Synchronous and induction motors operate in parallel. The total load is 385 kw. The power factor at the busbars is unity, for the induction motor 84 per cent and for the synchronous motor 70.7 per cent.

(a) Find the load in kilowatts and in kilovolt-amperes for the two motors.

(b) Draw the vector diagram for the currents with the busbar voltage as the reference vector.

6.15. The machine given in problem 8.14., Chap. XIV, is run as a synchronous motor at a mechanical load of 90 hp. from a 2,300-volt, three-phase line. Find the V-curve for this load.

7.15. A certain cement plant with an output of 2,000 bbl. of cement every 24 hr. has installed 2,000 hp. in motors. Its average demand over a 24-hr. day is 1,400 kw. and its maximum demand measured over a period of 15 min.

1,010,000 @ .01 11,500

B.B.L./mo = 30(2000) = 60,000

$\frac{11500}{60,000}$

is 1,700 kw. Power is supplied by the central station at a rate of $1 per month per kw. maximum demand plus 1 ct. per kw.h. based on an average power factor of 85 per cent. For a power factor below 85 per cent the energy charge will be increased in the inverse ratios of the power factors, and for power factors above 85 per cent a rebate will be allowed on the same basis up to a power factor of 100 per cent.

(*a*) On the basis of 85 per cent power factor, what is the total cost of energy per bbl. of cement? .19¢

(*b*) If slow-speed induction motors are used for most of the drives, the power factor will not be over 75 per cent. What will be the added energy charge per bbl. of cement? How many wattless kv.a. will be required to eliminate the penalty?

(*c*) Assume a synchronous condenser will be installed to correct the power factor and that the following commercial machines are available:

Kilovolt-amperes	Kilowatt losses, including exciter losses	Approximate cost installed complete with panel, etc.
500	32	$4,950
750	42	6,300
1,000	52	7,500
1,500	69	9,500

Assuming the interest and depreciation amounting to 17 per cent per year, what size machine should be installed to obtain minimum cost of operation on the basis of the present power factor being 75 per cent?

(*d*) When the plant was built, a bank of four 667-kv.a. transformers (one spare) was supplied to furnish the power to the plant. Assuming operation of the plant without a synchronous condenser to be at 75 per cent power factor, which of the machines mentioned in (*c*) would make it possible to reduce the size of the transformers to the next smaller commercial rating, namely, 500 kv.a.?

CHAPTER XVI

ROTARY CONVERTERS

The synchronous or rotary converter consists of a direct-current generator and synchronous motor combined into one machine. The direct-current commutator at one end of the armature and the alternating-current slip rings at the other are connected to the same winding. Through the slip rings the converter receives power as a synchronous motor; and at the same time the commutator delivers energy as a direct-current generator. The motor and generator currents flow simultaneously in the same armature conductors and are acted upon by the same field. The synchronous converter is therefore equivalent to a motor-generator set, and is used extensively for converting alternating into direct currents, especially in electric railway systems. When the process is reversed and the machine converts direct into alternating currents, it is known as an *inverted synchronous* or *rotary converter*.

(*a*) **Voltage Relations.**—In the preliminary discussion let the converter be separately excited and consider the losses negligible so that the same amount of energy is delivered through the brushes on the direct-current end as is received at the slip rings from the alternating-current circuit. The alternating-current side of the rotary converter has characteristics similar to the synchronous motor. As in the synchronous motor, the phase relations of the voltage and alternating current in the rotary converter may be changed by varying the field excitation. Let the excitation be of such value as to give unity power factor for the alternating current. Hence the maximum value of the current in the group of armature coils under consideration occurs when the maximum voltage is developed. If the fields are not distorted the maximum voltage is generated when the center of the group of coils is passing at right angles to the lines of force from the field; that is, opposite the center of the field pole. When the center of the same group of coils is midway between two poles, no voltage is generated; and at intermediate posi-

tions the instantaneous values are represented by $e = {}^{M}E \sin(\omega t)$. Diagrammatic representations of the commutator and slip-ring connections are shown in Figs. 1.16, 2.16, and 3.16 for a

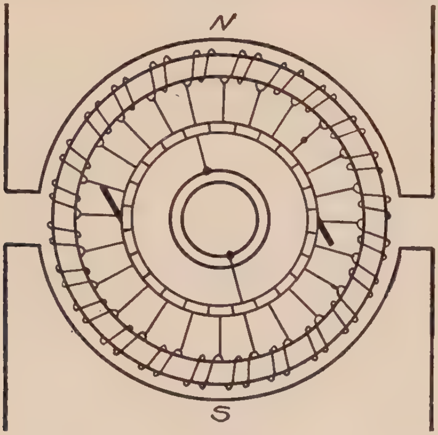


FIG. 1.16.—Single phase.

two-pole machine. The voltage between the direct-current brushes is the sum of the voltages of one-half of the coils on the

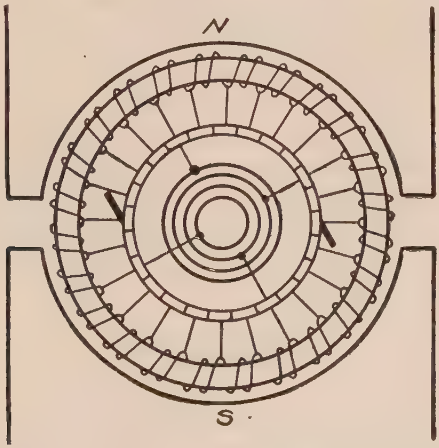


FIG. 2.16.—Two phase.

armature. The instantaneous voltages in each coil may be represented by vectors drawn through some point O as in Fig. 4.16 or arranged in a polygon as in Fig. 5.16. The number of

sides of the polygon corresponds to the number of coils; and the vertices lie in a circle, as the coils are equal. When the armature revolves, the points connected to the slip rings in Fig. 1.16 pass under the commutator brushes at the point of maximum voltage

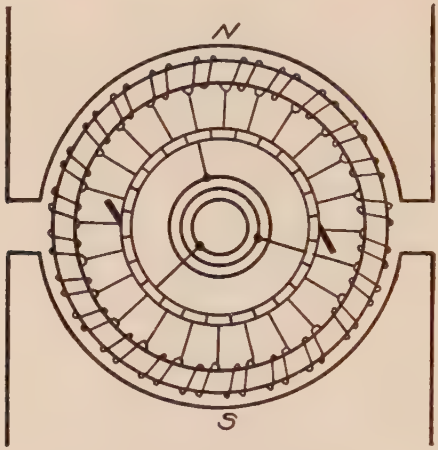


FIG. 3.16.—Three phase.

of the alternating wave. Hence the direct-current voltage E_0 must be equal to the maximum value of E_2, the alternating-current, single-phase voltage.

$$E_2 = \frac{E_0}{\sqrt{2}} = 0.707E_0 \tag{1.16}$$

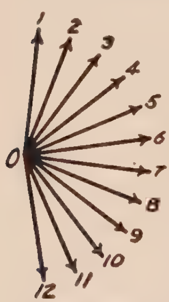


FIG. 4.16.

In the two-phase or quarter-phase converter, Fig. 2.16, the slip-ring connections are 90° apart, and the voltage relations for the single-phase and quarter-phase circuits are shown in Fig. 6.16 by the relative lengths of the diameter and the chord for 90°.

$$E_4 = E_2 \sin 45° = \frac{E_0}{\sqrt{2}} \sin 45° = 0.500E_0 \tag{2.16}$$

Similarly for a three-phase converter, Figs. 3.16 and 6.16, the slip-ring connections are 120° apart and the relative length of the chord and diameter gives the relation between the magnitude of the three-phase voltage E_3 and single-phase voltage E_2.

$$E_3 = E_2 \sin 60° = \frac{E_0}{\sqrt{2}} \sin 60° = 0.612E_0 \tag{3.16}$$

In the same manner the voltage E_6 between the slip rings of a six-phase converter is:

$$E_6 = E_2 \sin 30° = \frac{E_0}{\sqrt{2}} \sin 30° = 0.354E_0 \qquad (4.16)$$

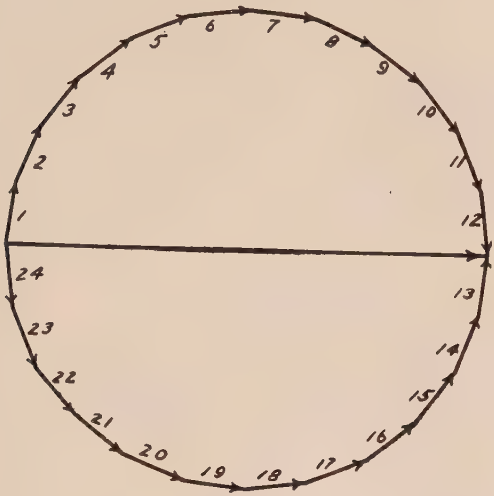


FIG. 5.16.

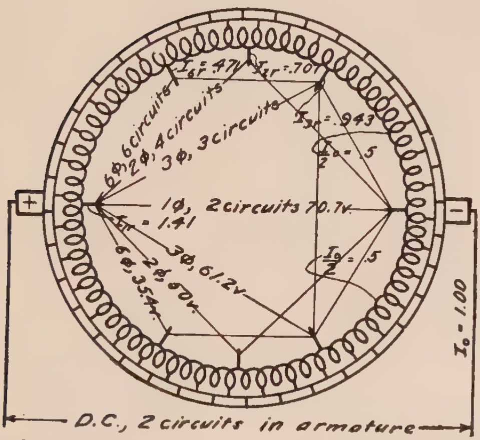


FIG. 6.16.—Showing relative current for same power.

These results may be expressed by one equation as in equation (5.16) by letting n represent the number of slip rings in the converter.

$$E_n = \frac{E_0}{\sqrt{2}} \sin \frac{\pi}{n} \qquad (5.16)$$

For single phase $n = 2$
For three phase $n = 3$
For quarter phase $n = 4$
For six phase $n = 6$

While under the stated conditions the voltage ratio is given by equation (5.16), other factors, especially the ratio of pole arc to pole pitch, and the losses in the machine, must be taken into consideration in finding the actual voltage ratio in commercial machines. When operating under normal direct-current voltage, commutating pole synchronous converters, Fig. 27.16, will have approximately the following voltage ratios:

	No load, per cent	Full load, per cent
Single phase, two phase and six phase, diametrical...	72	74
Three phase and six phase, double delta............	62	64

(*b*) **Current Relations.**—Under the assumptions of unity power factor and no losses, the current relations are readily found by equating the alternating-current input to the direct-current output and substituting the voltage ratios found in the preceding paragraph.

Let

I_0 = the direct current delivered through the brushes.
I_2 = the current in the single-phase winding.
I_3 = the current in the three-phase windings.
I_4 = the current for the quarter-phase windings.
I_6 = the current for the six-phase windings.

For single-phase converters:

$$E_0 I_0 = 2E_2 I_2 \therefore I_2 = 0.707 I_0 \qquad (6.16)$$

For three-phase converters:

$$E_0 I_0 = 3E_3 I_3 \therefore I_3 = 0.544 I_0 \qquad (7.16)$$

For quarter-phase converters:

$$E_0 I_0 = 4E_4 I_4 \therefore I_4 = 0.500 I_0 \qquad (8.16)$$

For six-phase converters:

$$E_0 I_0 = 6E_6 I_6 \therefore I_6 = 0.471 I_0 \qquad (9.16)$$

For a converter of n rings, the current in the *armature circuits*

$$I_n = \frac{\sqrt{2} I_0}{n \sin \frac{\pi}{n}} \qquad (10.16)$$

Since two circuits are connected to each collector ring, the currents in the mains are the vector differences of the currents in the circuits.

Letting the subscript r indicate *at the rings*, the *currents in the mains* at unity power factor will be for a:

Two-ring converter (single phase) $I_{2_r} = 2I_2 = 1.41I_0$ (11.16)
Three-ring converter (three phase) $I_{3_r} = \sqrt{3}I_3 = 0.943I_0$ (12.16)
Four-ring converter (two phase) $I_{4_r} = \sqrt{2}I_4 = 0.707I_0$ (13.16)
Six-ring converter (six phase) $I_{6_r} = I_6 = 0.471I_0$ (14.16)

The general expression for current in the *mains* or collector rings:

$$I_{n_r} = 2I_n \sin\frac{\pi}{n} = \frac{2\sqrt{2}I_0}{n} \quad (15.16)$$

In Table XII the respective values of the currents and voltages in the circuits and mains are given for 1.0 amp. and 100.0 volts in the direct-current mains.

TABLE XII.—VOLTS AND AMPERES FOR CONSTANT POWER

Direct current mains. E_0 = 100 volts. I_0 = 1.00 amp. W_0 = 100 watts

	Number of circuits in armature	Number of rings	Volts between mains		Amperes per armature circuit or per phase		Amperes per main
	n	n	$E_n = \frac{E_0}{\sqrt{2}} \sin\frac{\pi}{n}$	${}^{M}E_n = \sqrt{2}E_n$	$I_n = \frac{\sqrt{2}I_0}{n \sin\frac{\pi}{n}}$	${}^{M}I_n = \sqrt{2}I_n$	$I_{nr} = \frac{2\sqrt{2}I_0}{n}$
Single phase..	2	2	70.7	100.0	0.707	1.000	1.410
Three phase..	3	3	61.2	86.7	0.544	0.770	0.943
Quarter phase	4	4	50.0	70.7	0.500	0.707	0.707
Six phase.....	6	6	35.4	50.0	0.471	0.666	0.471

In Fig. 6.16 the effective value of current in the leads and voltage per phase are indicated corresponding to 1 amp. at 100 volts, direct current.

Thus, for a three-phase converter the current in the leads is 0.943 amp. and for a six-phase converter 0.471 amp. For the single-phase converter the current at the slip rings is 1.41 amp. while the voltage is only 70.7 volts.

(c) **Heat Losses at Unity Power Factor.**—From the previous discussion it is evident that the rotary converter may be used as a generator receiving power from a prime mover and delivering either direct or alternating currents, or both simultaneously. It may also be used as a synchronous or direct-current motor receiving the energy in the electric form and delivering mechanical energy by means of pulley and belt. As a rotary converter it receives energy from the alternating-current mains and delivers all but the amount lost in the machine to the direct-current mains. As an *inverted converter* or *inverted rotary*, power is received from the direct-current mains and delivered to the

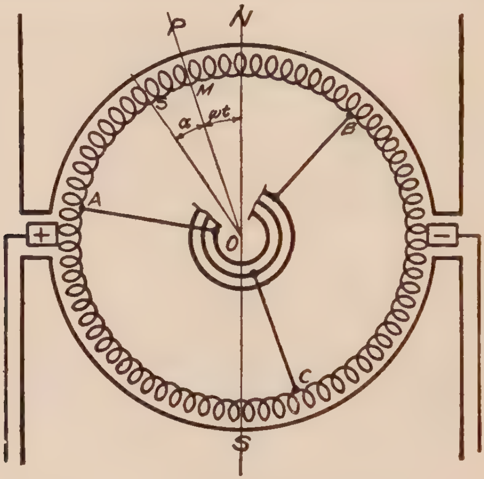


FIG. 7.16.

alternating-current mains of phase and frequency depending upon the number of slip-ring connections and the speed. Under the several kinds of service the armature heating is not the same for equal power passing through the machine. Since the rating of the synchronous converter depends upon the permissible temperature rise for the same reasons as for alternators and synchronous motors, the difference in heat loss in the armature affects the capacity of the machine. The difference in rating when used as a rotary converter as compared to a direct-current generator is determined by the RI^2 losses in the armature, since the eddy-current and hysteresis losses are approximately the same in both cases. Since the slip rings and commutator bars are connected to the same armature, the instantaneous value of the

current in any conductor will be the algebraic sum of the simultaneous values of the currents flowing in the direct-current and alternating-current circuits.

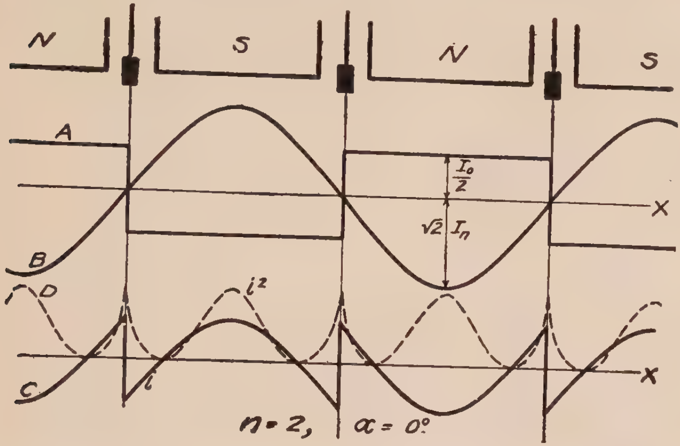


FIG. 8.16.

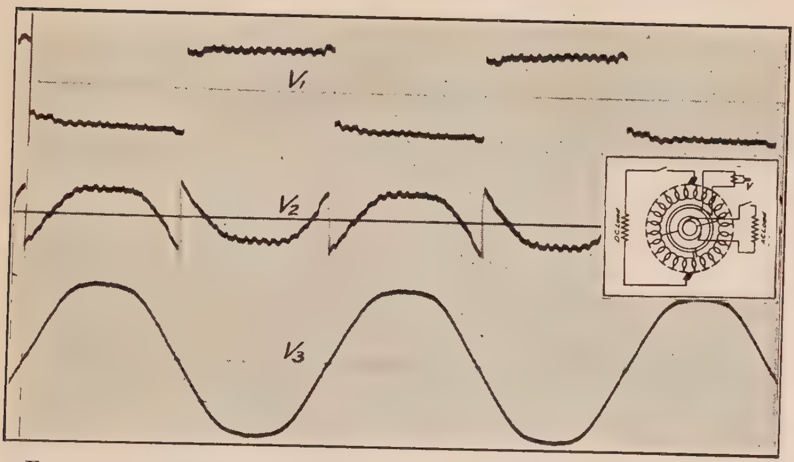


FIG. 9.16.—Oscillogram of current components in rotary converter. Vibrator inserted in armature winding at point midway between slip-ring taps. $\alpha = 0°$. V_1 = direct current only; V_2 = alternating current and direct current; V_3 = alternating current only. Power factor 100 per cent. Machine operated as an inverted converter.

Consider a two-pole, n-ring converter, as indicated by the diagram in Fig. 7.16. Let M be the conductor midway between the slip-ring connections A and B, and let S be the conductor at a distance of α from M. For the several conductors in the

circuit between A and B the value of α varies from 0 to $\pm\frac{\pi}{n}$. The maximum value of the alternating current, $\sqrt{2}I_n$, flows when conductor M passes under the center of the field pole (unity power factor assumed). Hence the instantaneous value for any other position of the armature is for conductor M:

$$i = \sqrt{2}I_n \cos \omega t - \frac{I_0}{2} \tag{16.16}$$

When conductor S reaches the positive brush,

$$\alpha = \frac{\pi}{2} - \omega t \tag{17.16}$$

and at the negative brush

$$\alpha = -\left(\frac{\pi}{2} + \omega t\right) \text{ or } \frac{3\pi}{2} - \omega t \tag{18.16}$$

The notation in equation (16.16) applies to multipolar machines by using electrical degrees and letting I_0 represent the direct current passing through each set of brushes. In Fig. 8.16 and in the corresponding oscillogram in Fig. 9.16 are shown the currents flowing through conductor M in a single-phase converter, *i.e.*, $n = 2$, $\alpha = 0$. The direct current reverses direction at the brushes forming a wave of rectangular shape. The neutral points of the alternating current coincide with the wave reversals of the direct current, and the maximum value is twice that of the direct current. The instantaneous value is the algebraic sum of the simultaneous values of the direct and alternating currents. Hence curve C, plotted as the sum of curves A and B, represents the wave shape and magnitude of the resultant current flowing in the armature conductor. The broken line, curve D, is plotted to the square of the instantaneous values in curve C and hence its ordinates are proportional to the heat generated in the conductor due to the RI^2 losses. In Fig. 10.16 and the corresponding oscillogram in Fig. 11.16 are shown the corresponding curves for the conductor at the point of connection to the slip ring, when $n = 2$, $\alpha = 90°$. By shifting the relative positions of the direct and alternating waves the instantaneous values, and hence the heating, become much greater for the conductor nearest to the slip-ring connection than for conductor M midway between the slip rings. Similar curves for $n = 2$ and $\alpha = 45°$ are shown in Fig. 12.16 and in the corresponding oscillogram in Fig. 13.16.

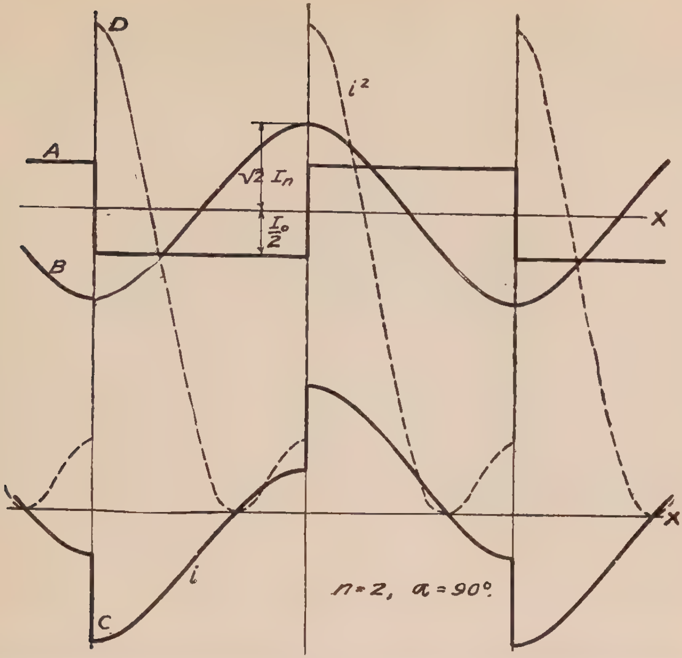


FIG. 10.16.

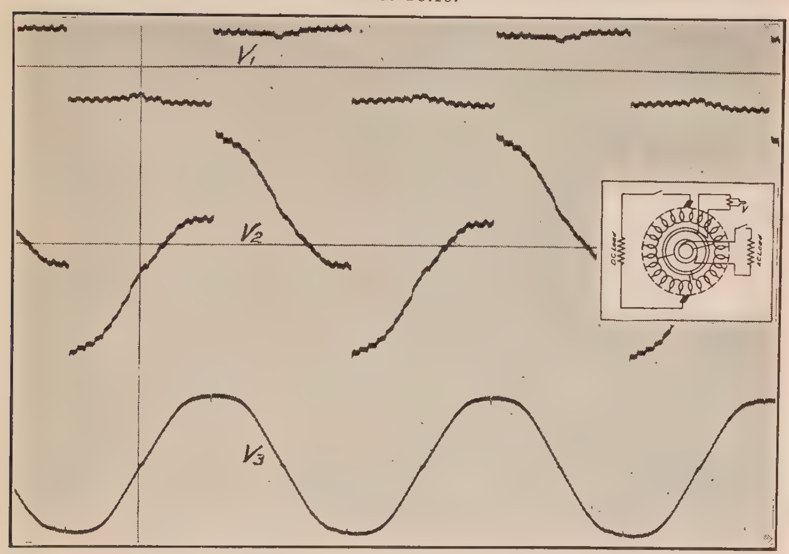


FIG. 11.16.—Oscillogram of current components in rotary converter. Vibrator inserted in armature winding next to slip-ring tap. $\alpha = 90°$. V_1 = direct current only; V_2 = alternating current and direct current; V_3 = alternating current only. Power factor 100 per cent. Machine operated as an inverted converter.

The curves for a three-phase converter are like Fig. 8.16 for $n = 3$, and $\alpha = 0$ and like Fig. 10.16 for $n = 3$ and $\alpha = \frac{\pi}{3}$.

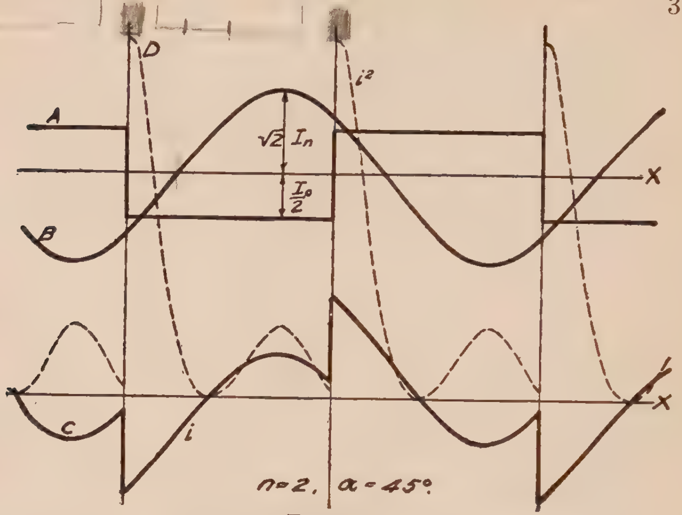


FIG. 12.16

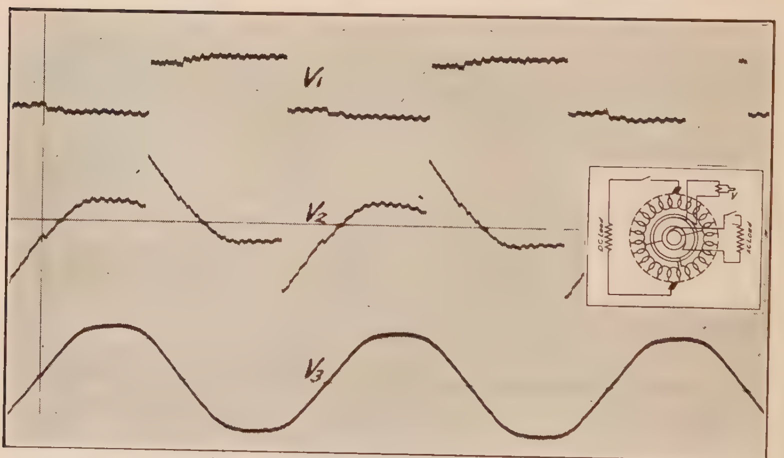


FIG. 13.16.—Oscillogram of current components in rotary converter. Vibrator inserted in armature winding one-fourth the distance between slip-ring taps. $\alpha = 45°$. V_1 = direct current only; V_2 = alternating current and direct current; V_3 = alternating current only. Power factor 100 per cent. Machine operated as an inverted converter.

Three sets of oscillograms similar to those in Figs. 9.16, 11.16 and 13.16 but for lagging load currents are shown in Figs. 14.16, 15.16 and 16.16.

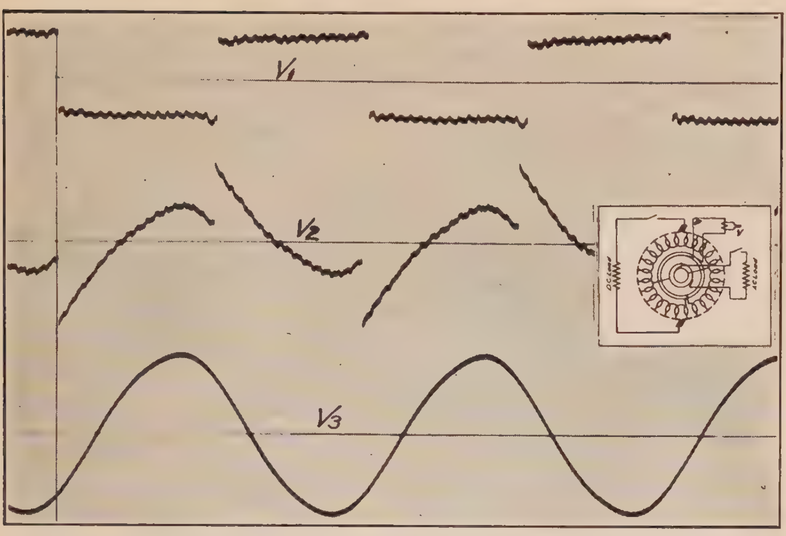


FIG. 14.16.—Oscillogram of current components in rotary converter. Vibrator inserted in armature winding at point midway between slip-ring taps. $\alpha = 0°$. V_1 = direct current only; V_2 = alternating current and direct current; V_3 = alternating current only. Power factor 75 per cent, current lagging. Machine operated as an inverted converter.

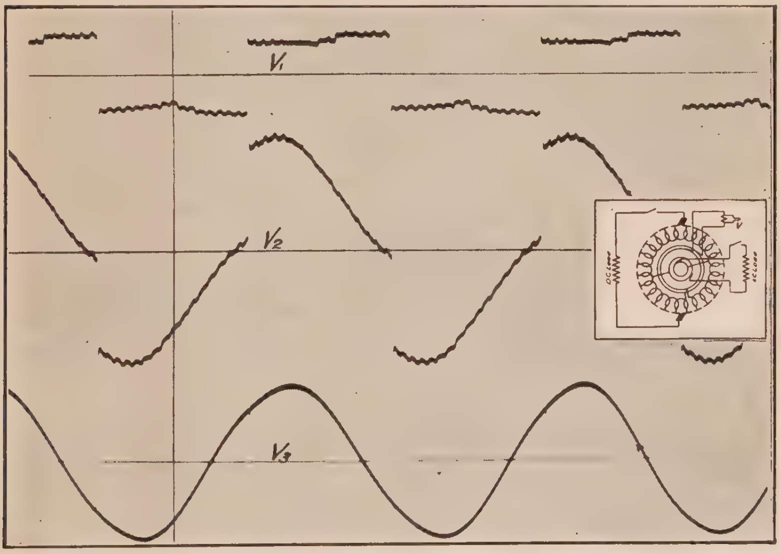


FIG. 15.16.—Oscillogram of current components in rotary converter. Vibrator inserted in armature winding next to slip-ring tap. $\alpha = 90°$. V_1 = direct current only; V_2 = alternating current and direct current; V_3 = alternating current only. Power factor 76 per cent, current lagging. Machine operated as an inverted converter.

From the figures and from the equation it is evident that in order to determine the heat loss in the armature it is necessary to find, first, the average loss in a conductor for a cycle, and, second, the average of the loss in the several conductors between each pair of leads.

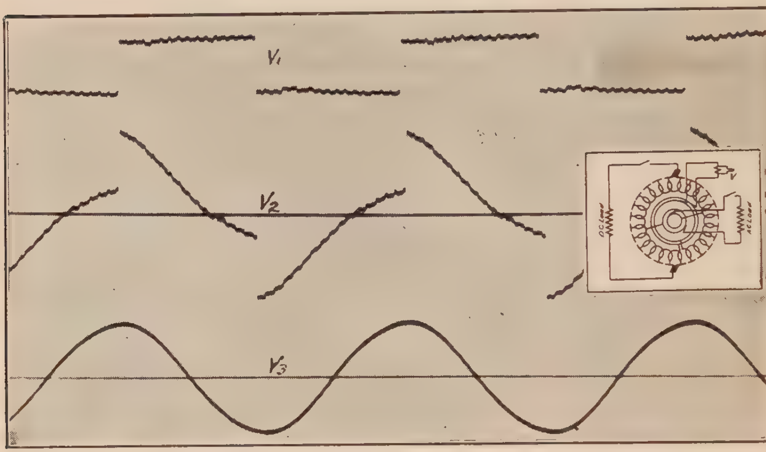


FIG. 16.16.—Oscillogram of current components in rotary converter. Vibrator inserted in armature winding one-fourth the distance between slip-ring taps. $\alpha = 45°$. V_1 = direct current only; V_2 = alternating current and direct current; V_3 = alternating current only. Power factor 64 per cent, current lagging. Machine operated as an inverted converter.

From equations (10.16) and (16.16):

$$i = \frac{2I_0 \cos(\omega t)}{n \sin\left(\frac{\pi}{n}\right)} - \frac{I_0}{2} \tag{19.16}$$

The heat loss is proportional to the square of the current.

$$i^2 = \frac{4I_0^2 \cos^2(\omega t)}{n^2 \sin^2\left(\frac{\pi}{n}\right)} - \frac{2I_0^2 \cos(\omega t)}{n \sin\left(\frac{\pi}{n}\right)} + \frac{I_0^2}{4} \tag{20.16}$$

$$= \frac{I_0^2}{4}\left[\frac{16 \cos^2(\omega t)}{n^2 \sin^2\left(\frac{\pi}{n}\right)} - \frac{8 \cos(\omega t)}{n \sin\left(\frac{\pi}{n}\right)} + 1\right] \tag{21.16}$$

The average heat loss per conductor in half a cycle

$$= r \int_{-(90°+\alpha)}^{(90°-\alpha)} \frac{i^2 d(\omega t)}{\pi} \tag{22.16}$$

in which r is the resistance of one armature conductor between taps.

From equations (21.16) and (22.16) the heat loss:

$$= \frac{rI_0^2}{4\pi}\int_{-(90°+\alpha)}^{(90°-\alpha)}\left[\frac{16\cos^2(\omega t)}{n^2\sin^2\left(\frac{\pi}{n}\right)} - \frac{8\cos(\omega t)}{n\sin\left(\frac{\pi}{n}\right)} + 1\right]d(\omega t) \quad (23.16)$$

$$= \frac{rI_0^2}{4}\left[1 - \frac{16\cos\alpha}{\pi n\sin\left(\frac{\pi}{n}\right)} + \frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right] \quad (24.16)$$

To find the average heat in the armature the average value per conductor as given in equation (24.16) must be integrated between the limits of $\alpha = -\frac{\pi}{n}$ and $\alpha = \frac{\pi}{n}$ and the result divided by $\frac{2\pi}{n}$.

Average heat in armature:

$$= \frac{rI_0^2}{4\frac{2\pi}{n}}\int_{-\frac{\pi}{n}}^{\frac{\pi}{n}}\left[1 - \frac{16\cos\alpha}{n\pi\sin\left(\frac{\pi}{n}\right)} + \frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right]d\alpha \quad (25.16)$$

$$= \frac{rI_0^2}{4}\left[1 - \frac{16}{\pi^2} + \frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right] \quad (26.16)$$

One-half of the armature is taken into the calculations both for the alternating current and the direct current, and hence the ratio is the same as for the whole armature.

The average armature heating is therefore $\left[1 - \frac{16}{\pi^2} + \frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right]$ times the heat generated by the same direct current. Hence for the same temperature rise an n-ring rotary converter transmits

$$\frac{1}{\sqrt{1 - \frac{16}{\pi^2} + \frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}}}$$

times as much current as when the machine operates as a direct-current generator.

From this ratio the power ratings of the synchronous converter are found in terms of the direct-current generator as stated in Table XIII.

TABLE XIII.—POWER RATINGS OF A SYNCHRONOUS CONVERTER

As a direct-current generator	As a single-phase converter $n = 2$	As a three-phase converter $n = 3$	As a two-phase converter $n = 4$	As a six-phase converter $n = 6$	As a twelve-ring converter $n = 12$
1.00	0.85	1.33	1.63	1.93	2.44

In the preceding discussion of the current relations, it was assumed that no energy was lost in the converter and that the

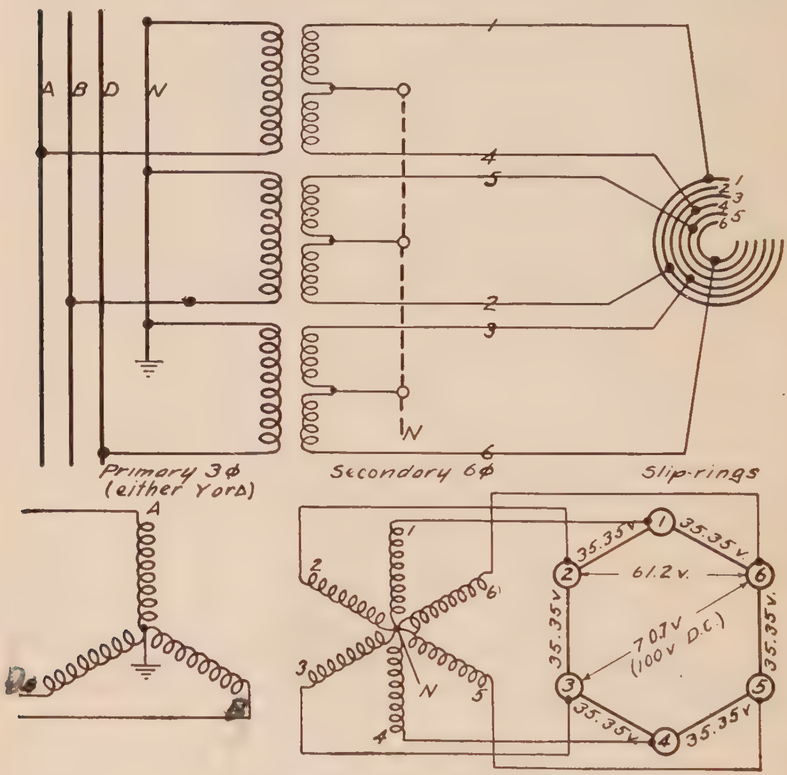


FIG. 17.16.—Diametrical connection. Three phase to six phase.

field excitation was adjusted for unity power factor. For power factors less than unity,[1] and taking into account the efficiency of the machine, the heat losses in the armature conductor are greatly increased. The load losses are also increased for power factors less than unity. Any variation of wave shape from the

[1] STAHL, *Elec. World*, p. 1090, Oct. 28, 1911.

sinusoidal form also affects the heat losses in the converter. The rating of the converter is computed on the basis of sine waves and unity power factor as given in Table XIII.

(*d*) **Transformer Connections for Rotary Converters.**—In most systems using rotary converters the energy is delivered to

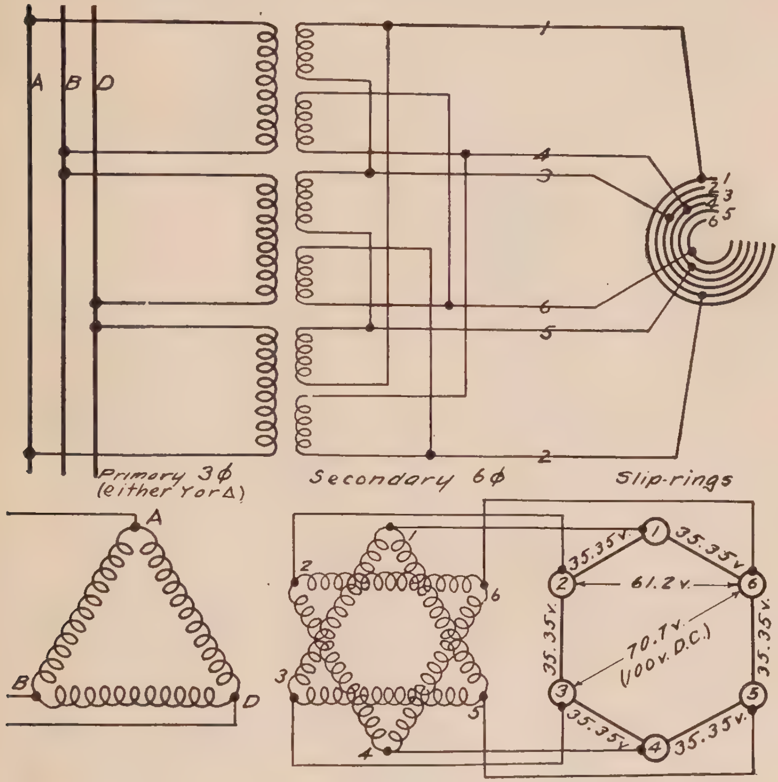


FIG. 18.16.—Double-delta connection. Three phase to six phase.

the converter substation in the three-phase system. From the relative ratings as given in Table XIII it is evident that the same machine will give a larger output for the same heating if connected as a six-phase converter than in the three-phase system. The ratio is 1.33:1.93, or 1:1.45, an increase of 45 per cent in the capacity of the converter. The change from three-phase to six-phase is readily accomplished. The converter itself requires six slip rings instead of three with connections to the windings at intervals of 60 instead of 120 electrical degrees. The trans-

formation from a three-phase to a six-phase system may be accomplished in several ways, and the following connections are in general use:

1. Diametrical	3 phase to 6 phase	Fig. 17.16.
2. Double Δ	3 phase to 6 phase	Fig. 18.16.
3. Double *Y*	3 phase to 6 phase	Fig. 19.16.
4. Double *T*	3 phase to 6 phase	Fig. 20.16.
5. Double *T*	2 phase to 6 phase	Fig. 21.16.
6. Ring	3 phase to 6 phase	Fig. 22.16.
7. Distributed *Y*	3 phase to 3 phase (with Edison three-wire system on direct-current side)	Fig. 23.16.

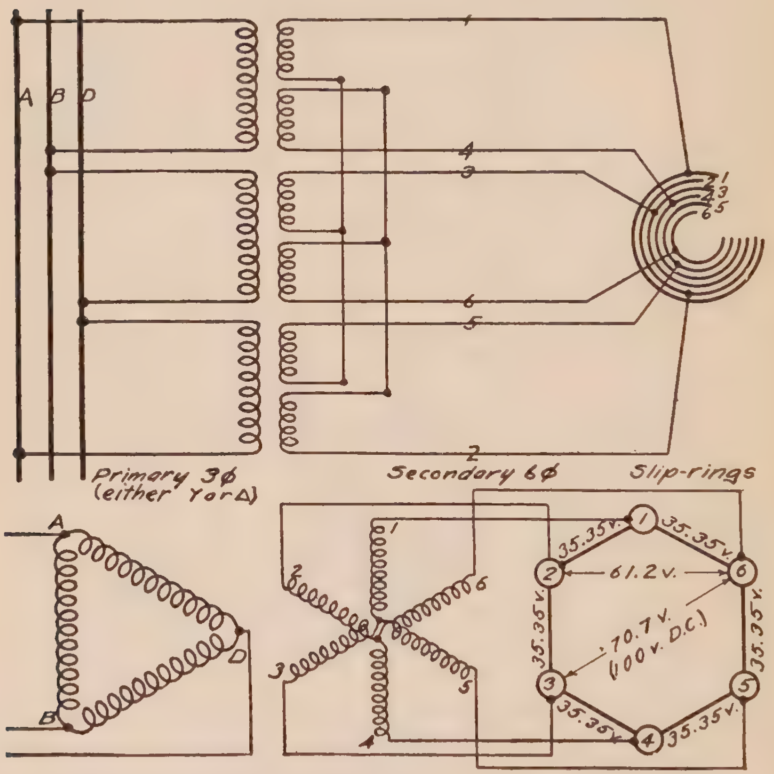


FIG. 19.16.—Double-*Y* connection. Three phase to six phase.

The diametrical or double-delta connections are most commonly used with rotary converters. A neutral can conveniently be secured in the diametrical connection and this arrangement is generally preferred whether the direct-current side is two-wire or three-wire Edison system.

The choice of connections for any system often depends on factors outside of the rotary converter, such as the type of direct-current distribution system, minimum insulation stress, voltage ratios, operating three-phase in connection with six-phase, ground connections, etc. By means of the double-*T* connection, Fig. 21.16, the transformation from two-phase is made directly to six-

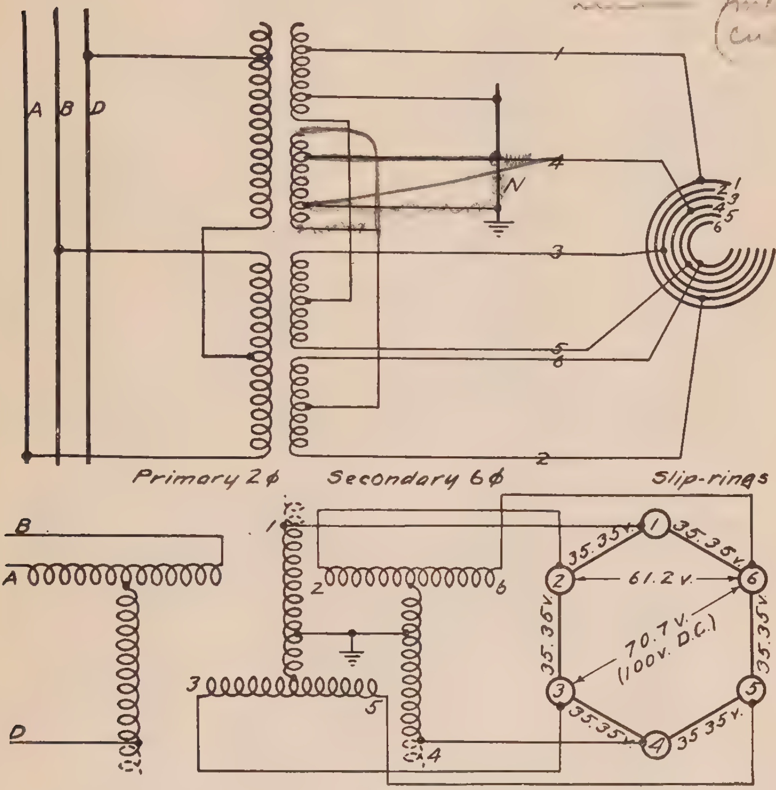


Fig. 20.16.—Double-*T* connection. Three phase to six phase.

phase. The *teaser* transformer must either have a tap at 86.7 per cent of the full voltage or a proportionate number of turns on the secondary windings. The double-*T* connection, Fig. 20.16 may also be used for three-phase to six-phase transformation in connection with the three-wire Edison system on the direct-current side. The distributed *Y* eliminates flux distortions in the transformers when operating a three-phase rotary converter

connected to a three-wire Edison system. The two windings are interconnected as shown in Fig. 23.16. The unbalanced neutral current flows through the two coils in each transformer in opposite directions, and hence has no magnetizing effects. With the simple Y connection even a 10 per cent unbalanced neutral current magnetizes the transformer cores beyond the saturation

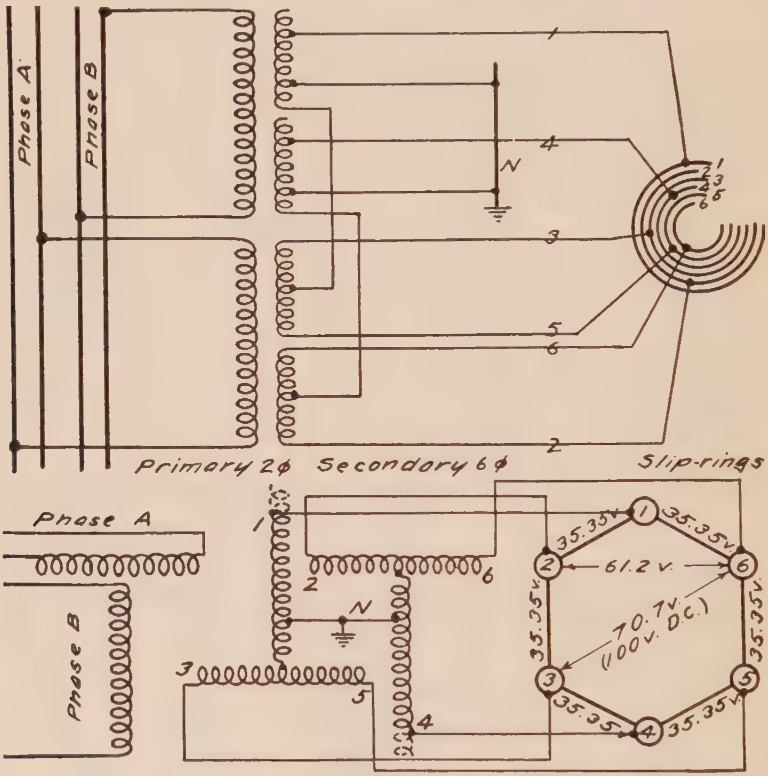


FIG. 21.16.—Double-T connection. Two phase to six phase.

point and thus causes abnormally large magnetizing currents to flow, with accompanying excessive losses. In the distributed Y connection the primary may be connected in Δ and it is therefore often preferable to the double-T. The double-T connection requires only two transformers, but has also the disadvantage that the triple harmonic exciting current is not short-circuited.

(*e*) **Synchronous Booster, Split-pole, Auxiliary-pole and Commutating-pole Synchronous Converters.**—As has already been

shown, the ratio of the alternating-current to the direct-current voltage in a rotary converter is a constant, depending upon the number of phases and the maximum value of the alternating-current wave. The ratio is not affected by changes in the field

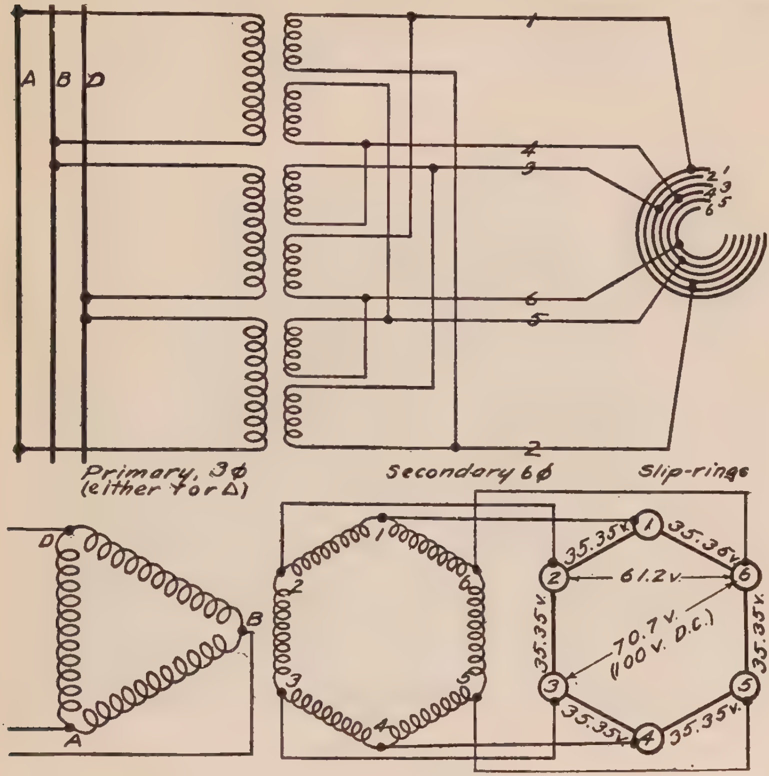


FIG. 22.16.—Ring connection. Three phase to six phase.

excitation, as the alternating-current part of the converter acts like a synchronous motor and hence changes in field excitation merely cause variation in the power factor without affecting the voltage. In the operation of the converter it is often necessary to have the direct-current voltage under control of the operator, although the power is supplied from constant-potential, alternating-current mains. The control of the direct-current voltage is usually secured in one of two ways:

1. By varying the alternating-current voltage impressed on the rotary converter by means of a booster, or change in transformer ratio, or by using reactive leads.

2. By varying the wave shape of the alternating-current voltage.

The synchronous booster rotary converter, as illustrated in Fig. 24.16, consists of a simple rotary converter in combination with an alternating-current generator mounted on the same shaft and having the same number of poles. By varying the field of the alternating-current generator, the voltage impressed

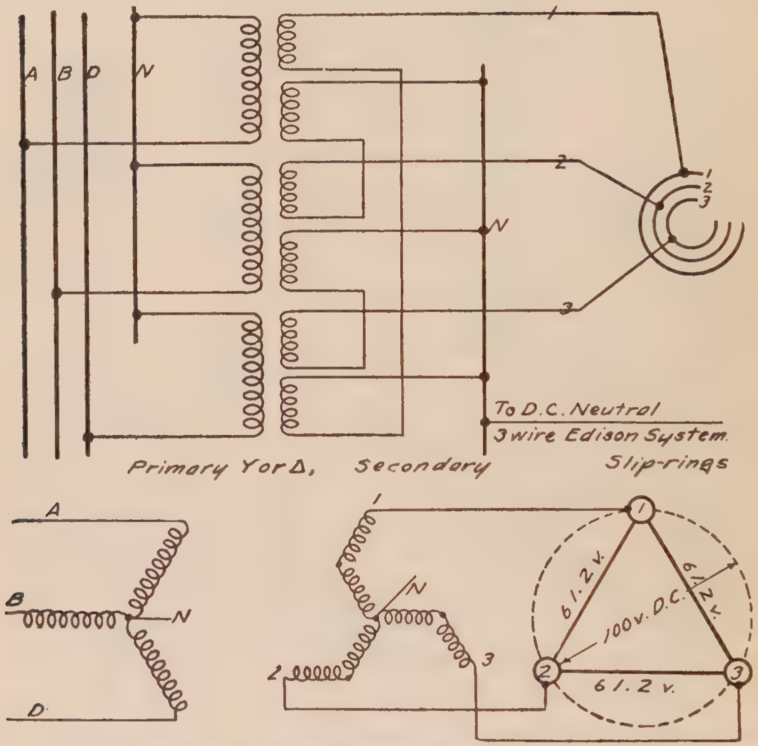


FIG. 23.16.—Distributed-*Y* (interconnected star) connection. Three phase to six phase.

on the rotary-converter part of the machine may be increased or decreased at will. The direct-current voltage is thereby under direct control of the operator. In commercial machines the range of voltage regulation is usually from 15 per cent above to 15 per cent below the line voltage. The principle of the booster converter is therefore simple and easily understood.

The change in the alternating voltage impressed on the rotary converter may likewise be obtained by changing the ratio of the

voltage transformation between the machine and the constant-voltage transmission line by means of induction regulators. To a certain extent a similar variation may be secured by using inductive reactances in the alternating-current leads to the converter. The choice of apparatus depends upon many factors, but their effect on the rotary converter is the same; namely, to vary the impressed voltage and thereby control the value of the direct-current voltage delivered by the converter.

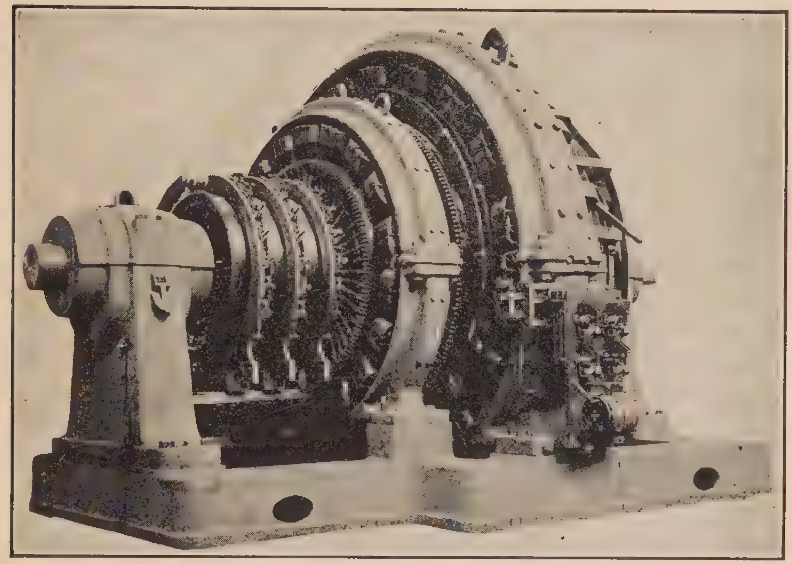


FIG. 24.16.—Synchronous booster rotary converter. (*Westinghouse Electric & Mfg. Co.*)

The action of the *split-pole converter* is based on an entirely different principle. The direct-current voltage is equal to the maximum value of the impressed alternating-current voltage wave. Hence, by changing the wave shape the direct-current voltage may be varied several per cent while the effective value of the alternating current remains constant. This change in wave shape may be secured by splitting the pole parallel to the shaft into three narrow sections, each having a separate winding. By making the field excitation of the side sections stronger than for the middle part, the voltage wave is peaked, as in Fig. 25.16, giving a direct-current voltage higher than for the corresponding

sine wave of the same maximum value. On the other hand, if the middle section is stronger the wave shape is flattened, as in Fig. 26.16, and the direct-current voltage is less than for the sine wave of equal effective value.

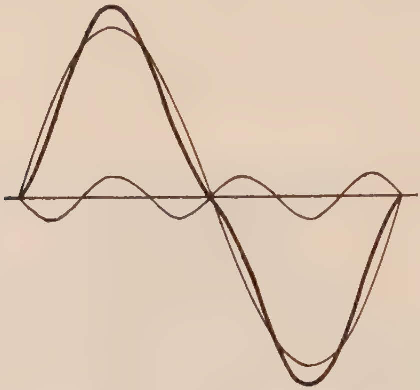


FIG. 25.16.—Peaked wave with equivalent sine wave.

The action of the sectional-pole converter is to some extent dependent upon the change in wave shape, but depends chiefly on the shifting of the field flux relative to the direct-current brushes. The direct-current voltage of the converter, as in any direct-current generator, varies with the shifting of the brushes relative

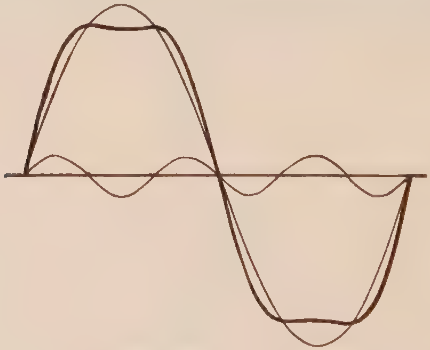


FIG. 26.16.—Flat-top wave with equivalent sine wave.

to the field poles. Hence, by changing the relative field excitation of the main and auxiliary poles the magnetic flux is shifted while the brushes remain fixed. Thereby the direct-current

voltage may be raised or lowered without changing the impressed alternating-current voltage. It should be noted that the sectional pole of the rotary converter merely provides a means for varying the direct-current voltage and is not an interpole for commutation. In fact, the range over which the voltage may be varied by means of the sectional pole is chiefly limited by commutation troubles. For this reason the sectional pole is placed nearer the trailing tip so as to leave the necessary space for commutation.

The *commutating-pole synchronous converter*, Fig. 27.16, is a standard design for electric railway service. The outstanding features of this type of converter are:

1. High-reluctance commutating poles.
2. Improved amortisseur winding.
3. Radial-type brush rigging.
4. Flash barriers.

The *high-reluctance commutating pole* is constructed with a comparatively short section of iron core placed next to the armature and with the remainder of the pole of a non-magnetic material. This design enables the converter to withstand many disturbances, which would cause severe flash-overs on the earlier designs. On account of the high reluctance of this pole piece, a higher excitation is required to give the flux density required for commutation.

The high-reluctance pole has the desirable characteristics of following with great accuracy the direct-current variations in load, maintaining under rapidly changing conditions the proper flux to insure sparkless commutation. Moreover, on account of the small amount of magnetic material in the core, these poles are less affected by the magnetizing current produced in the armature by alternating-current line disturbances. On account of their inherently more stable characteristics, 25-cycle converters do not require high-reluctance poles.

Synchronous converters of the type shown in Fig. 27.16 have the *amortisseur* or *damper windings*, which permit the converter to be started from the alternating-current side. These windings have the additional advantage of stabilizing the operation and preventing hunting of the converter by checking sudden distortions of the magnetic field during alternating- or direct-current disturbances. The copper bars or rods constituting the amortisseur winding or grid are brazed together with a silver alloy, which

gives both conductivity and strength equal to a continuous copper conductor.

The radial-type brush rigging used on synchronous converters for railway service is constructed so as to give ample spacing between brush holders and to eliminate overhanging parts. Exposed surfaces where an arc might form are protected by arc-resistant insulation. Each group of brushes is completely enclosed by covers and insulation so as to prevent the burning of springs or other parts. The attachment for supporting the

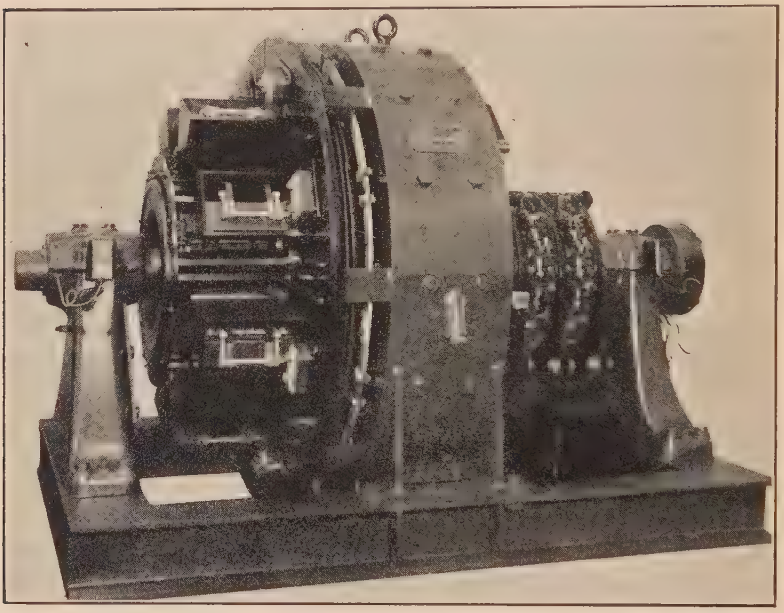


FIG. 27.16.—Commutating-pole, synchronous converter. (*Gen. Elec. Co.*)

brushes is at a radial point near the inner end of the commutator where an arc is least likely to form.

Flash barriers are used to give additional protection from flashing that might be caused by either alternating- or direct-current disturbances. Each barrier consists of several members or scoops, each of which is made up of a rectangular-shaped board of fireproof insulating material set at an acute angle to the commutator surface and parallel to the commutator segments. This device deflects and cools the series of arcs formed between the segments on the commutator when flashing occurs and thus prevents the arc being carried to the stud of opposite polarity.

On account of the iron cores of the commutating poles, the sparking at the direct-current brushes is severe when starting the converter from the alternating-current side. In order to eliminate this sparking, the machines are provided with a device for raising all but two of the direct-current brushes. These two pilot brushes are used for obtaining the field current for proper polarity and for reversing the field if necessary.

All converters for railway service are equipped with a *speed-limit switch* and a *mechanical end-play device.* The *speed-limiting mechanism* is attached to the armature shaft and its operation automatically opens the direct-current circuit-breaker when the speed of the machine exceeds a predetermined value. Excessive speed might occur when the alternating-current power is off, and energy flowing back to the converter through the direct-current feeders or from a storage battery causes the machine to run as a differential motor. The speed-limiting device consists of a switch operated by a centrifugal governor so designed that it operates at practically the same speed irrespective of the rate of acceleration. Large synchronous converters, Fig. 27.16, are supplied with a four-pole, double-throw *field break-up switch* for the shunt field. The service rendered by this switch is twofold. First, it opens the field in several places in order to prevent excessive voltage stress on the insulation, which would be induced in the shunt field windings when starting from the alternating-current side; and, second, it provides a means for obtaining proper polarity. The switch is normally in the up position. When the switch is in the down position the direction of the field is opposite to

Table XIV.—Rotary Converters, Six-phase, 600 Volts, Direct Current

	Rated full load	f	Speed	Iron loss	Friction and windage	Shunt field loss	Series and commutating field loss	Armature resistance from direct-current end
	Kilowatts	Cycles	R.p.m.	Kilowatts	Kilowatts	Kilowatts	Kilowatts	Ohms
1	500	25	750	3.6	5.0	0.5	3.3	0.0180
2	500	60	1,200	8.4	5.3	1.3	3.2	0.0140
3	1,000	25	750	5.7	7.5	1.0	5.0	0.0098
4	1,000	60	900	9.8	12.0	2.5	7.6	0.0068
5	2,000	25	375	13.0	12.0	2.0	9.0	0.0041
6	2,000	60	600	17.5	17.5	5.6	13.0	0.0034
7	3,000	25	250	19.0	20.0	3.0	14.0	0.0032
8	3,000	60	400	22.0	35.0	6.5	17.5	0.0024

that induced by the alternating current which causes the armature to "slip," thus reversing the polarity. As soon as the polarity has been corrected, the switch is thrown to the operating position.

When the load is variable, as in a railway system, *compounding of synchronous converters* is necessary in order to maintain constant direct-current voltage. Since overexcitation of the fields causes a low power factor, overcompounding is undesirable.

The transformers used with synchronous converters in railway service have a high inherent reactance produced by a magnetic bridge between the primary and secondary windings. Compensation for the voltage drop in the transformers and converter is automatically produced by the series field, provided the shunt field is properly adjusted. With the adjustment of the shunt field rheostat so that unity power factor is obtained at some point (depending on the load factor of the machine) between 60 and 100 per cent full load, the converter will take lagging currents at all lower loads, or leading currents at all loads above the point of adjustment for unity power factor, and give approximately constant direct-current voltage.

(*f*) **Starting Rotary Converters.**—Rotary converters may be started, either from the alternating-current side as synchronous motors, or from the direct-current side as shunt motors.

If a compound-wound rotary converter is started as a direct-current motor the series field must be short-circuited during the starting period to prevent weakening or even reversal of the field flux, as the compound winding acts differentially to the shunt winding when the machine is running as a direct-current motor. The rotary slip rings are usually directly connected to the transformer secondary windings, as the switching is generally done on the high-tension side. Under this condition, the transformer secondary windings form a partial short-circuit on the armature, which greatly increases the starting current and which may cause complications that may make it difficult to start the rotary converter as a shunt motor. Rotary converters, started from the alternating-current side, operate as induction motors during the starting period, in much the same manner as explained for synchronous motors. The field break-up switch opens the shunt field and reduced alternating-current voltage is applied to the armature by means of a starting compensator.

For converters with interpoles the brushes must be lifted during the starting period by some form of brush-lifting device to prevent excessive sparking. One narrow brush on each stud is left on the commutator to provide field excitation for the shunt field. The same precautions must be taken on the polarity of rotary converters, when started as induction motors, as for synchronous motors.

(*g*) **Hunting of Rotary Converters.**—Rotary converters are subject to hunting for the same reason as synchronous motors, but, as the armature "floats" on the line without mechanical connection to either the prime mover or the load, the oscillations, unless effectively damped, become greater and the undesirable effects more pronounced. The damping winding must therefore be designed with lower resistance so as to stop more effectively hunting in rotary converters than is necessary in synchronous motors. When hunting takes place the rotor oscillates, so as to be alternately ahead and behind its mean position. This oscillation produces variations in the alternating-current power component, without an equivalent change in the direct-current output. The pulsations in the energy input are balanced by changes in the kinetic energy of the armature accompanied by a fluctuating-armature reaction, which causes field distortions and undesirable induced voltages. For machines having narrow commutating zones even moderate hunting may cause vicious sparking, while excessive hunting leads to flash-over. The effect produced on commutation by hunting is essentially the same as if the brushes were moved rapidly backward and forward about the natural operating position. The remedy for hunting of rotary converters is the same as for synchronous motors and lies largely in design. Naturally, any source of periodic impulses that produces hunting should be removed, if possible.

(*h*) **The Inverted Converter.**—Rotary converters are generally used for changing alternating currents into direct currents. In special cases the process is reversed; that is, the same machine is used for converting direct currents into alternating currents. This change in process affects the operating characteristics of the rotary. The inverted converter operates primarily as either a shunt or compound-wound, direct-current motor. Hence the speed and, as a consequence, the frequency of the alternating-current output depend chiefly on the strength of the field excitation. The armature reaction produced by an inductive load weakens the

field, and thus increases the speed, which in turn increases the frequency. The increase in frequency makes the load more reactive and this again causes a still greater lagging armature reaction. That is, the effect is cumulative and the inverted converter has inherently a tendency to race. All inverted converters must therefore be provided with an effective speed-limiting device.

If two or more rotary converters are operated in parallel, particularly on storage-battery load, one or more of the machines may reverse and start racing as an inverted rotary. To prevent increase in speed above a predetermined maximum, it is customary to provide speed-limiting devices on all rotary converters. Other characteristics as heating, efficiency, and rated output are essentially the same for the inverted converter as for the more generally used rotary converter.

PROBLEMS

1.16. From the data in Table XIV calculate the efficiency for 25, 50, 75, 100, 125 and 150 per cent full load. Assume the load losses at full load to be equal to 50 per cent of the iron losses. Neglect losses in rheostat. Field excitation in all cases adjusted for unity power factor.

2.16. For 600 volts on the direct-current end what are the theoretical alternating voltages between adjacent slip rings for two-, three-, four and six-ring converters? Neglecting losses, what alternating-line currents would give 100 amp. on the direct-current end for the above?

3.16. For a six-ring rotary, problem 2.16, what is the voltage on the transformer secondary terminals in each case?

(a) Diametrical connection. $600(.707)\frac{1}{2}$

(b) Double-delta connection.

(c) Double-star connection. Same as (a)

(d) Ring connection. Same as (a)

(e) Double-T, two-phase connection. Same as b

(f) Double-T, three-phase connection.

(g) Interconnected star (using three rings).

CHAPTER XVII

COMMUTATOR MOTORS

Simplicity is the notable feature of constant-speed, alternating-current machines, when compared to the corresponding direct-current apparatus—simplicity in design, in construction and in operation. Alternators, transformers, synchronous and induction motors cost less to manufacture, have lower operating and maintenance expense and a lower depreciation than the corresponding direct-current machines. While the transformer is the foundation of the alternating-current system, the alternator, the squirrel-cage induction motor and the synchronous motor comply almost perfectly with the requirements in their respective fields. For constant-speed service the alternating-current machinery offers many advantages.

For variable-speed loads, however, the reverse is true. Direct-current motors can operate efficiently under a wide range in speed, while the corresponding alternating-current motors are complicated and comparatively inefficient. To provide a strong starting torque the polyphase induction motor needs a wound rotor with an adjustable resistance; and if operated continuously at half synchronous speed its efficiency would be less than 50 per cent. The single-phase induction motor has inherently no starting torque and hence the devices necessary to bring it up to speed introduce complications, increase first cost and maintenance and lower the efficiency.

In electric railway service the speed variation is an important feature and the starting torque is very large. The requirements of street railway loads can be complied with, in a highly satisfactory manner, by the direct-current series motor. For interurban and trunk lines the problem of power transmission may make it more desirable to use a single-phase system, although a comparison of the motors alone is decidedly in favor of the direct-current machine.

(*a*) **The Straight Series Motor.**—The basic principle of the single-phase series motor is the same as that of the direct-current

series motor; and the simplest method for explaining the characteristics of the former is by direct comparison with the known characteristics of the latter. The field and armature windings are connected in series and the circuit diagram, Fig. 1.17, is the same as for a direct-current series motor. Since the field flux and armature current reverse direction simultaneously, the torque continues in the same direction for successive half cycles. Superimposed upon the series-motor principle is the transformer action inherent in all alternating-current apparatus. A direct-current series motor could be operated as an alternating-current motor if it were not for excessive heating produced by the eddy currents and hysteresis losses in the poles, yoke and armature, and by the induced currents in the short-circuited armature coils, and for destructive sparking at the commutator.

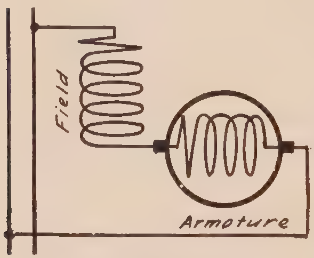


Fig. 1.17.

To reduce the eddy currents to a minimum, the entire magnetic circuit of the alternating-current series motor is laminated; and the iron used must have low hysteresis loss. The alternating field flux necessitates other changes in the design, as the transformer action is the source of four undesirable features in the commercial operation of the motor, in comparison to the corresponding direct-current series motor, namely:

1. The commutator troubles are increased.
2. The armature heating is greater.
3. The starting torque is weaker.
4. The power factor is always less than unity, and very low during the starting period.

The brushes cover two or more commutator bars and the corresponding armature coils are short-circuited in the same manner as for direct-current series motors. In the alternating-current motor these coils form a secondary circuit interlinked with the alternating field flux. The resulting induced voltage causes heavy currents to flow in the short-circuited coils, which increase the heating and produce excessive sparking. To improve commutation, the voltage induced per coil must be made as small as possible; or the product of the number of turns in the short-circuited coil and the interlinking alternating flux must be

sufficiently small to give satisfactory commutation. This is obtained by several modifications in the design as compared to the standard direct-current motor.

1. By reducing the number of turns per coil. Except for very small motors the least possible number is used, or one turn per coil.

2. By using more poles, hence more paths, and therefore less flux per coil. This increase is limited by other features of the design; but in alternating-current railway motors six poles are usually used instead of four in the direct-current motor.

3. By increasing the number of commutator segments.

4. By reducing the magnetic flux density.

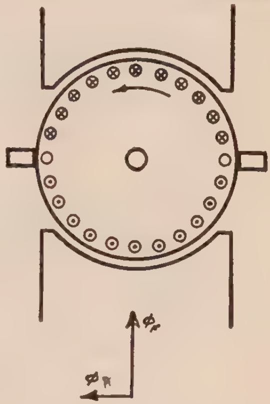


FIG. 2.17.

To compensate for the weaker field and fewer turns the armature current must be increased in order to have the same torque, and this, in turn, necessitates a larger armature diameter and a longer commutator. The low field densities require only a few field turns and hence only a small space for the field coils. Alternating-current series motors have, as a result, short stubby field poles of larger cross-section than the corresponding direct-current motors. The few field turns and large cross-section give a low reluctance in the magnetic circuit. For the same total field flux the inductance is directly proportional to the number of turns. In order to have a fairly good power factor, the reactive power component must be small and hence the inductance in the circuit must be kept as low as possible. The relative value of the field and armature ampere-turns has an important bearing on the commutation of the motor. In a direct-current series motor the total flux in the armature is the resultant of the main-field flux and the cross-flux produced by the armature current. Illustrating by a two-pole motor as in Fig. 2.17, the *main-field* may be represented by the vector ϕ_F, and the armature *cross-field* by ϕ_A. The conductors short-circuited by the brushes lie in the cross-field and hence, when the armature rotates, the flux ϕ_A induces a voltage which, in turn, produces a current in the short-circuited turns. Consequently, if the armature ampere-turns

are relatively large, the cross-field ϕ_A is strong and the induced voltage and current in the short-circuit turns may create serious difficulties in commutation. Therefore, in order to secure satisfactory commutation, ϕ_F is made stronger than ϕ_A, especially in direct-current motors without commutating poles. In motors supplied with commutating poles the ratio may approach unity although the distortion of the resultant field is undesirable as it causes larger core losses and higher maximum voltages between the commutator segments. While a strong field is desirable in the direct-current series motor, it is not permissible in the alternating-current series motor, as already explained, and the strength of the cross-field may be much larger than the main field, as indicated in Fig. 3.17.

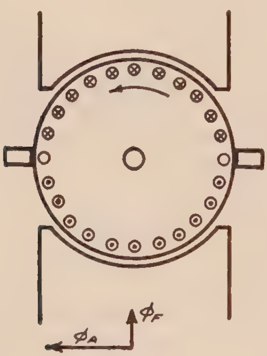


FIG. 3.17.

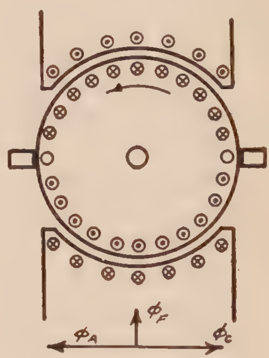


FIG. 4.17.

(*b*) **Compensated Straight Series Motor.**—In order to prevent destructive sparking on the commutator the cross-field is neutralized by means of an auxiliary or compensating winding, placed in slots in the pole faces as indicated in Fig. 4.17. The compensating winding, in the straight series motor, is connected in series with the armature and has the same number of turns, Fig. 5.17. The currents in the armature under the field pole and in the parallel conductors of the compensating windings flow in opposite directions, and hence the magnetic fluxes in the two windings are of equal strength but opposite in direction, as indicated in Fig. 4.17. Compensation for the conductors under the brushes and the open space between the field poles may be secured by fractional pitch windings or by the use of commutating poles.

Power Factor.—The compensating winding improves the power factor as well as the commutation. In Fig. 6.17 is shown the vector diagram of a plain alternating-current series motor, and in Fig. 7.17 the corresponding vector diagram for a single-phase motor having a compensating winding but omitting the

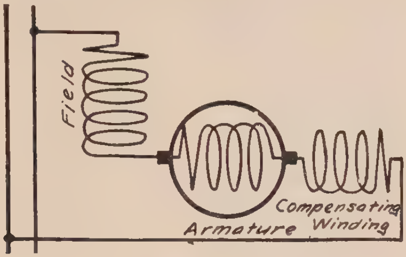


FIG. 5.17.

currents induced in the coils short-circuited by the brushes. The current is represented by a vector I along the X-axis and the respective volts consumed by the real and reactive power components in the circuit are shown in both magnitude and phase position by the voltage vectors.

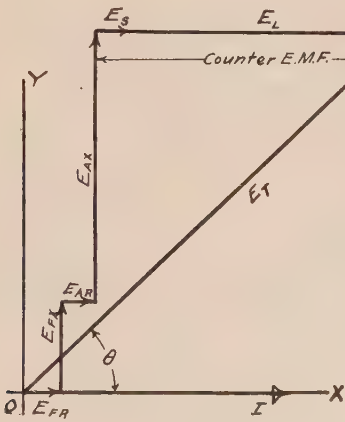


FIG. 6.17.

E_{FR} = voltage for field resistance.
E_{FX} = voltage for field reactance.
E_{AR} = voltage for armature resistance.
E_{AX} = voltage for armature reactance.
E_S = voltage for speed loss (friction, windage and iron).

E_L = voltage for output or mechanical load.
$E_S + E_L$ = the counter e.m.f.
E_{CR} = voltage for resistance of compensating winding.
E_{CX} = voltage for reactance of compensating winding.
E_T = total impressed voltage.
$E_{AX} - E_{CX}$ = voltage for leakage between armature and compensating windings.

For straight series motor, Fig. 6.17:

$$\dot{E}_T = \dot{E}_{FR} + \dot{E}_{AR} + \dot{E}_S + \dot{E}_L + \dot{E}_{FX} + \dot{E}_{AX} \tag{1.17}$$

$$E_T = \sqrt{(E_{FR} + E_{AR} + E_S + E_L)^2 + (E_{FX} + E_{AX})^2} \tag{2.17}$$

A. series motor
Conductively comp.-motor
Inductively comp. "
BA
SCR motor (2 windings) Rep
K motor
-Weichel

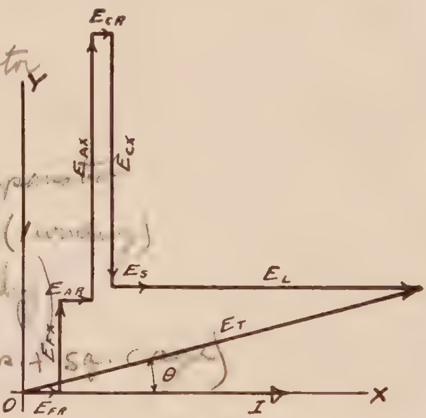


FIG. 7.17.

For compensated straight series motor, Fig. 7.17:

$$\dot{E}_T = \dot{E}_{FR} + \dot{E}_{AR} + \dot{E}_{CR} + \dot{E}_S + \dot{E}_L + \dot{E}_{FX} + \dot{E}_{AX} + \dot{E}_{CX} \tag{3.17}$$

$$E_T = \sqrt{(E_{FR} + E_{AR} + E_{CR} + E_S + E_L)^2 + (E_{FX} + E_{AX} - E_{CX})^2} \tag{4.17}$$

It is readily seen from the vector diagrams that the power factor is greatly improved by use of the compensating winding. In commercial designs the power factor without the compensating winding is about 35 to 40 per cent, while with a compensating winding it may reach 90 per cent. While the inductive reactance due to the main field cannot be eliminated, it may be reduced to a comparatively small value, by providing a mag-

netic circuit of low reluctance. This is, in part, gained by the larger cross-section of fields and armature necessary for obtaining good commutation, as already explained, but *to a larger degree by using a smaller air gap than in the direct-current motor.*

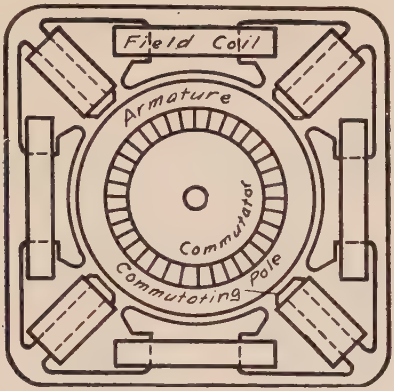


FIG. 8.17.—Direct-current series motor.

Comparative Size.—In Figs. 8.17 and 9.17 is shown an approximate comparison between a 600-volt, direct-current, railway

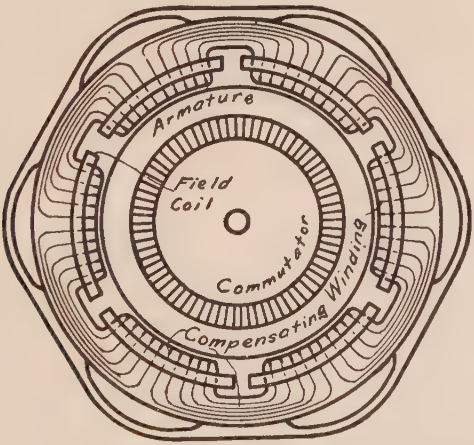


FIG. 9.17.—Alternating-current series motor.

motor and the corresponding 25-cycle, alternating-current compensated motor. The dimensions along the shaft are approximately the same. In general, the 25-cycle motor is from 25 to 45 per cent, and a 15-cycle motor from 15 to 30 per cent, heavier

than the corresponding direct-current motor. For higher direct-current voltages the comparison is more favorable to the single-phase motor.

Resistance Leads.—As already stated, the alternating field flux induces heavy currents in the armature coils when short-circuited by the brushes. These currents produce two very undesirable effects:

1. Destructive sparking on the commutator.
2. Excessive losses in the armature.

By inserting *resistance leads* between the commutator segments and the armature coils the commutation is improved

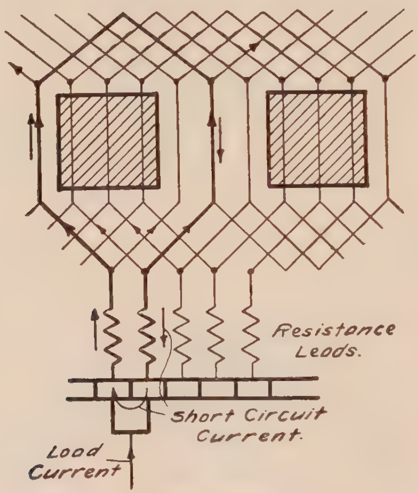


Fig. 10.17.

and the heat losses are reduced. From Fig. 10.17 it should be noted that two resistance leads are in series in the short-circuit carrying the induced currents, while the same two resistance leads are in parallel in the load-current circuit. By this means the short-circuit currents and the accompanying commutator sparking are reduced until satisfactory commutation is secured. With the decrease of the short-circuit current the RI^2 losses in the armature are also reduced, while the resistance leads add a new heat-loss factor to the circuit. In well-designed motors the sum of the losses in both the resistance leads and the armature winding is less than in the armature alone, if the resistance leads were removed. The resistance leads also improve the starting torque and provide a certain protection against higher harmonics

and other disturbing influences. The chief objection to the resistance leads is the increased complication in the design. The armature must needs be compact and the introduction of a resistance section between the commutator and the active portion of the armature winding under the field flux is a serious handicap to the designer. In motors operating on low frequencies, 15 cycles or less, satisfactory commutation can be secured without the use of resistance leads. At 25 cycles, the standard fre-

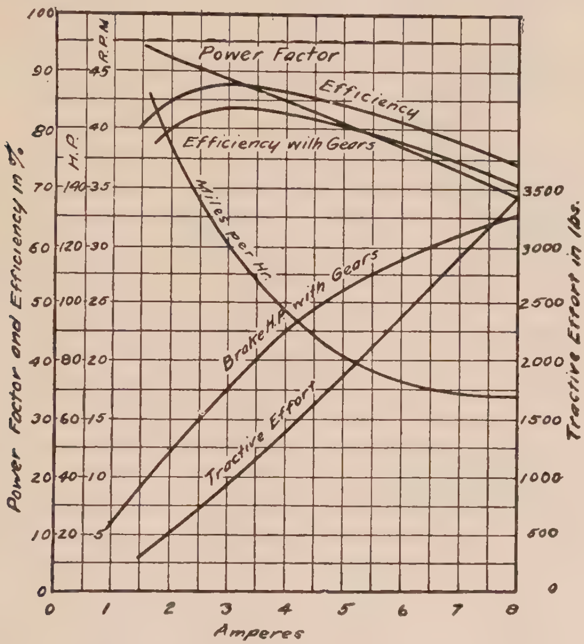


FIG. 11.17.—Characteristic curves of 100-hp., single-phase, series, compensated, railway motor. 245 volts, 25 cycles. (*Westinghouse Electric and Manufacturing Company.*)

quency for electric railways, most motors are supplied with resistance leads, although some designers consider their use an unnecessary complication. For circuits having 40 or more cycles per sec., resistance leads are necessary for successful operation.

Summarizing the more important features in the design of the compensated straight series motor as compared to the corresponding direct-current series motor we have:

1. Both field and armature cores are laminated.
2. Larger armature diameter.

3. Field poles are short and stubby; more in number and of greater cross-section.

4. Commutator longer, of larger diameter and of greater number of segments.

5. Small air gap.

6. Compensating winding.

7. Resistance leads.

Typical characteristic curves of a compensated series, alternating-current, railway motor are shown in Fig. 11.17. It should be noted that this motor will operate in a highly satisfactory manner on direct-current circuits. It is, in fact, a better but more expensive machine than the ordinary direct-current, series motor.

Notable improvements have lately been made in the design of single-phase, series motors for trunk railway service.[1] The advance in design, produced by more effective proportioning and arrangement of the essential parts, and resulting in better power-factor, efficiency, commutation and speed-torque characteristics, is based on many years of practical experience in alternating-current railway electrification.

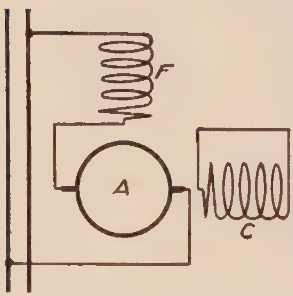


Fig. 12.17.

(*c*) **Inductively Compensated Series Motor. Primary Excitation.**—In the discussion of the induction motor it was shown that, aside from delivering mechanical power, the motor operated like a transformer. Similarly in a series motor, the alternating fields induce voltages in the several circuits in the same manner as in the stationary transformer. With the compensating winding placed in grooves, parallel to the armature conductors, currents for neutralizing the armature cross-field may be obtained by short-circuiting the compensating winding as shown in Fig. 12.17. The inductively compensated series motor possesses practically the same operating characteristics on alternating-current circuits as the straight series motor.

(*d*) **Inductively Compensated Series Motor, Secondary Excitation.**—By connecting the field in series with the compensated

[1] Jungk, H. G., "Design of Single-phase Series Motors," *Proc. Am. Inst. Elec. Eng.*, Vol. 49, p. 1013.

winding as in Fig. 13.17 the motor field is excited by the secondary circuit. The armature only is in the primary connected to the supply mains, while the voltage induced in the compensating coil sends current through both the main field and compensating windings. The current in the armature leads the main field flux on account of the large inductive reactance in the secondary, and the power factor of the motor is very low. Obviously, the power factor may be improved by inserting a shunt across the field as indicated in Fig. 14.17, but this arrangement

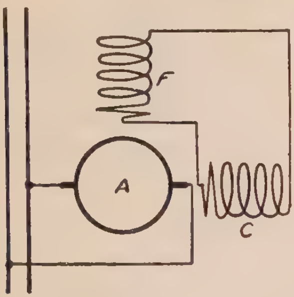


Fig. 13.17.

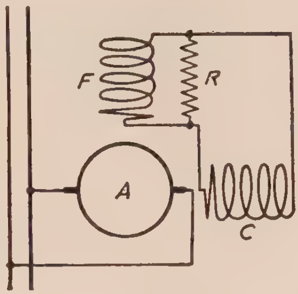


Fig. 14.17.

also causes a marked loss in efficiency. The time phase of the armature current with respect to the main field produces a commutating field, which at synchronous speed is in magnitude and phase position of the required value to give good commutation. This motor cannot be operated on direct-current circuits and its commercial use seems to be limited to certain types of induction meters.

(*e*) **The Repulsion Motor.**—The main field or stator is connected to the supply mains while the wound armature is short-circuited through brushes placed 180 electrical degrees apart, as indicated by the circuit diagrams in Figs. 15.17 and 16.17; a compensating winding may be connected in series with either the stator or armature winding. The repulsion motor is fundamentally a transformer, with the primary winding and core stationary. The secondary is like an ordinary direct-current armature, in which the armature conductors and core rotate, while the short-circuited brushes are stationary. Hence, while the conductors in the secondary winding rotate, the secondary circuit remains in a definite position with respect to the stator. In the repulsion motor as in the single-phase

induction motor, there are transformer and speed fields, whose reactions with the armature currents produce the desired torque.

At standstill the speed field is zero and the starting torque is produced by the reactions between the induced armature current and the stator field. The magnitude of the starting torque

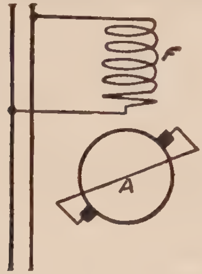


FIG. 15.17.—Thomson repulsion motor.

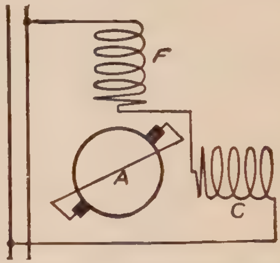


FIG. 16.17.—Compensated repulsion motor.

depends upon the relative position of the armature brushes and the direction of the stator field. If the brushes are in line or at right angles to the direction of the stator field flux, no torque is produced, as may be seen by inspection of Figs. 17.17 and 18.17. The magnetic poles are along the Y-axis and the maximum trans-

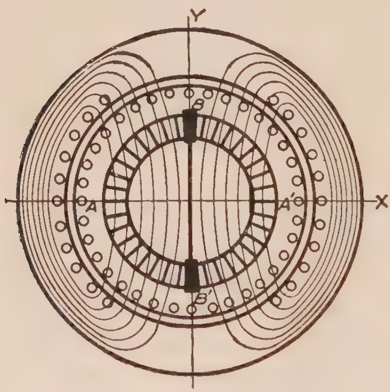


FIG. 17.17.

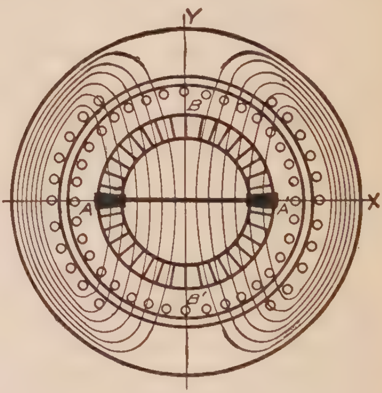


FIG. 18.17.

former action occurs at AA' along the X-axis. Any reaction between the field and the induced armature current in sector AB, Fig. 17.17, is at all instants balanced by the equal and opposite torque in the BA' sector. Similarly, the torque in AB' is at all instants equal and opposite to the reaction in the $B'A'$

sector. The voltage induced from A to B, Fig. 18.17, is neutralized by the induced voltage from B to A'. Similarly, AB' is neutralized by $B'A'$. Hence there is no difference in voltage between the brushes, Fig. 18.17, no armature current flows and therefore there is no torque.

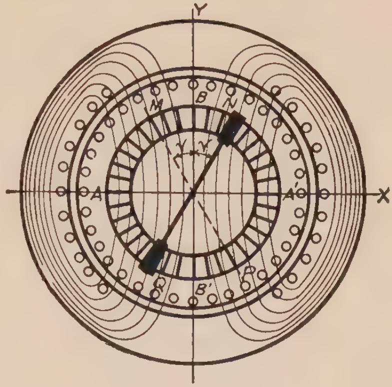


Fig. 19.17.

Let the short-circuited brushes be displaced by an angle γ from the axis of the stator field, as in Fig. 19.17, with the line NQ and the symmetrically placed line MP 2γ apart. Comparing Fig. 19.17 with Figs. 17.17 and 18.17 it is seen that the conditions for the conductors in the areas $NA'P$ and MAQ in Fig. 19.17 are the same as for all the conductors in Fig. 17.17, while the

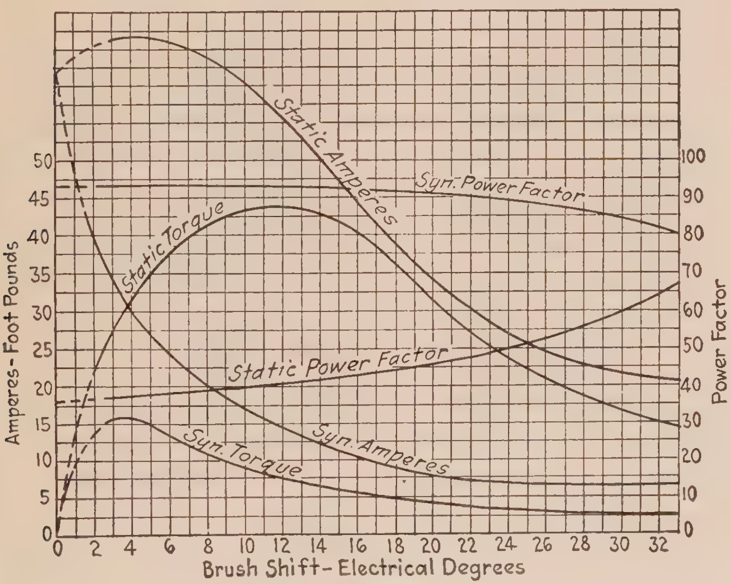


Fig. 20.17.

conductors lying in the areas MBN and $PB'Q$ have the same conditions as all the conductors in Fig. 18.17. Hence in the arcs

NP and MQ the transformer action of the primary field induces voltages that cause currents to flow through the armature and the short-circuited brushes. No torque is, however, produced as the reaction in the arc NA' is balanced by the equal and opposite action in PA', and similarly for MA and QA. In the arcs MBN and $PB'Q$ the resultant transformer action is zero, since the voltage induced in arc MB is neutralized by the equal and opposite voltage induced in BN; likewise QB' is balanced by $B'P$. However, the currents flowing in the arcs MBN and $QB'P$, Fig. 19.17, produced by the transformer action in MAQ and $NA'P$ react upon the stator field and produce a starting torque. In the conductors between MB and BN the currents flow in the same direction and react upon the same field, while on the opposite side between Q and P the relative directions of both the current and field flux are reversed. Hence the starting torque produced at B' is in the same direction as at B and both tend to cause rotation in the same direction. For successive half waves of the impressed voltage both the primary field and the induced armature currents reverse in direction, hence producing a torque continuously in the same direction.

Experimental data showing the effect of brush shift are given by the curves in Fig. 20.17. Three of the curves show the variation of torque, amperes and power factor as functions of brush position under static or locked-rotor conditions. The other three curves are plotted for the synchronous speed, 1,800 r.p.m.

In analyzing the reactions between the armature and the field it is simpler to let the line joining the brushes be the reference axis and to separate the primary field into components in phase and in quadrature with the armature winding, as shown in Fig. 21.17. The component ${}_t\phi_p$ of the primary field ϕ_p in line with the brushes is the transformer field which induces voltages that cause currents to flow in the armature. The quadrature component ${}_f\phi_p$ is the main field; the reactions between the armature currents and the main field ${}_f\phi_p$ produce the starting torque.

$$ {}_t\phi_p = \phi_p \cos \gamma \tag{5.17} $$

$$ {}_f\phi_p = \phi_p \sin \gamma \tag{6.17} $$

$$ \frac{{}_t\phi_p}{{}_f\phi_p} = \cot \gamma \tag{7.17} $$

The induced armature currents also produce a field ${}_t\phi_a$ in opposition to the primary transformer flux ${}_t\phi_p$. If the trans-

former action were perfect with both resistance and leakage reactance zero, that is, neglecting all losses, then ${}_t\phi_a = {}_t\phi_p$ and the total flux in the motor would be ${}_f\phi_p$. Hence for an *ideal* repulsion motor at standstill the total voltage impressed on the primary circuit is balanced by the voltage induced by the flux ${}_t\phi_p$ in the stator. In the actual motor the armature resistance, leakage reactance and iron losses absorb part of the impressed voltage and the flux ${}_t\phi_p$ is correspondingly larger than ${}_t\phi_a$. The armature current is in space phase and practically in time phase with the flux ${}_f\phi_p$, and the reaction produces a strong starting torque.

When the armature rotates, a speed field is produced in the same manner as explained for the single-phase induction motor in Chap. XIII. Neglecting losses and assuming an ideal motor, the speed field is in space quadrature to the primary field as in Fig. 22.17. Under the assumed conditions the speed field must produce a counter-e.m.f. that just balances the armature speed

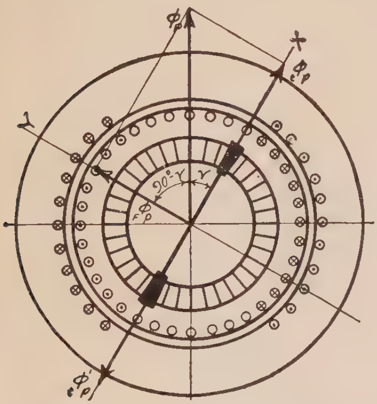


Fig. 21.17.

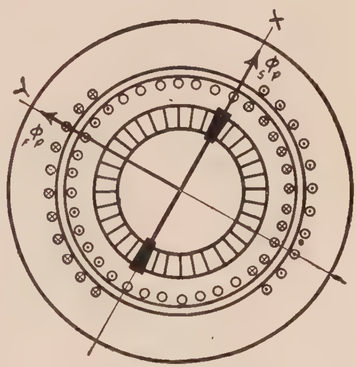


Fig. 22.17.

voltage. At synchronous speed S the speed field is equal to the primary field. At any other speed s the speed field is:

$$ {}_s\phi_p = \frac{s}{S}\,{}_f\phi_p \tag{8.17} $$

The speed voltage is in time phase with the flux ${}_f\phi_p$ and hence in time quadrature with ${}_s\phi_p$. Simultaneous values are therefore expressed by equations (9.17) and (10.17).

$$ {}_f\phi_p = {}^M_f\phi_p \cos \omega t \tag{9.17} $$

$$ {}_s\phi_p = \frac{s}{S}\,{}^M_f\phi_p \sin \omega t \tag{10.17} $$

The total field:

$$\phi_{total} = {}_f\phi_p + j{}_s\phi_p \qquad (11.17)$$

The voltage impressed upon the motor must be proportional to the corresponding fluxes, and consists of two components in time quadrature; one, ${}_fE_p$, for ${}_f\phi_p$, and the other, ${}_sE_p$, for ${}_s\phi_p$.

$$\dot{E}_o = {}_f\dot{E}_p + {}_s\dot{E}_p;\ \text{or}\ E_o = \sqrt{{}_fE_p^2 + {}_sE_p^2} \qquad (12.17)$$

The repulsion motor has the characteristics of a straight series motor. To a limited extent it has been used for commercial work on loads requiring series motors. It is widely used as a device to give starting torque to single-phase induction motors. A typical starting and operating torque curve is shown in Fig. 23.17 for a motor that starts as a repulsion motor and automatically changes to a squirrel-cage induction motor when the speed approaches synchronism.

(*f*) **Single-phase Constant-speed Motor** (*Wagner Type BA*).—The circuit diagram is shown in Fig. 24.17. The stator has a main inducing winding, while the motor is provided with a commuted winding short-circuited by the permanently interconnected brushes along an axis displaced from the stator winding. A centrifugally operated device short-circuits the commutator segments when the rotor approaches synchronous speed. A device is also provided that automatically lifts the brushes off the commutator as soon as the short-circuiting mechanism has interconnected the commutator segments, and thereby completely short-circuited the rotor winding. The characteristics of the motor are thus changed from series to shunt and it operates at nearly constant and nearly synchronous speed for all loads. In practice, the centrifugal device is located within the rotor at the opposite end from the commutator. Lifting the brushes reduces the wear on the commutator and practically eliminates the noise under normal operation. The motor is usually started and stopped by means of a single double-pole switch, except in some cases. An ordinary direct-current resistance starter is sometimes used with the larger motors.

The starting performance curves are shown in Fig. 24.17 for the machine directly connected to the mains. From the start until the commutator is short-circuited the motor has series characteristics, the torque varying with the speed as shown by the curve *Torque*-2. The corresponding current and power

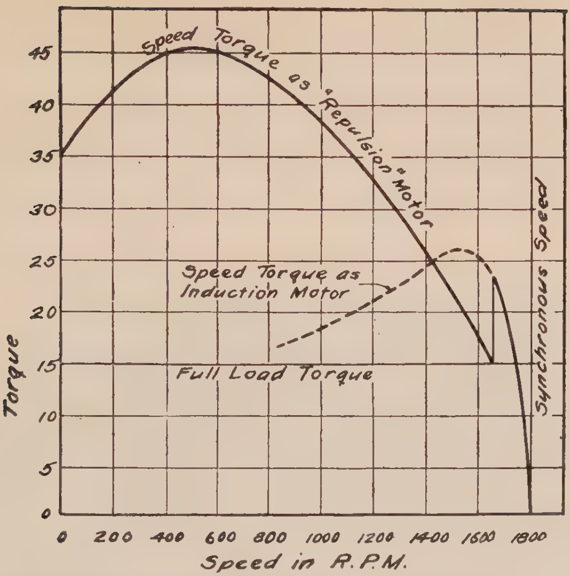


FIG. 23.17.—Speed-torque curves for single-phase induction motor (repulsion starting).

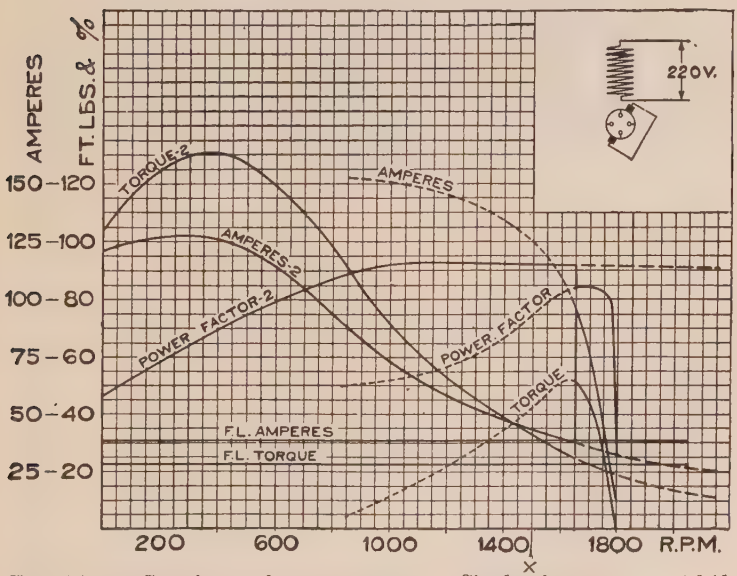


FIG. 24.17.—Starting performance curves. Single-phase motor. 7½ hp., 60 cycles, four poles, 220 volts. Type BA. (*Wagner Electric Corporation.*)

factor values are shown by curves marked *Amperes*-2 and *Power Factor*-2 respectively. After the commutator has been fully short-circuited, the performance of the machine is indicated by the curves marked *Torque, Amperes* and *Power Factor.* The centrifugal device is set so as to operate at the speed marked x in the figure, 1,500 r.p.m. When the short-circuiting of the commutator takes place, the torque, from a value slightly in excess of full-load torque (*Torque*-2 *curve*) suddenly rises to over 50 ft.-lb. (*Torque curve*) and then rapidly drops to whatever

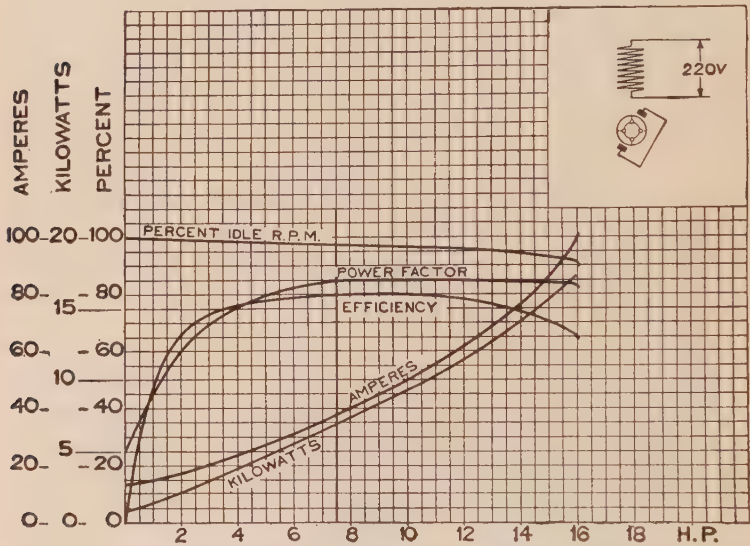


FIG. 25.17.—Characteristic curves. Single-phase motor. 7½ hp., 60 cycles, four poles, 220 volts. Type BA. (*Wagner Electric Corporation.*)

value the load requires. This change takes place in the short time required by the motor to rise in speed from 1,500 to 1,750 r.p.m. Likewise the current rises momentarily from 38 to about 94 amp. and then rapidly falls to the value corresponding to the load on the motor.

The operating characteristics are shown in Fig. 25.17.

(*g*) **Single-phase Repulsion-induction Motor** (*General Electric Company*).—The SCR motor has two rotor windings, one commutated, corresponding to that of the repulsion motor, and the other a grid similar to the squirrel cage of the induction motor.

From the former, which is essentially equivalent to a series motor, the SCR motor obtains a high starting torque and from the latter good speed regulation.

The completed motor, 3-hp. size, is shown in Fig. 26.17 and in disassembled form in Fig. 27.17, while the characteristic curves are given in Fig. 28.17. The combined repulsion-induction characteristics are readily seen in the speed-torque curve, Fig. 28.17.

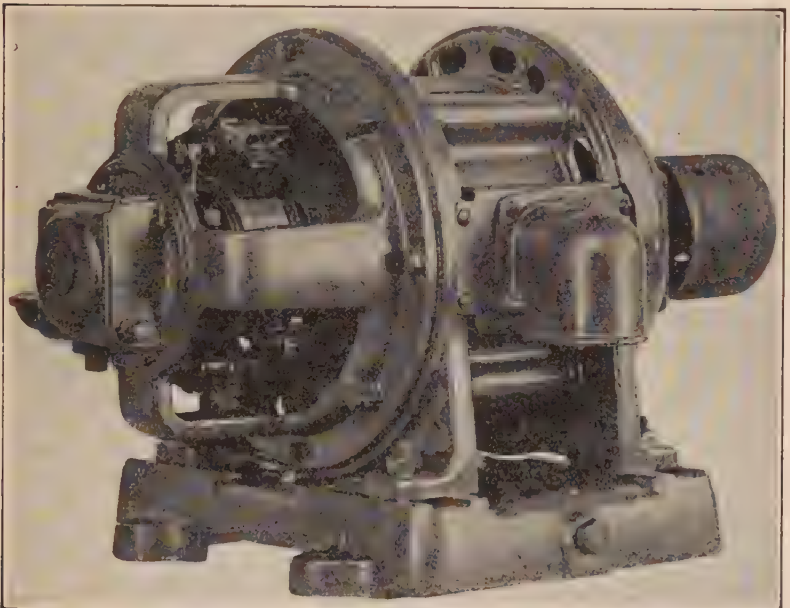


FIG. 26.17.—Single-phase repulsion-induction motor. Type S.C.R. four-pole, 3-hp., 1,800-r.p.m., $^{110}/_{220}$-volt, 60-cycle motor with base, pulley and conduit box. (*General Electric Company.*)

In starting, the full-line voltage is applied to the motor terminals. The starting torque varies from 300 per cent of full-load torque for 3-hp. and smaller sizes, Fig. 28.17, to 175 per cent for 10-hp. and larger motors. The accelerating or pull-up torque, as well as the maximum pull-out torque, is approximately 200 per cent of full-load torque. The running free speed of the SCR motor is slightly above synchronism and the full-load regulation is approximately 6 per cent.

The squirrel cage is placed deep enough below the commutated winding of the rotor to allow a large portion of the main flux of

the motor to pass between the two windings. The slits connecting the slots of the two windings are of such dimensions that under starting conditions most of the field flux that enters the rotor passes across these slits between the two sets of windings.

The currents induced in the squirrel cage at starting are sufficient to prevent all but a small portion of the flux from entering the squirrel cage. Altogether, then, the squirrel cage has only a minor effect on the starting conditions, so that a high

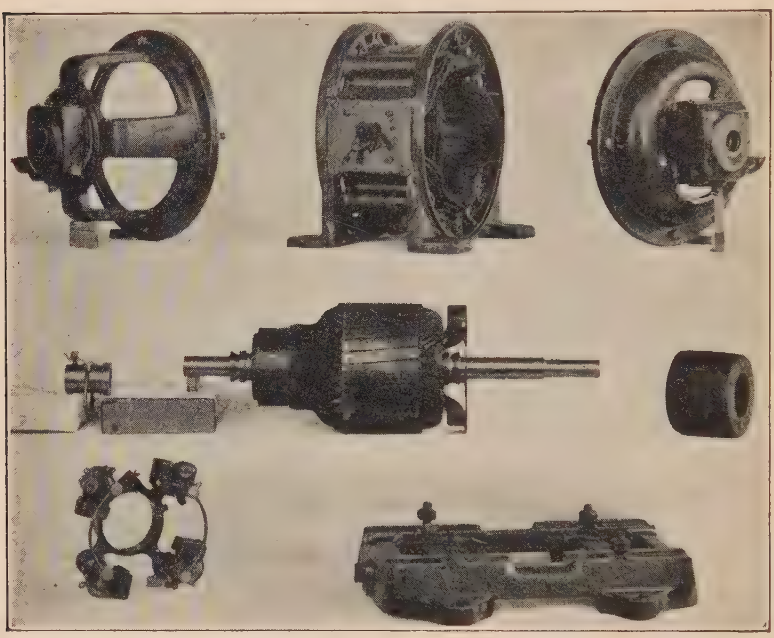


FIG. 27.17.—Disassembled view of a type S.C.R., four-pole, 3-hp., 1,800-r.p.m., 110/220-volt, 60-cycle motor.

starting torque, comparable to that of the plain repulsion motor, is obtained.

When the motor is running in the neighborhood of synchronous speed, the air-gap field is a rotating field, practically the same as in the polyphase induction motor. In a general way, at speeds near synchronism the resultant of the currents in the stator and rotor windings sets up a revolving field which works on the squirrel cage to develop induction-motor torque. The maximum torque thus developed in the squirrel cage in normal operation is approximately twice as great as the maximum

torque that would be developed if the repulsion winding of the rotor were removed.

When the speed is above synchronism, the action of the squirrel cage corresponds to that of a polyphase-induction motor; *i.e.*, it develops generator torque. The no-load speed is that at which this generator torque is exactly equal to the motor torque developed by the repulsion winding currents, and ordinarily is about 2 per cent above synchronism.

The maximum horsepower output is obtained at a speed roughly 10 per cent below synchronism, where the maximum

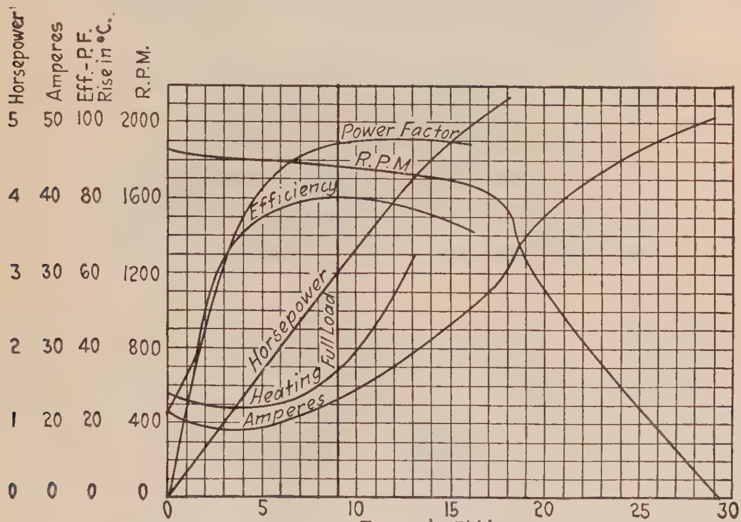


FIG. 28.17.

induction-motor torque is added to the repulsion-motor torque, giving a maximum output of approximately twice full load.

Briefly, the rotating field produced by the repulsion-motor windings works on the squirrel cage to develop induction-motor torque, which holds the speed of the motor close to synchronous speed throughout the entire range of normal operation.

(*h*) **Single-phase Squirrel-cage Motor** (*Wagner Type BK*).—The disadvantages due to the lack of starting torque and low power factor at all loads inherent in the single-phase induction motor have been eliminated in the design of the single-phase, squirrel-cage motor having both commuted and squirrel-cage windings with a double set of brushes spaced 90 electrical

degrees on the commutator. Photographs of an assembled and disassembled motor are shown in Figs. 29.17 and 30.17. The circuit

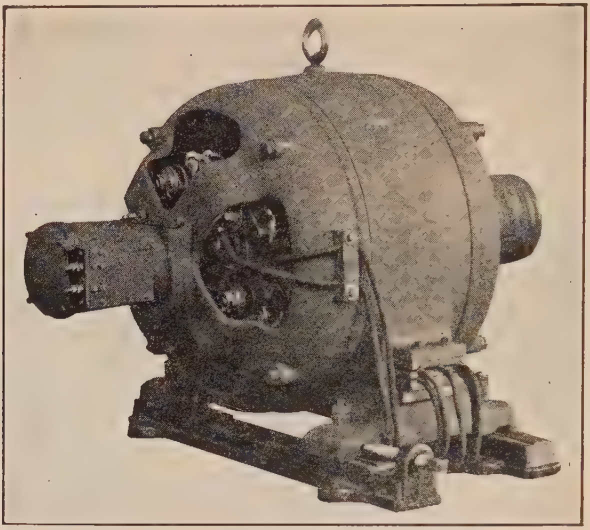


FIG. 29.17.

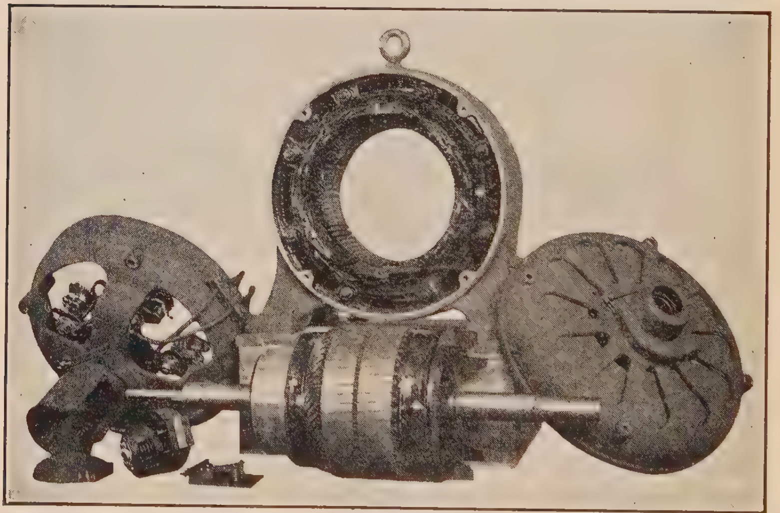


FIG. 30.17.—Single-phase, squirrel-cage motor with commuted winding. Type BK. (*Wagner Electric Corporation.*)

diagram is shown in Fig. 31.17*a* and a section of the iron core showing the slots for the two windings and the intervening magnetic layer in Fig. 31.17*b*.

The motor[1] consists of a rotor carrying a commuted winding 6, located in a number of partly open slots 18, placed near the outer periphery of the rotor laminations 16, as shown in Fig. 31.17, and also a squirrel-cage winding, the bars 11 of which are located in holes 17, provided in the rotor punchings and interconnected at each end by

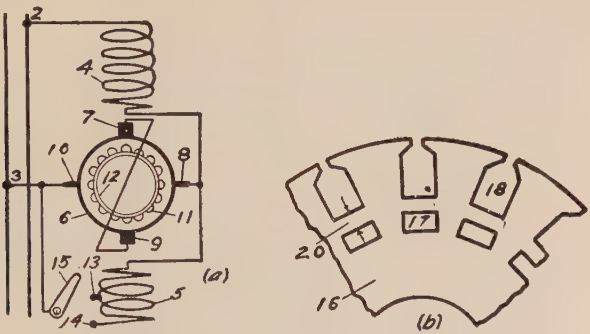


FIG. 31.17.

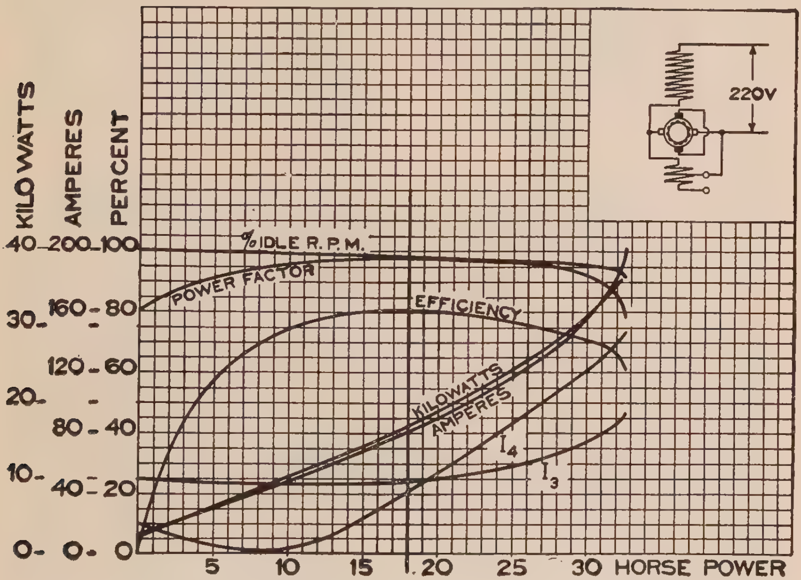


FIG. 32.17.—Characteristic curves. Single-phase motor. 18 hp., 60 cycles, six poles, 220 volts, 1,200 r.p.m. Type BK. (*Wagner Electric Corporation.*)

means of conducting end rings 12. The holes 17 accommodating the squirrel-cage winding are so placed as to be separated by a certain

[1] FYNN, "Single-phase, Squirrel-cage Motors," *Trans. Am. Inst. Elec. Eng.*, Vol. 34, p. 2483.

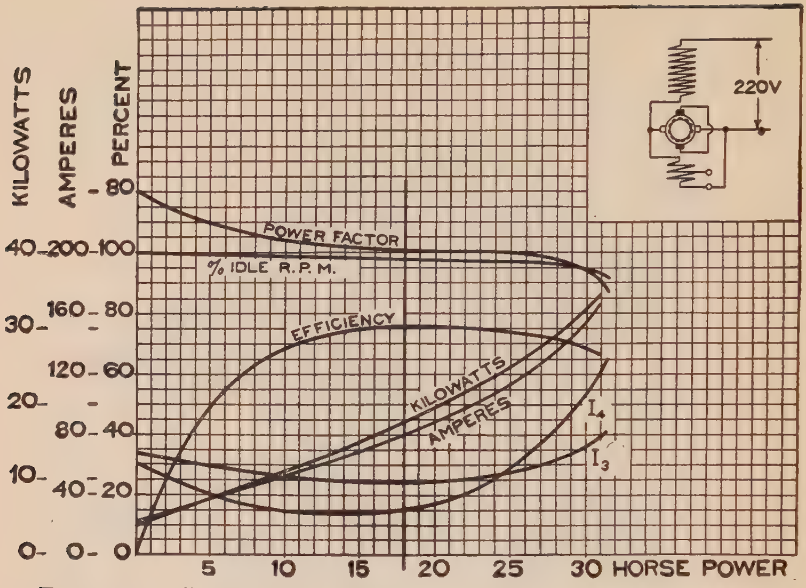


FIG. 33.17.—Characteristic curves. Single-phase motor. 18 hp., 60 cycles, six poles, 220 volts, 1,200 r.p.m. Type BK. (*Wagner Electric Corporation.*)

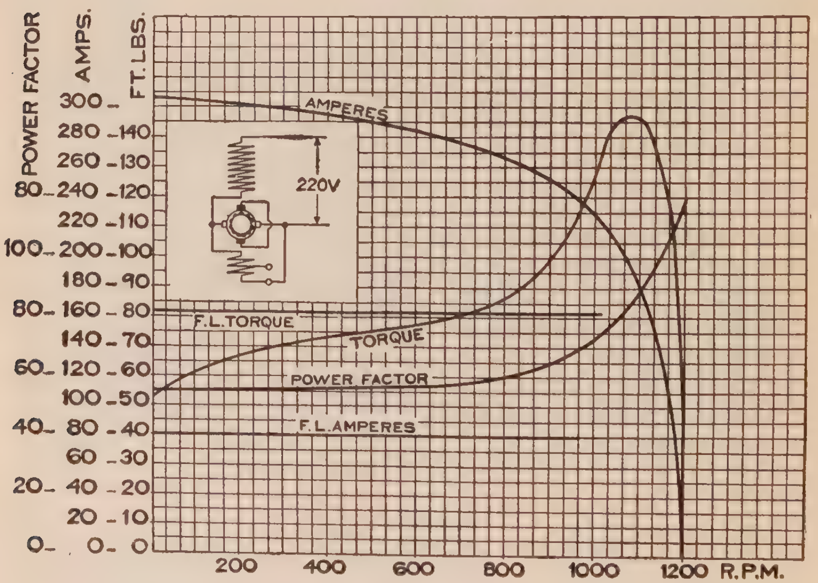


FIG. 34.17.—Single-phase motor. 18 hp., 60 cycles, six poles, 220 volts, 1,200 r.p.m. Type BK. (*Wagner Electric Corporation.*)

amount of magnetic material 20, from the slots 18, carrying the commutated winding. The stator carries a main inducing winding 4, and a coaxial compensating winding 5, usually provided with a tap 13. The main or working brushes 7, 9 are permanently short-circuited and placed in line with the axis or the stator windings. The auxiliary or exciting brushes 8, 10 are displaced by 90 electrical degrees from the short-circuited brushes. At starting, the main stator winding 4 is connected across the mains 2, 3 in series with the commuted rotor winding 6, by way of the exciting brushes, 8, 10. The brushes 7, 9 are short-circuited, and the circuit of the winding 5 is open, this being indicated in Fig. 31.17*a* by showing the switch 15 in its "off" position. After the motor has reached its normal speed, switch 15 is placed on either point 13 or point 14. If connected to the tap in the compensating winding 5, the machine will operate with unity power factor under most loads. If switch 15 connects brush 10 to the end of 14 of the compensating winding 5, then the machine will operate with leading power factor at no load and with unity power factor at full load.

The corresponding characteristic operating curves are shown in Figs. 32.17, 33.17 and 34.17. The curves for the two connections differ mainly in the power factor; for maximum compensation the current leads at starting and at light loads. The efficiency is slightly higher for the normal compensation.

(*i*) **Polyphase Commutator Motors.**—Several types or designs of polyphase commutator-induction motors have been developed to meet special industrial load requirements. The machines are self-exciting and, in fact, as self-contained as the ordinary induction motor.

High starting torque and leading power factor are the outstanding advantages, while higher first cost and larger maintenance charges than for induction motors are the main disadvantages.

The *Fynn-Weichsel* motor,[1] developed by the Wagner Electric Corporation, has a starting performance equal to, and its starting equipment as simple as, that of a standard slip-ring induction motor. This motor has a synchronizing, or pull-in, torque of from 150 to 200 per cent normal running torque; a leading power factor over the whole working range; and requires no external exciter. Its maximum horsepower as a synchronous machine is 150 to 200 per cent of normal, Fig. 37.17, depending on the design of the unit. When loaded beyond this point, the machine ceases

[1] Weichsel, H., "A New Alternating-current General-purpose Motor," *Trans. Am. Inst. Elec. Eng.*, Vol. 44, p. 7.

to function as a synchronous motor, but continues to operate as an induction motor. When the excessive overload is reduced,

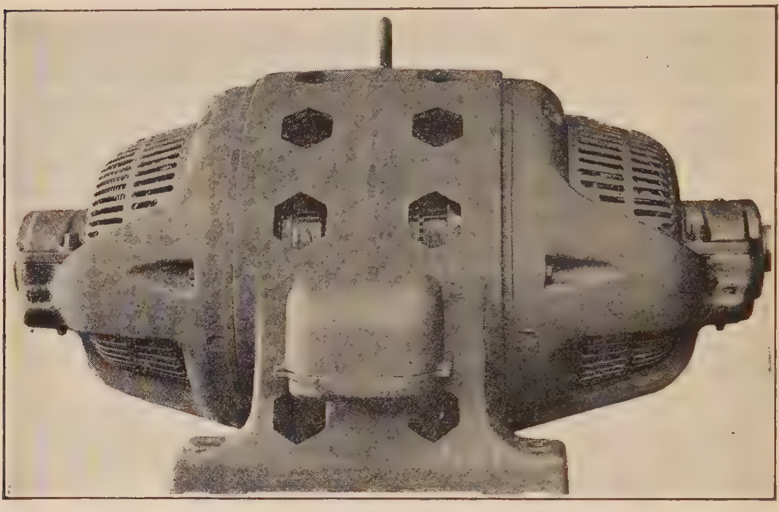


Fig. 35.17.—Fynn-Weichsel three-phase commutator motor. 220 hp., 60 cycle; 440 volt; eight-pole; 900 r.p.m. (*Wagner Electric Corporation.*)

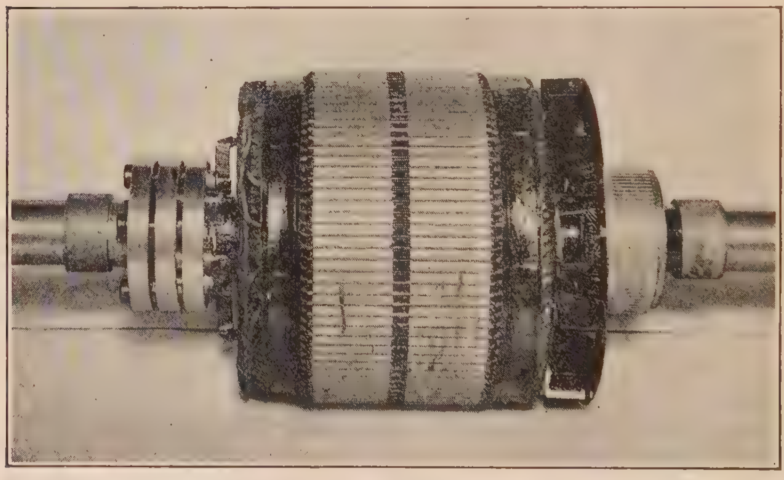


Fig. 36.17.—Armature for the Fynn-Weichsel motor shown in Fig. 35.17.

to about 150 per cent of normal, the machine automatically returns to synchronous speed and operates as a synchronous motor.

A view of an assembled machine of this type is given in Fig. 35.17. The rotor carries the main alternating-current winding, connected to the line by means of slip rings. Also a small direct-current winding is connected to the commutator. The armature for the Fynn-Weichsel motor, Fig. 36.17, has an appearance similar to that of a rotary-converter armature. The stator is built up of punchings similar to those of a standard induction motor, and carries two windings which are displaced 90° from each other. The circuit diagram is shown in Fig. 37.17. During the starting period, variable resistance is inserted in both circuits of the two stator windings, A and F, while the F circuit is also connected to the direct-current armature winding. With sufficient resistance inserted, a good starting torque is produced by a small starting current. When the machine accelerates, the resistances are gradually decreased, until both resistances are short-circuited when the rotor has reached synchronous speed. Under normal operating conditions the brushes of the commutator send direct current into the field winding F, which, therefore, produces a magnetic action identical with that of the rotating direct-current winding of the synchronous motor with pronounced poles.

When the motor operates in synchronism, a direct-current voltage is obtained across the commutator brushes for the same reason as explained in Chap. XVI on Rotary Converters. In rotary converters, however, the axis of the direct-current brushes is at right angles to the axis of the field winding F. In the Fynn-Weichsel motor, the brush axis forms an angle with the axis of the field winding, which is less than 90°. Therefore, at no load the brush axis is not at right angles to the resultant field. With increasing load on the machine, the space position of the resultant field changes. Consequently, the angle between the axis of the resultant magnetic field becomes larger with increasing load and approaches 90° at maximum synchronous motor load.

This is caused by the effect explained in Chap. XV on Synchronous Motors, that with increasing load the resultant field changes its relative position. This change in field position affects the voltage across the commutator brushes, thereby automatically regulating the exciting current.

At speeds below synchronism, the voltage across the commutator brushes always has the same frequency as the voltage induced in the secondary windings A and F, Fig. 37.17, and when

the angle between the brush axis and the field axis is small the induced voltage in F and the voltage across the commutator brushes are nearly in phase; and hence, for a given speed, the current flowing through the F winding is larger than would be the case if the machine operated as an induction motor. This increase in current is an important factor in giving the Fynn-Weichsel motor excellent synchronizing-torque characteristics.

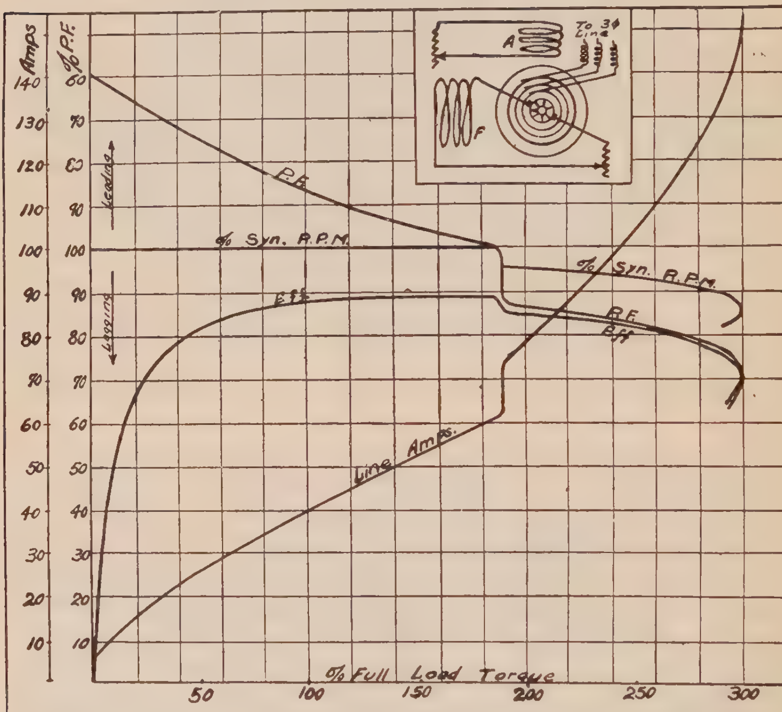


FIG. 37.17.—Characteristic curves for Fynn-Weichsel motor. 15 hp., 60 cycle, four-pole, three-phase.

The operating performance curves of a 15-hp., four-pole motor are shown in Fig. 37.17. The alternating current increases with increasing load and the power factor is leading over the whole working range. At about 190 per cent of normal torque, the power factor reaches unity, and the machine is automatically converted into an induction motor but continues to operate up to about 300 per cent full-load torque. When operating as an induction motor the power factor is lagging.

The Fynn-Weichsel motor can also operate as a generator, but a shift in the position of the brushes is necessary to obtain the best results. A series of interesting tests on this type of machine when operating as generator were conducted at the University of Washington.[1]

The working characteristics of the Fynn-Weichsel machine operating as either a generator or motor can be represented by a

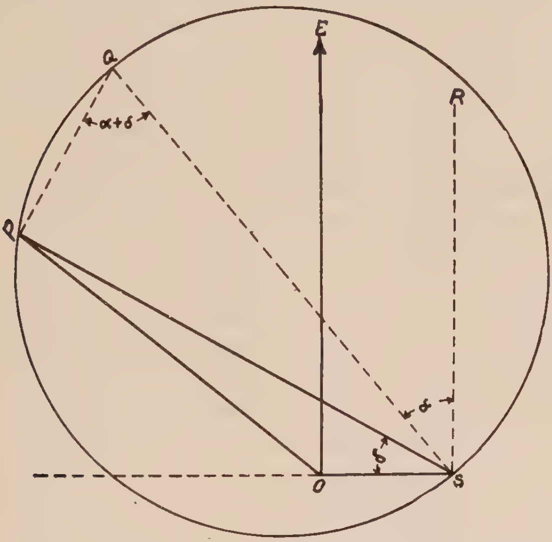


FIG. 38.17.—Circle diagram for Fynn-Weichsel motor.

circle diagram. Figure 38.17 shows a simple circle diagram, developed by Weichsel, based on the assumption that the ohmic resistance and leakage reactance are negligible.

With a fixed voltage and frequency impressed on the armature member, a rotating magnetic field of a definite strength must exist in the machine independent of the current flow in the stator and rotor. This field must be sufficiently strong to produce a voltage equal and opposed to the impressed voltage. The resultant ampere-turns necessary to produce this field are the same as those ampere-turns which are required to magnetize the motor when running idle as an induction motor. In the vector diagram, Fig. 38.17, let these ampere-turns be represented by OS. The line voltage leads 90° and is represented by OE.

[1] SMITH, GEORGE S., Discussion, *Trans. Am. Inst. Elec. Eng.*, Vol. 44, p. 29.

The primary ampere-turns produced by the alternating current are given by OP. The ampere-turns produced by the secondary or direct-current circuit are given by PS.

The input to the machine at any time is proportional to the distance of point P from the base line OS. At no load the axis of the resultant field of any synchronous motor, in which self-induction and ohmic resistance are negligible, coincides with the axis of the direct-current field winding, *i.e.*, in the diagram, Fig. 38.17, at no load the point P lies on the base line OS. With increasing load, the axis of the resultant field is displaced, at an angle δ, from the axis of the direct-current field winding. If the axis of the direct-current brushes forms an angle α with the axis of the direct-current field winding, then under load conditions the axis of the resultant field forms an angle α plus δ with the axis of the direct-current brushes, and hence the direct-current voltage is expressed by equation (13.17):

$$E = {}^{M}E \sin (\alpha + \delta). \tag{13.17}$$

The maximum direct-current voltage, which can be obtained, occurs when $\alpha + \delta = 90°$. The corresponding ampere-turns produced by this voltage in the stator winding are in the diagram represented by the line SQ, which is drawn in such a manner that the angle $PQS = \alpha + \delta$ and angle $QPS = 90°$. Under these conditions the vector SP is proportional to ${}^{M}E \sin (\alpha + \delta)$ or $SQ \sin (\alpha + \delta)$, and hence point P must lie on a circle, the diameter of which forms an angle α with the voltage vector OE or the line SR drawn perpendicular to the base line OS. If, for any load condition, point P lies on that part of the circle which is located below the base line OS, the machine operates as a generator and is returning energy to the line. It will readily be seen from Fig. 38.17 that the ratio of maximum energy which can be returned to the line when operating as a generator and the maximum energy which can be taken from the line when operating as a motor depend on the position of the brushes with respect to the field winding. Under the condition that the axis of the brushes is displaced 90° from the axis of the field winding, the circle becomes symmetrical to the base line OS, and the maximum output as generator becomes equal to the maximum input as a motor.

Machines of this type when connected to a supply line in combination with induction motors not only improve the power factor of the load and, therefore, decrease the copper losses in the

system, but also greatly improve the regulation. These machines have a further stabilizing effect on the line, because with increasing line voltage the leading reactive current taken by the motors decreases, while with decreasing line voltage the leading reactive-current component increases.

(*i*) **Other Types.**—Over 40 distinct types[1] of alternating-current commutator motors are known and quite a number have been manufactured and found to give satisfactory service. Compared to the polyphase, squirrel-cage induction motor all are complicated in design and, as a consequence, the first cost and maintenance charges are higher.

PROBLEMS

1.17. Draw a vector diagram for a series-excited, inductively compensated, alternating-current, commutator motor; circuit diagram as in Fig. 12.17.

2.17. Draw the vector diagram for an inductively compensated and excited and series-fed, alternating-current, commutator motor; circuit diagram as in Fig. 13.17.

[1] FLYNN, "Characteristics of Alternating-current Motors," *Trans. Am. Inst. Elec. Eng.*, Vol. 34, p. 1349.

CHAPTER XVIII

INDUCTION GENERATOR OR GENERAL TRANSFORMER

(*a*) **The Induction Generator.**—In the discussion of the induction motor, Chap. XIII, it was shown that the speed of the rotating field of an ordinary squirrel-cage motor is slightly greater than the speed of the rotor. The slip of the rotor makes the magnetic lines of force cut the conductors, thereby inducing voltages which cause currents to flow in the rotor circuits. The reaction between the rotating field and rotor currents transfers energy from the stator to the rotor. The magnetic flux is therefore the connecting link by which electric energy is transferred from the stator circuit and transformed into mechanical energy in the rotor.

Let the rotor of the induction motor be belted to a direct-current motor or any prime mover whose speed can be varied; and let the stator be connected to the mains of the supply circuit. If the speed is increased until the rotor runs in synchronism with the rotating field, no voltage is induced and no current flows in the rotor circuits. The energy necessary to overcome the friction and windage comes from the prime mover and the energy dissipated by the losses in the stator from the supply mains. If the speed is still further increased, the rotor revolves faster than the field, or above synchronism, and voltage is again induced and the rotor currents flow in the opposite direction from what was the case when operating below synchronous speed. If the rotor currents are reversed with respect to the field, the resulting reaction is also in the opposite direction, or energy flows from the rotor to the stator. Hence, when operating above synchronous speed the mechanical energy supplied to the rotor is transmitted magnetically across the air gap and delivered as electric energy in the stator circuit. At speeds above synchronism, therefore, the induction motor is a generator, and transfers and transforms the mechanical energy from the prime mover into electric energy in the stator or primary circuit.

Primary Frequency.—When operating as an induction motor the rotor slip depends on the load, but variations in load and slip nowise affect the primary frequency. In precisely the same way, when the machine is operated above synchronism, as a generator, the amount of power transmitted depends on the excess over synchronous speed, but the relative rotor speed does not affect the frequency in the stator circuit.

Therefore, if operated in parallel with ordinary alternators, the speed of the rotor varies with the load and the machine does not run in synchronism with the other machines. The frequency of the current in the stator of the induction generator is the same as for the other alternators, and the division of the load depends upon the governors of the prime movers.

Excitation.—The induction generator is not self-exciting; the reactive energy for the rotating magnetic field must be supplied from an outside source to the stator winding. When operating in parallel with other alternators delivering power to constant-potential mains, the exciting current automatically adjusts itself to the requirements for variations in the load. The power component is supplied through the rotor by the prime mover of the induction generator, but the quadrature component or reactive power required for the magnetic field comes from the other machines through the mains. Hence, whether operating as an induction motor or as an induction generator, the reactive power in the stator circuit must be supplied from an outside source. Operating in connection with over-excited synchronous motors is a desirable arrangement for induction generators as well as for induction motors. It is evident that the required reactive power could be supplied by static condensers. The reactive power would oscillate between the condenser and the field of the induction generator with double the frequency of the voltage, appearing alternately in the magnetic field of the generator and the dielectric field of the condenser. The cost of the condensers would in many cases prohibit the use of this arrangement in commercial plants, since the required reactive power is approximately 25 per cent of the generator capacity.

Circle Diagram.—Since the induction generator is simply a reversed induction motor, having the same electric and magnetic circuits, the current locus for the generator is merely an extension of the locus for the induction motor. Draw the circle

diagram for the machine as an induction motor, Fig. 26.13, and complete the circle as shown in Fig. 1.18. The upper half of the diagram is the current locus of the induction motor, or for speeds below synchronism; while the lower half gives the corresponding current locus for speeds above synchronism, or for the machine operating as an induction generator.

$MO = {}_mI$, the magnetizing current.
$P'M = I_1$, the stator current.
$P'O = I_{21}$, the equivalent rotor current.

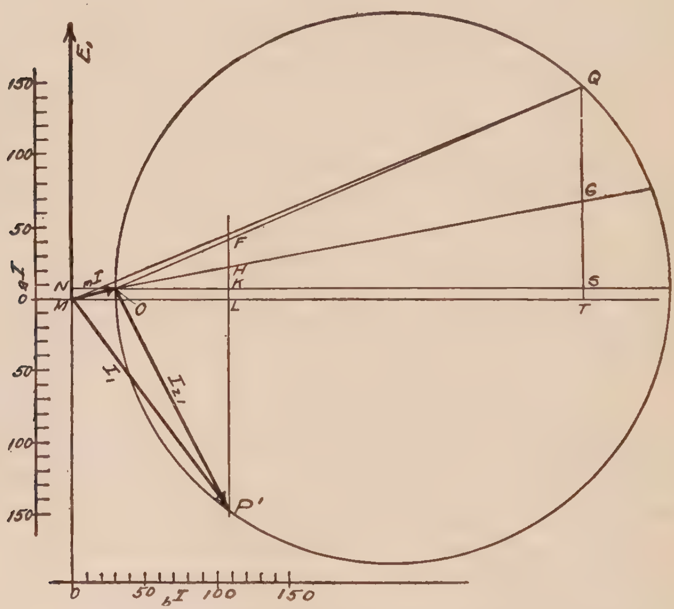


FIG. 1.18.—Circle diagram for asynchronous generator.

If the diagram represents values per phase, to obtain the corresponding quantities for the motor or generator multiply by n, the number of phases.

Under the same assumptions and approximations as for the induction-motor diagram, Fig. 26.13, the corresponding values in watts may be obtained for the induction generator from the diagram in Fig. 1.18.

$$nE_1(P'F) = \text{mechanical input} \tag{1.18}$$
$$nE_1(MN) = \text{core loss, windage and friction} \tag{2.18}$$

$$nE_1(HK) = \text{primary copper loss} \tag{3.18}$$

$$nE_1(HF) = \text{secondary copper loss} \tag{4.18}$$

$$nE_1(P'F - FL) = \text{electrical output} = nE_1(P'L) \tag{5.18}$$

$$\text{Cos } P'ME_1 = \text{power factor of stator circuit} \tag{6.18}$$

$$\frac{P'F - FL}{P'F} = \text{efficiency} = \frac{P'L}{P'F} \tag{7.18}$$

$$ML = jb_1E_1 = \text{quadrature component of } I_1 \tag{8.18}$$

$$nE_1(ML) = \text{reactive power supplied by the mains} \tag{9.18}$$

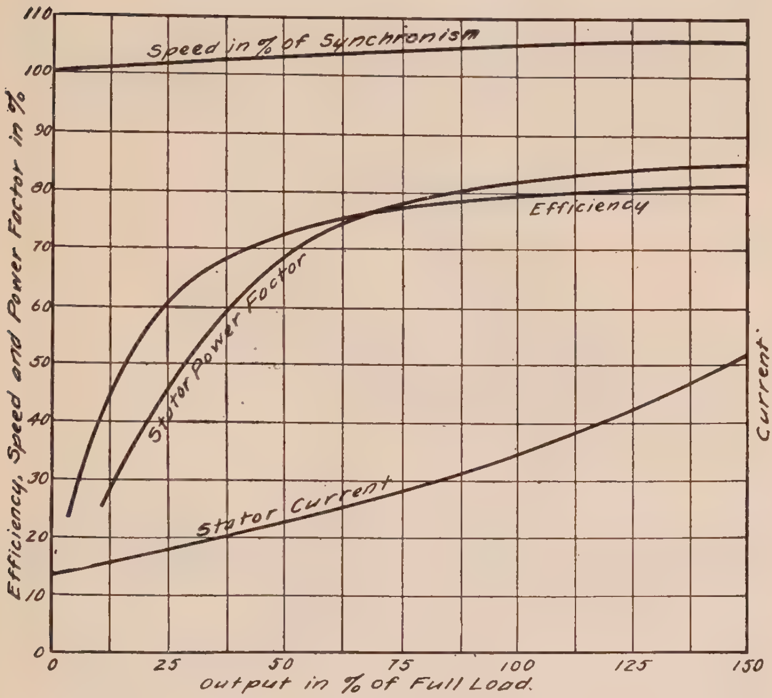


FIG. 2.18.—Performance curves of a three-phase asynchronous generator.

From the circle diagram the performance curves, Fig. 2.18, may be calculated in the same manner as explained for the induction motor.

(*b*) **The General Transformer.**—In the stationary transformer the voltage is raised or lowered but the frequency remains the same in the two circuits. The electric energy received in the

primary is transmitted by the magnetic circuit to the secondary and transformed to electric energy at a higher or lower voltage but without change of frequency. In the induction motor the electric energy received in the stator is transmitted by the magnetic field across the air gap and transformed into the mechanical form and delivered through the rotor to the load. From the slip rings of a wound-rotor induction motor, electric energy may be obtained in the same manner as from the secondary of the stationary transformer. Connecting the slip rings to an outside electric circuit, Fig. 3.18, the machine is both a motor and a transformer, and both mechanical and electric power may be obtained simultaneously. The frequency of the voltage and current in the secondary circuit from the slip rings need not be the same as in the stator, but depends upon the relative speed of the rotating field and rotor. If the rotor is

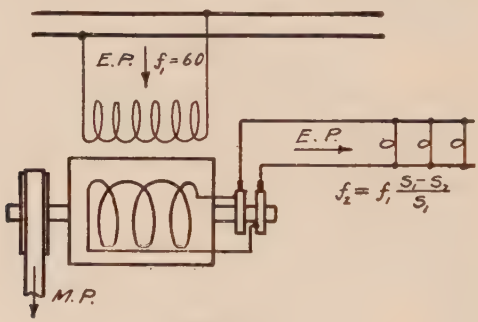


FIG. 3.18.

stationary the frequency of the secondary current from the slip rings is the same as in the primary, and the machine is a stationary transformer with a large leakage flux.

Let the speed of the rotor be controlled independently of the rotating magnetic field, by belting to a variable-speed motor which can operate in either direction. If the rotor revolves in the same direction as the stator field, the frequency of the current from the slip rings is proportional to the difference in the speeds. If the rotor is forced to rotate in the opposite direction, that is, run backward, the frequency of the secondary current is proportional to the sum of the speeds. Hence, if the backward rotation is considered negative, the frequency of the secondary currents is in all cases proportional to the algebraic difference in the speeds of the rotating field and the rotor.

f_1 = frequency of the primary or stator circuit.
s_1 = speed of the primary or rotating field.
f_2 = frequency of the secondary or rotor slip-ring circuit.
s_2 = speed of the rotor.

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{10.18}$$

The voltage in the secondary depends upon the relative number of turns in the stator and rotor circuits and upon the relative speeds. Neglecting losses in the machine:

$$E_2 = E_1 \frac{n_2(s_1 - s_2)}{n_1 s_1} \tag{11.18}$$

Let ${}_{ss}x_2$ = reactance of the secondary circuit at standstill, and x_2 the corresponding reactance at speed s_2.

$$x_2 = \frac{(s_1 - s_2)}{s_1} {}_{ss}x_2 \tag{12.18}$$

$$\dot{I}_2 = \frac{\dot{E}_2}{Z_2} = \frac{\dot{E}_1 \dfrac{n_2}{n_1} \dfrac{(s_1 - s_2)}{s_1}}{\left[r_2 + j\, {}_{ss}x_2 \dfrac{(s_1 - s_2)}{s_1} \right]} \tag{13.18}$$

The power relations of the several circuits or the direction of the energy flow depends upon the relative speed of the rotating field and rotor. The reactive power, for both the stator and the rotor circuits, comes from the primary mains at all rotor speeds.

The reaction between the rotating field and the rotor currents necessarily consists of two equal and opposite forces. With the forces equal, the work done by the field or by the rotor is directly proportional to their speeds. Neglecting losses in the machine, the power and speed relations may be grouped into four divisions.

1. For rotor speeds above synchronism

$$s_2 > s_1. \tag{14.18}$$

The machine is an induction or asynchronous generator and the mechanical power supplied through the rotor pulley is transformed into electric energy in both the primary and secondary circuits. The relative frequency in the two circuits is given in equation (15.18).

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{15.18}$$

2. For speeds between standstill and synchronism

$$0 < s_2 < s_1 \tag{16.18}$$

The total power is supplied from the primary circuit. The rotor delivers mechanical power. The frequency of the current in the rotor circuit is proportional to the slip.

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{17.18}$$

3. At standstill

$$s_2 = 0 \tag{18.18}$$

With both windings stationary the machine operates as a stationary transformer. On account of the large air gap the magnetizing current is comparatively large, but all the power is supplied by the primary and appears in the electric form in the rotor secondary.

$$f_2 = f_1 \tag{19.18}$$

4. With the rotor running backward

$$s_2 < 0 \tag{20.18}$$

The machine is a combination generator and transformer. Electric energy is received from the primary and mechanical energy through the rotor pulley. From both sources the energy flow is through the rotor and slip rings to the secondary circuit. The frequency of the secondary circuit is greater than in the primary and equal to the sum (algebraic difference) of the speeds. s_2 is negative.

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{21.18}$$

In cases 2 and 3 all the losses in the machine are supplied from the primary circuit. In case 1 near synchronous speed and until the mechanical power supplied through the pulley is equal to or greater than the losses, the difference comes only from the primary circuit. In case 4 the losses are supplied from both circuits.

With a single winding on the rotor a single-phase current is delivered through the slip rings. From a rotor wound polyphase, the corresponding polyphase currents are delivered from the slip rings. The machine can be used as a phase transformer since the secondary winding is independent of the primary.

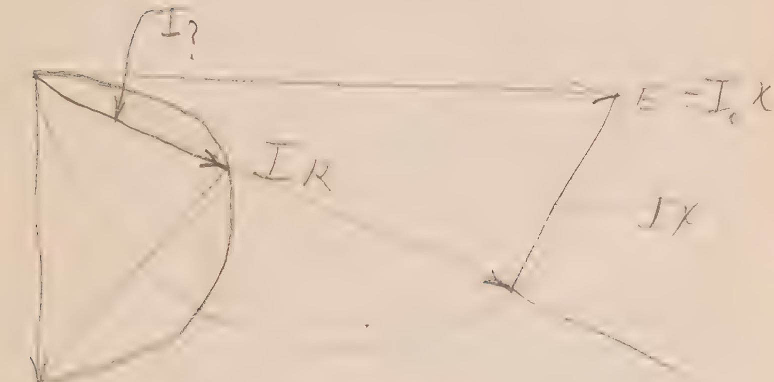
The extreme flexibility of a machine consisting of a primary and a rotating secondary is apparent. The general transformer can operate as a motor, generator, transformer, phase converter or frequency changer, and may even perform most of these transformations simultaneously.

PROBLEMS

1.18. The machine in problem 3.13, Chap. XIII, is run as an induction generator. Find the variations of torque, current, power factor and efficiency by means of the circle diagram from 100 to 125 per cent speed, and plot curves.

2.18. Show that in the transition in changing an induction machine from a motor to a generator by increasing the speed, for certain speeds power is supplied both mechanically and electrically. Calculate by McAllister's method the losses at no load when they are supplied electrically and when they are supplied mechanically.

$$I_2 = \frac{s E_{ss}}{\sqrt{R_2^2 + s^2 X_{ss}^2}} = \frac{E_{ss}}{\sqrt{\frac{R_2^2}{s^2} + X_{ss}^2}}$$



$$\frac{I_?}{I_0} = \frac{I_x}{I_x}$$

tangent
to where
curve

$I_? = I$

CHAPTER XIX

RECTIFIERS, OSCILLATORS AND INVERTERS. AMPLIFIERS

In the application of electric energy to industrial power requirements both direct and alternating currents are necessary. Electric power can be generated and transmitted for considerable distances more economically by alternating than by direct currents. Hence, all large electric power developments are basically alternating-current systems. However, the inherent properties of certain industrial power loads and processes require direct currents; as for example, electrolysis in the production of aluminum, zinc, hydrogen, etc.; the electrolytic refining of copper and other metals, and the charging of storage batteries. In other fields, notably electric railways and certain industrial drives, direct currents are in many cases more desirable than alternating currents.

For the conversion of alternating currents into direct currents several methods and many types of equipment have been invented and developed. The bulk of direct-current power available for industrial purposes is obtained from alternating-current power systems by means of rotary converters or by synchronous-motor, direct-current generator sets. However, a stationary type of converter, the steel-enclosed, mercury-arc power rectifiers, and hot cathode rectifiers, have lately become formidable competitors to the rotating machines for converting alternating currents into direct-current power.

I. MERCURY-ARC RECTIFIERS

The development of mercury-arc rectifiers,[1] to which Steinmetz and Cooper-Hewitt made important contributions, has been in progress for more than 30 years. In the early stages the problems were largely of scientific interest, but during the past few years industrial applications have become possible and the work has

[1] For a comprehensive treatise, including bibliography, see "Mercury Arc Power Rectifiers," Marti and Winograd, McGraw-Hill Book Company, Inc.

centered on the design, construction and operation of increasingly larger commercial units.

The essential elements of the mercury-arc rectifier are:

1. A steel-enclosed high-vacuum chamber.
2. Inside this chamber, a pool of mercury, forming the electron-emitting cathode.
3. Two or more carbon or steel electrodes, forming the non-electron-emitting anode.
4. Some form of ignition and excitation apparatus.
5. Air-tight-insulated leads, for conducting the current to the anode and from the cathode.

In addition to the above basic elements many other parts or accessory appliances, as vacuum pumps, vacuum-measuring and regulating devices, backfire shields, a cooling system, transformers, high-speed circuit-breakers and various indicating instruments, are necessary for the practical operation of mercury-arc rectifiers. When the rectifier is in operation, a mercury arc, consisting mainly of a stream of electrons flowing either continuously or intermittently, spans the high-vacuum space from the mercury-pool cathode to the carbon anodes.

(*a*) **Mercury-arc Characteristics.**—In gaseous conduction of electricity the movement of electrons toward the anode, and of positive ions towards the cathode, constitutes the flow of current between the electrodes. Each electron and each positive ion carries a charge of $1.59 \cdot 10^{-19}$ coulomb; or for each ampere $629 \cdot 10^{16}$ electrons, or positive ions, must pass the given cross-section of the circuit per second. The voltage required for gaseous conduction between two electrodes depends mainly on the gas or vapor pressure, and on the temperature and material of the electrodes.

The changes in the gas or vapor pressure affect not merely the interference *resistance* encountered by the electrons in moving from the cathode to the anode, and the positive ions in the reverse direction, but also greatly modify the *ionization-by-collision* and *space-charge* effects. Under very low gas pressures the space-charge effects are greatly augmented, while the ionization by collision is decreased. For comparatively high gas pressures the resistance to the motion of the electrons by molecular interferences becomes very large. From extended investigations it has been found that an optimum pressure (a few microns) for mercury-arc rectifiers exists, at which ionization by collision produces an abundant supply of electron carriers while under the

same conditions both the gas-flow resistance and the space-charge effects are comparatively small.

The use of mercury as the electron-emitting cathode offers many advantages:

1. Less voltage is required to release the electrons from the mercury surface, at the operating temperature, than for other metals.

2. Mercury vapor in the vacuum chamber greatly increases the desired ionization by collision.

3. The ionized mercury atoms are attracted to the mercury-pool cathode and the heat produced by collision increases the cathode temperature.

4. Any condensation or change in quantity of the mercury vapor in the chamber is automatically adjusted by the mercury in the pool forming the cathode.

5. The mercury arc, terminating on the cathode surface, produces an intensely hot *cathode spot*, which moves rapidly over the surface of the mercury pool. The high temperature (approximately 2000°C.) of the *cathode spot* is of very great importance in producing a rapid emission of electrons without causing deterioration of the cathode surface. The valve action, that is the property of permitting the current to flow in one direction only, is not a unique characteristic of mercury. The valve action of the mercury-arc rectifier is largely due to the difference in temperature at the two electrodes. In the rectifier the cathode is brought to a state of rapid electronic emission while the anode is maintained at a temperature below that at which electrons can be liberated.

The *energy loss* in the mercury-arc rectifier can be separated into three parts, that may be represented by the respective *voltage drops.*

1. The voltage drop at the cathode surface, approximately 7 volts, is essentially the same for all types of mercury-arc rectifiers.

2. The voltage drop in the arc is fairly constant and directly proportional to the length of the arc. Although other factors cause variations, it may be taken as approximately 0.1 volt per cm. length of arc.

3. The voltage drop at the anode surface is approximately 5 volts, but varies with the material used, the shape of the anode and other factors.

In general the voltage drop in the rectifier may be expressed as the sum of 1, 2 and 3, above. Hence, for an arc 3 ft. long the approximate voltage drop would be:

$$7 + 5 + 36 \cdot 2.54 \cdot 0.1 = 21 \text{ volts} \tag{1.19}$$

Under operating conditions the temperature of the mercury arc varies over a wide range; estimated limits, from 1,000° to 10,000°C. This high arc temperature necessitates a cooling system for the rectifier container and lead-in terminals. As there are no rotating parts that would carry away heat by mechanical connection, a cooling system becomes all the more necessary.

(*b*) **Rectification.**—It is customary to connect the phase terminals or free ends of the transformer secondary windings

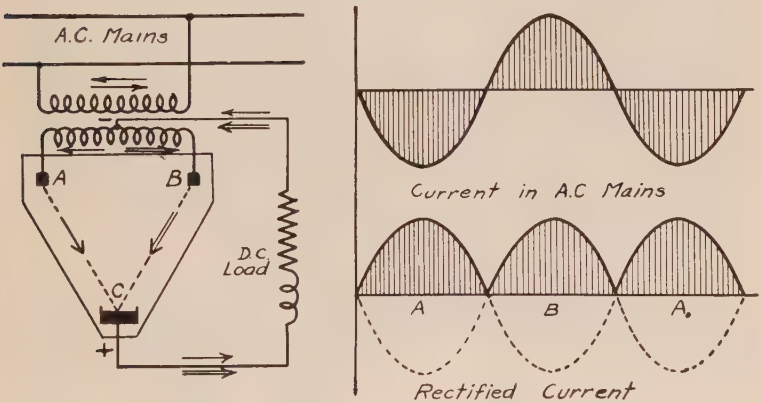


FIG. 1.19.—Single-phase circuit diagram.

to the anodes of the mercury-arc rectifier. The *cathode* thereby becomes the *positive terminal*, and the *neutral point* of the transformers the *negative terminal* of the external, direct-current circuit. A simple, elementary representation of circuit connections and the resulting rectified currents for single-phase, three-phase and six-phase rectifiers are shown in Figs. 1.19, 2.19 and 3.19, respectively. The actual currents and voltages differ widely in wave shape, depending on the transformer connections used, the properties of the mercury arc and other factors.

The arrows in the diagram indicate the direction of current flow. The rectified current is pulsating but flows in one direction only, half of the time from each of the two anodes A and B.

The single arrow indicates current passing through the *A* anode and the double arrow the current from the *B* anode.

In the three-phase rectifier the three anodes reach maximum potential in succession and at any instant the current flows from

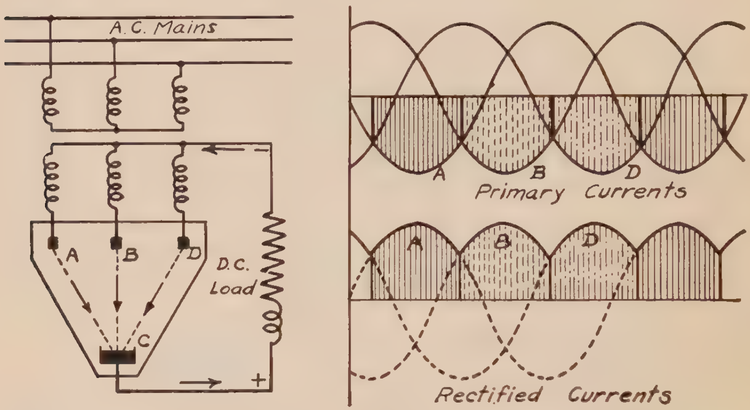


FIG. 2.19.—Three-phase circuit diagram.

the anode having the highest positive potential. In six-phase rectifiers the undulations of the direct current are less than for three-phase units; and it is evident that the larger the number

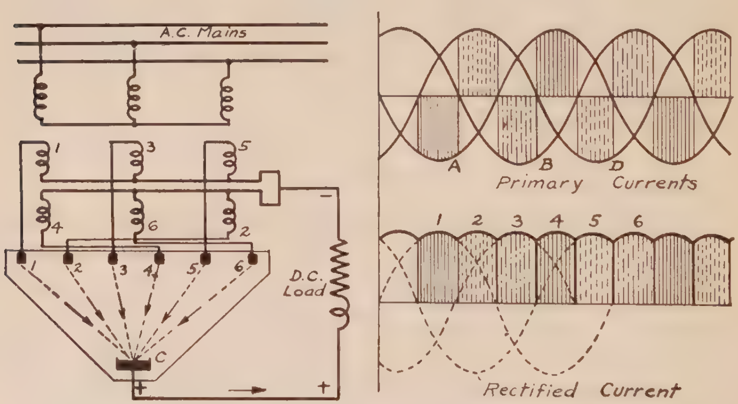


FIG. 3.19.—Six-phase circuit diagram.

of phases the smaller the undulations of the rectified direct-current output.

Since the current flows only from the anode at the highest positive potential it follows that the greater the number of phases

the shorter the periods of active conduction for each anode, unless two or more electrodes are held at the same potential.

(c) **Rectifiers.**—Many types and designs of mercury-arc rectifiers have been developed by American and European manufacturers for industrial service. An external view of a medium-sized unit for electric railway loads is shown in Fig. 4.19.

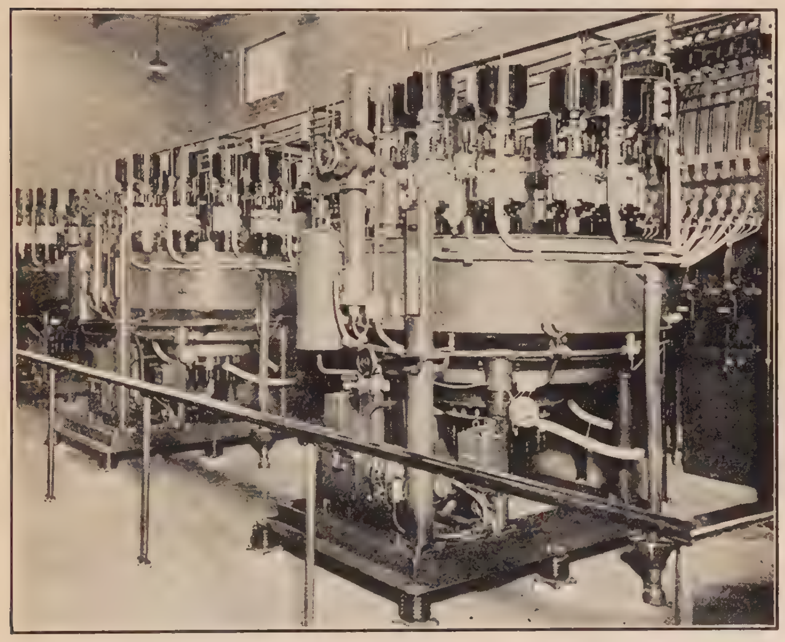


FIG. 4.19.—Three 1,000 kw., 600-volt, mercury-arc rectifiers. (*General Electric Company.*)

A better appreciation of the arrangement of the essential parts may be obtained from a cross-sectional view, as in Fig. 5.19. The figure shows the positions and relative magnitudes of the cooling jacket, vacuum chamber, mercury cathode, main-load anodes, ignition and excitation anodes, vacuum equipment and other essential elements of a steel-enclosed rectifier.

The inner tank, Fig. 5.19, forms the vacuum chamber, and the water-filled space between the two tanks, the cooling system or water-jacket, necessary for keeping the leads, electrodes and inner tank within prescribed temperature limits.

The anodes as well as the cathodes are insulated from the tanks. Moreover, the rectifier and all the accessory apparatus are insu-

lated from the ground, as the tank must be kept at a slightly higher potential than the cathode.

(*d*) **Ignition and Excitation.**—The first step in the process of loading the mercury-arc rectifier is to bring the cathode into an active state of electronic emission. This is accomplished by means of the ignition and excitation equipment, which starts the arc in the vacuum chamber. The excitation equipment also

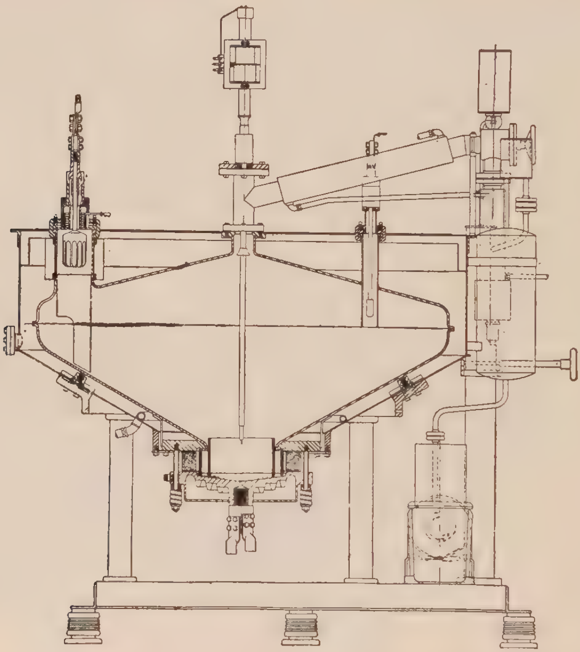


FIG. 5.19.—Cross-section of a 1,000-kw., 600-volt, mercury-arc rectifier. (*General Electric Company.*)

maintains the mercury arc during sudden changes in load or other momentary disturbances that may break the main electron stream, or arc, from the cathode to the load anodes. The equipment and methods of operation differ in various types of manufacture but the basic principle, which is essentially the same for all, may be made clear by referring to the specific circuit diagram shown in Fig. 6.19.

On closing the switch from the auxiliary bus to the ignition and excitation transformer, the ignition contactor closes, exciting the solenoid of the ignition anode. This anode is mounted directly over the cathode

on top of the vacuum chamber and has a movable plunger which is forced into the mercury pool against the action of a spring when the solenoid is excited. When the plunger strikes the mercury, current flows through the plunger, the mercury cathode, resistance R and the ignition relay. At the same time, the contactor is short-circuited,

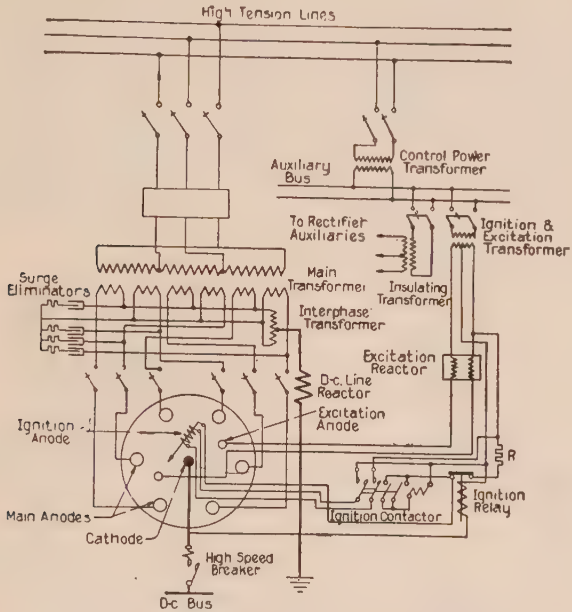


FIG. 6.19.—Diagram of connections for a six-phase, 600-volt, mercury-arc rectifier. (*General Electric Company.*)

allowing the plunger to return to its normal position. As the plunger leaves the mercury, an arc is drawn and the cathode emits electrons, provided its polarity is negative. The excitation anodes then pick up the arc and supply direct current to the ignition relay, which opens and deenergizes the ignition system. If the polarity is not correct when the anode leaves the mercury, the contactor closes after time delay, and the ignition process is repeated until the excitation anodes pick up the arc. The excitation system is merely an auxiliary, single-phase, full-wave rectifier. In order to prevent the current from falling to zero at the end of each cycle a reactor is inserted in the excitation anode lines.

(*e*) **Transformer Connections.**—Many factors enter into the problem of selecting the most desirable transformer connections for mercury-arc rectifiers. The number of anodes in the rectifier, efficient utilization of the transformers, the regulation of the

direct-current voltage, the wave shapes of the voltages and currents, the simplicity of the wiring and other factors must be given consideration. In the same manner as in the case for rotary converters, Chap. XVI, several transformer connections are used in commercial installations. Only the five more common systems will be mentioned:

1. Six-phase, diametrical connection. Delta primary. This connection is shown in Fig. 19.16 under Rotary Converters.

2. Six-phase, diametrical connection. Star primary. This connection is shown in Fig. 17.16 under Rotary Converters.

3. Six-phase, fork connection. Star or delta primary. Figure 7.19.

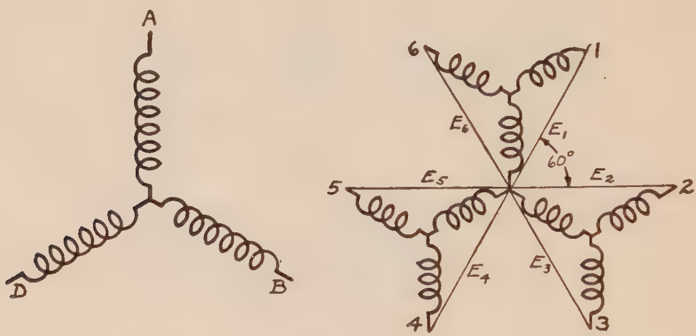


FIG. 7.19.—Six-phase, fork connection.

4. Six-phase connection from double three-phase with interphase transformers. Star primary. Figure 8.19.

5. Twelve-phase connection from double six-phase with interphase transformers. Star primary. Figure 9.19.

Various other circuit connections have been developed by introducing accessory devices, as tertiary windings, primary reactances, anode reactances, supply-line reactances, tap changing, etc., to gain better voltage regulation or other advantages in the operation of the rectifier.

The secondaries of the six-phase, fork connection consist of three, star-connected windings with a common neutral point, which forms the negative terminal of the direct-current circuit, as shown in Fig. 7.19. The voltages for all the secondary windings are equal and displaced 60° from each other, hence forming a balanced six-phase system. Each anode and branch winding carries the full direct-current load during one-sixth of

each cycle and each of the star windings next to the neutral point the full current for one-third of each cycle. A more economical use of both the anodes and the transformer sections are obtained by the connections shown in Figs. 8.19 and 9.19.

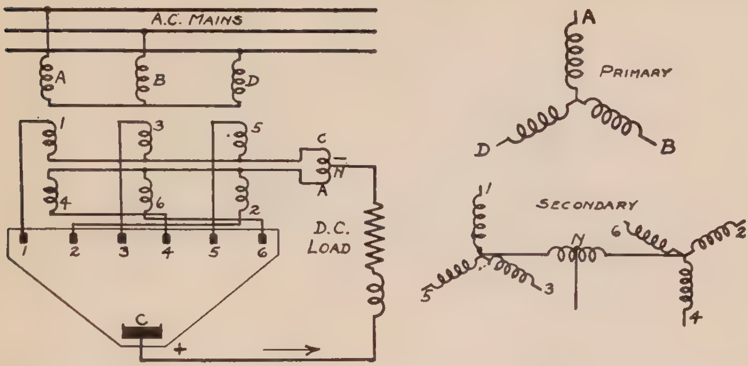


FIG. 8.19.—Six-phase rectifier with interphase transformer. Double, three-phase transformer connection.

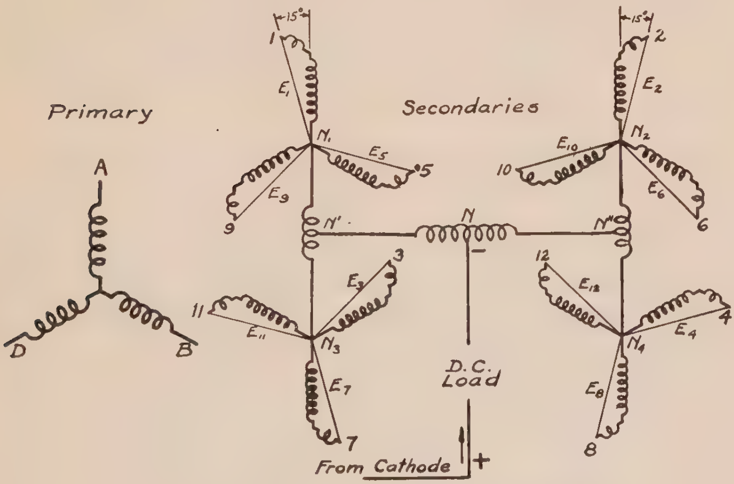


FIG. 9.19.—Twelve-phase rectifier with interphase transformer. Double, six-phase transformer connection.

By means of the interphase transformer, which serves as a voltage equalizer it is possible to operate the two, three-phase transformers in parallel, as illustrated by the circuit connections in Fig. 8.19. As a consequence two anodes carry current simul-

taneously, so that each anode is active for one-third of each cycle instead of one-sixth as for the fork or diametrical connections. This connection also provides a more economical use of the transformers and improves the direct-current voltage regulation.

The twelve-phase connection shown in Fig. 9.19 is based on the same principle as the six-phase system in Fig. 8.19 and illustrates the extension of the use of interphase transformers and parallel operation to twelve-anode rectifiers. Each of the two sets of six-phase connections in Fig. 9.19 is basically the same as the six-phase system shown in Fig. 8.19. The two sets of six-phase transformer anode terminals must be displaced 30° with respect to each other to form the desired twelve-phase arrangement. This displacement is obtained by means of the combined sectional windings which shift the two six-phase sets 15° in opposite directions, as shown in Fig. 9.19.

(*f*) **Vacuum System.**—In the operation of the mercury-arc rectifier a very high vacuum must be maintained in the arc chamber. Moreover, the vacuum must be kept automatically within narrow, definite limits, preferably at a pressure between 1 and 5 microns; that is, from 0.001 to 0.005 mm. of mercury. An effective exhaust system, usually consisting of a motor-driven pump, receiver tank, mercury condensation pump and vacuum valves, is therefore, essential to the operation of mercury-arc rectifiers.

One of the very difficult problems in the development of mercury-arc rectifiers was to obtain a satisfactory seal for the vacuum chamber. To meet service requirements a good seal must possess, in a high degree, the following properties: (*a*) air-tightness; (*b*) resistance to high temperatures; (*c*) little if any deterioration or low maintenance; (*d*) good insulator; (*e*) simplicity in construction and assembly. Several types of seals have been developed and in each case the practical application depends not only on the material used but also on the design of the joint. The material used in the *mycalex seal* is a composition of ground mica and barium borate. In the *mercury seal* the joints, having solid gaskets of special design, are made air-tight by a layer or filler of mercury. In the *two-stage seal* or *cascade joint* the packing material is a combination of aluminum and lead.

The high temperatures generated by the mercury arc and the enclosed, stationary type of equipment necessitate the use of a cooling system to keep the temperatures of the several parts of

the rectifier within permissible limits. The requirements are met by surrounding the vacuum chamber by a water-jacket in which water is circulated by an automatically controlled pump.

(*g*) **Backfire.**—The operation of the rectifier is based on the valve action of the mercury arc. Under normal conditions the mercury pool is always the cathode, and the several carbon electrodes, connected to the transformer terminals, the anodes. Under certain conditions, which are not fully understood, it sometimes happens that one of the carbon electrodes develops a cathode spot and suddenly becomes a cathode, thereby short-circuiting the transformer secondaries. This is known as *backfire or arc-back*, a violent and extremely undesirable disturbance in the operation of rectifiers. The conditions or factors that produce backfire have not been fully determined but the immediate cause appears to be the development of a hot spot on an anode electrode. Possibly gases liberated by excessive local heating or impurities in the electrodes or glow discharge or low vacuum may be factors in the process. Mercury condensing on the anode electrodes, when inactive, may become the source for a cathode spot to form on the anode, resulting in backfire. In order to prevent the condensation of mercury on the anodes, when inactive, *backfire shields* and *screens* are inserted in the rectifier. No complete preventive for backfire has yet been found.

(*h*) **Power Factor.**—The over-all power factor of the mercury-arc rectifier depends to a considerable extent on the transformer connections used. Moreover, the measured ratio of the watts to the volt-amperes is largely affected by the voltage and current wave shapes. The reactive power is largely in the harmonics and not directly comparable to the full-frequency, reactive power of synchronous motors. For large polyphase rectifiers the power factor at full load is about 95 per cent. The power factor for any given rectifier is not adjustable, as is the case for rotary converters and synchronous motors.

(*i*) **Efficiency.**—The voltage drop in the mercury-arc rectifier, proper, is essentially constant, and hence the loss is directly proportional to the load current. On the basis of efficiency the rectifier has, therefore, two distinct advantages over rotary converters or synchronous-motor, direct-current generator sets.

1. High efficiency for high-voltage, direct-current loads.
2. High efficiency over a wide range in load.

In Fig. 10.19 are given the comparative efficiencies of two comparable rectifier and converter units. The over-all efficiency

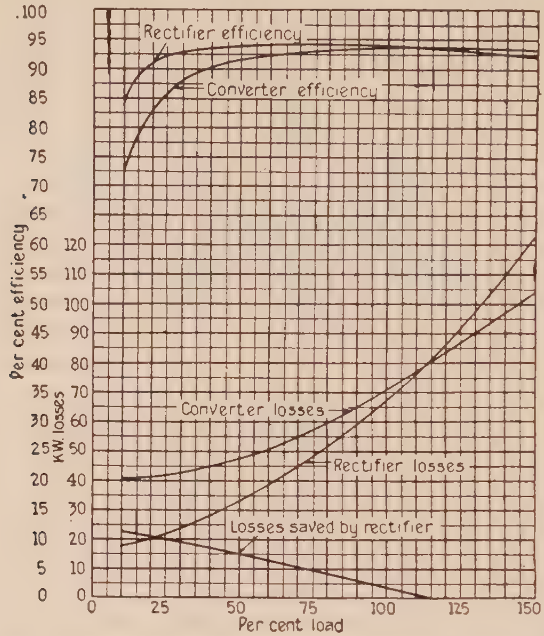


FIG. 10.19.—Efficiency-load curves and losses-load curves of 1,000-kw., mercury-arc rectifier and 1,000-kw. rotary-converter stations supplying direct current at 600 volts. (*General Electric Company.*)

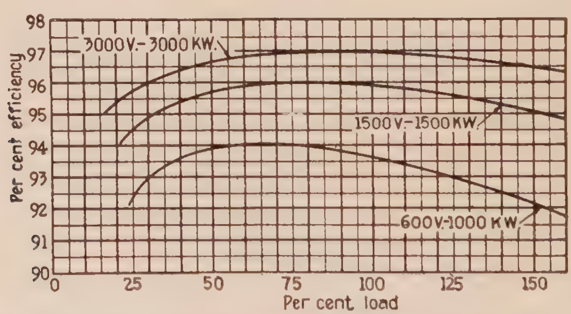


FIG. 11.19.—Over-all station efficiencies for mercury-arc rectifiers for 600, 1,500, and 3,000 volts. (*General Electric Company.*)

of the rectifier unit includes losses in the transformers and auxiliary apparatus, and hence is considerably lower for light loads than at full load. It should be noted that although the

full-load efficiencies of 600-volt rectifiers and rotary converters are essentially the same, the efficiency of the rectifier is markedly higher at the lower loads. At higher voltages as shown in Fig. 11.19 the over-all efficiency of the rectifier is considerably higher at all loads than can be obtained from rotary converters or synchronous-motor, direct-current generator sets. Mercury-arc rectifiers are noiseless, can carry momentary overloads and operate at any power frequency.

II. HOT-CATHODE RECTIFIERS

All vacuum tubes are, in principle, rectifiers and can be used for converting alternating currents into direct currents. However, for most types the efficiency of conversion is low and the power output small. Five types of hot-cathode vacuum tubes are used directly as either rectifiers or inverters, or use indirectly the rectifying power in the control of other circuits.

1. *Kenotrons.*—Two-element, high-vacuum tubes, used for rectifying small currents in high-potential circuits.

2. *Pliotrons.*—Three- or four-element, high-vacuum tubes; in essence, a kenotron with a control grid or third element and, in some cases, a space-charge grid or fourth element.

3. *Tungars.*—Two-element, low-voltage tubes rectifying alternating currents for charging batteries and similar small power requirements.

4. *Hot-cathode Rectifiers.*—Two-element tubes with mercury-vapor conduction. For rectifying alternating currents to meet large, direct-current power requirements.

5. *Thyratrons.*—Three-element tubes with mercury-vapor conduction. Used in oscillators or inverters, rectifiers, amplifiers and for many types of control circuits.

The operating characteristics of kenotrons, pliotrons and tungars are discussed in Chap. XXI of "Direct Currents," the first volume of this work.

(*a*) **Hot-cathode Rectifiers (Mercury).**—The essential elements of this unit are a hot cathode, a cold anode and a highly evacuated tube in which is placed a small amount (single drop) of mercury. The temperature of the mercury, which does not form the cathode but is kept in the coldest part of the tube, determines the pressure of the conducting mercury vapor. The current passes through the tube as an arc, always in one direction, when the

voltage exceeds the critical voltage. An elementary diagram for a single-phase circuit is illustrated in Fig. 12.19. Similarity of the circuits in Figs. 12.19 and 1.19 for the single-phase, mercury-arc rectifier should be noted. The hot cathodes replace the mercury pool in Fig. 1.19 while each anode in the mercury-arc rectifier is replaced by the plate of the hot-cathode rectifier in Fig. 12.19.

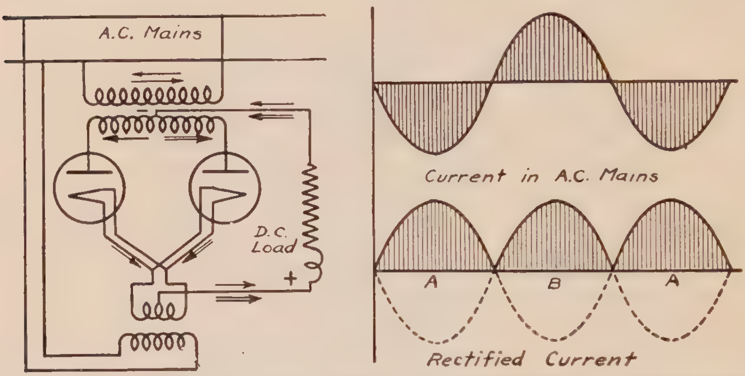


FIG. 12.19.—Single-phase circuit diagram for rectification by hot-cathode rectifier.

Aside from the special transformer for heating the cathode filament the two circuits are electrically identical. It is evident that the three-phase and six-phase circuit diagrams, Figs. 2.19, 3.19, 6.19, 7.19, 8.19 and 9.19, or any connections adapted to mercury-arc rectifiers, can also be used for hot-cathode rectifiers, by merely replacing each anode with a hot-cathode rectifier and connecting all the cathodes in parallel.

The hot-cathode mercury rectifiers offer several advantages over the mercury-arc rectifiers, notably:

1. Smaller size for the same rated capacity.

2. Shorter distance between cathode and anode, and hence lower voltage drop in the arc.

3. Lower cathode voltage drop for the hot, barium-coated cathode.

4. Higher efficiency of rectification as the total voltage drop in the hot-cathode rectifier is about 12 volts as compared to about 21 volts in the mercury-arc rectifier.

5. Freedom from backfiring.

(*b*) **Thyratrons.**—The thyratron[1] is a three-element tube and consists of a hot cathode, a grid, a cold anode and usually a drop of mercury which provides the conducting vapor. Argon, neon or helium gas may be used in place of the mercury vapor. The starting of the current, which passes through the tube as an arc, is controlled by the grid voltage. After the current starts, the grid voltage has no influence until the anode voltage becomes

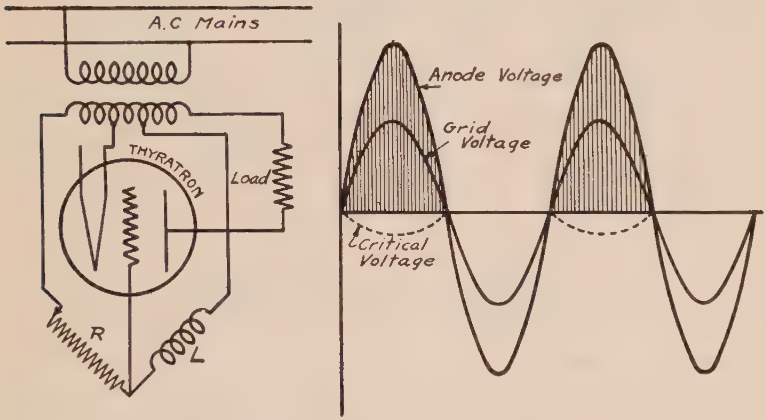


FIG. 13.19.—Control characteristics of a thyratron on which alternating currents impressed on both the anode and the grid are in time phase.

zero after which the starting process repeats. With alternating currents the anode voltage is zero twice for each cycle. The current flows in one direction only. Hence each thyratron produces a pulsating unidirectional current. If alternating currents are impressed on the grid, the magnitude and the time phase of the grid voltage, with respect to the anode alternating current, determine the fraction of each positive half wave that current will flow in the tube, *i.e.*, the *average value* of current output can be controlled by varying either the time phase or the magnitude of the grid voltage. By changing the resistance in the grid circuit from infinity to zero the grid voltage is retarded 180°. Current starts as soon as the grid voltage is less negative than the critical voltage and the flow continues practically for the full positive half cycle, and repeats in successive cycles. By changing the time phase of the grid circuit so that it lags 80° behind the anode voltage, the start of the current flow will be

[1] HULL, DR. A. W., Hot Cathode Thyratrons, *Gen. Elec. Rev.*, Vol. 32, p. 390, (1929).

delayed and the average current reduced in proportion to the shaded areas in Fig. 14.19 as compared with Fig. 13.19.

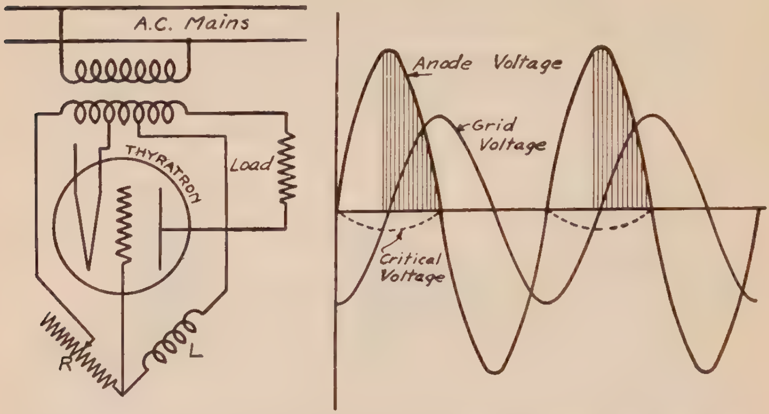


FIG. 14.19.—Control characteristics of a thyratron in which the time phase of the grid voltage lags 80° behind the load-current voltage.

From the above it is evident that by *phase control* of the grid voltage with respect to the load or anode voltage the average value of the current can be varied from a maximum to zero, by

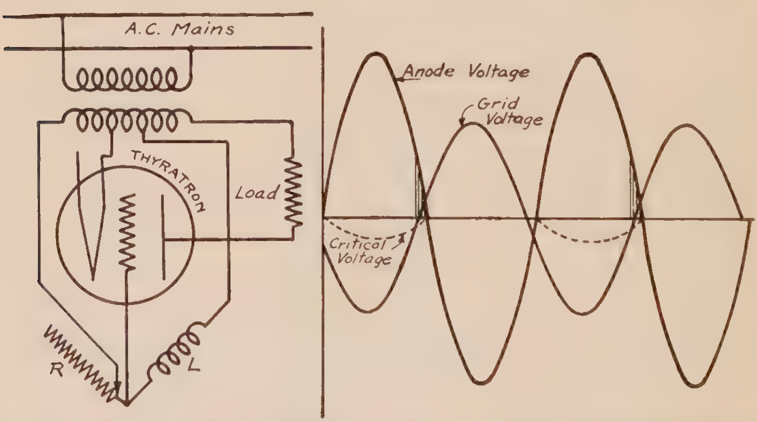


FIG. 15.19.—Control characteristics of a thyratron in which the grid voltage is nearly in opposition to the anode voltage.

changing the resistance in the grid circuit from infinity to zero as illustrated in Figs. 13.19 to 15.19.

The phase control of the grid voltage may be obtained by a combination of condensance and inductance in the grid circuit, in

place of the inductance and resistance shown in Figs. 13.19 to 15.19. The vacuum required for operation is from 1 to 100 microns. The internal voltage drop of the mercury-vapor thyratron is about 12 volts; the same as for the two-element hot-cathode rectifier. The cathode excitation requires less than a watt per ampere so that the over-all efficiency of the thyratron is higher than for mercury-arc rectifiers or rotary converters.

III. VACUUM-TUBE OSCILLATORS AND INVERTERS

When alternating currents of frequencies higher than a few hundred cycles per second are desired, they are usually obtained by means of vacuum-tube oscillators. The vacuum tube consists essentially of a hot cathode or *filament*, a cold anode or *plate*, and a control element or *grid* (see Figs. 16.19 and 17.19) all contained in a highly evacuated bulb. The filament is operated at a high temperature so that the velocity of the free electrons present becomes sufficiently great to enable them to break through the surface of the filament and so pass out into the surrounding space. This phenomenon is called *emission.* If the plate is charged positively with respect to the filament, the electrons, being negatively charged, will be drawn toward the plate. Ordinarily all the electrons emitted by the filament will not be drawn over to the plate, some dropping back into the filament. As the potential of the plate is increased with respect to the filament, fewer electrons drop back into the filament and more pass over to the plate. If the grid, which consists of a screen or mesh of wires, is made positive with respect to the filament, it will help the plate to draw over the electrons thereby increasing the electron flow or plate current. Some electrons will also go to the grid under these conditions, but as its area is relatively small, the grid current will be small. If, on the other hand, the grid is charged negatively with respect to the filament, it will tend to force the electrons back into the filament and so decrease the plate current. The grid thus acts as a valve, opening and closing the circuit of the plate current to a greater or less degree.

In Fig. 16.19 is shown the Hartley circuit for the *Hartley vacuum-tube oscillator.* In this circuit the *plate* and *grid inductances* together with the *tank condenser* form an *oscillatory circuit,* known as the *tank circuit.* If the condenser in this circuit be charged and then allowed to discharge through the plate and

grid inductances as shown, the current flow would be alternating and of decreasing magnitude. The frequency is determined by the size of the condenser and inductances and is equal to $\frac{1}{2\pi\sqrt{LC}}$ cycles per sec.; L in henrys and C in farads. The decrease in amplitude is due to losses in the tank circuit and to the energy delivered to the output. If sufficient energy be supplied to this circuit, during each cycle, to supply the losses and power output per cycle, the amplitude of the current would remain unchanged. The function of the vacuum tube is to deliver the required energy to the tank circuit.

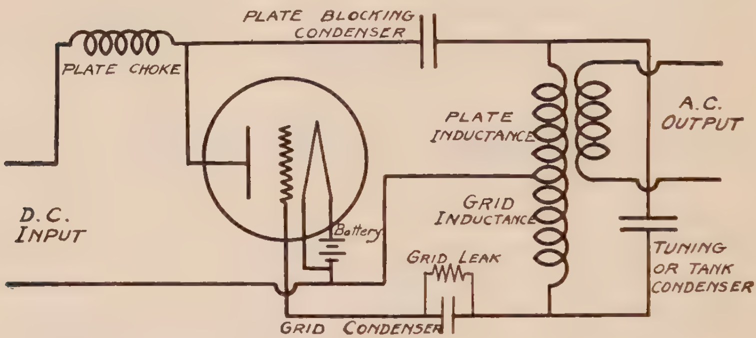


FIG. 16.19.—Circuit diagram for the Hartley vacuum-tube oscillator.

As the energy stored in the tank circuit alternates between the inductance and condensance, there will be a time when the grid will be charged positively with respect to the filament. A large direct-current plate current will flow under the influence of this positive grid potential, building up a field and storing energy in the plate choke (inductance). At a time of one-half cycle later the grid will be negative, thereby shutting off or greatly decreasing the flow of plate current, and causing the plate choke (inductance) to discharge its energy into the tank circuit. This discharge or pumping action occurs once each cycle and thereby the necessary energy is delivered to the oscillatory or tank circuit to maintain the oscillations at constant magnitude.

The grid condenser and leak, shown in the circuit, Fig. 16.19, increase the efficiency of the circuit, but do not affect the general principles of operation. The plate-blocking condenser serves to prevent short-circuit of the direct-current supply through the plate inductance, but permits the passage of the alternating

current which delivers the energy to the oscillatory circuit. It is not necessary to have the tank condenser charged first before this circuit will start oscillating, it being assumed so only for purposes of explanation. In actual practice the closing of a switch to the direct-current supply will, first, store energy in the inductance which, then, discharges into the condenser, starting the oscillation.

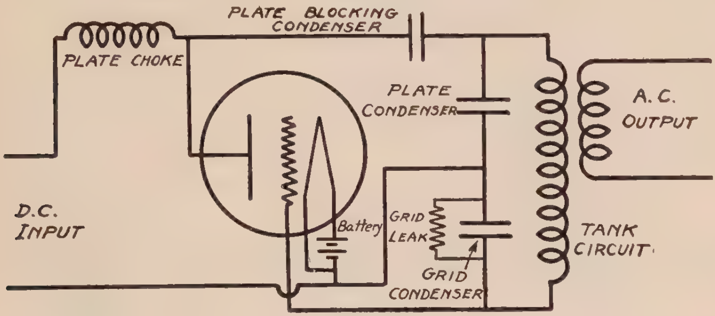


FIG. 17.19.—The Colpitts circuit for vacuum-tube oscillator.

In Fig. 17.19 is shown the circuit for the *Colpitts vacuum-tube oscillator.* The theory for this circuit is very similar to that for the Hartley circuit in Fig. 16.19, the principal difference being that the division of voltage between the plate and grid is obtained by using a *plate* and a *grid condenser* in series instead of a *plate* and a *grid inductance.* The grid leak, in combination with the condenser, serves to increase the efficiency of operation.

Another type of oscillator[1] or *inverter,* due to Sabbah, is shown in Fig. 18.19. The tubes used in this circuit, called *thyratrons,* are distinguished from the ordinary vacuum tube by the presence of mercury vapor inside the bulb. The effect of this mercury vapor is such that no plate current will flow while the grid is negative with respect to the filament but that once the plate current has started the grid no longer controls it. The plate current can then be stopped only by removing the plate potential.

The operation of the circuit is as follows. Suppose that thyratron A draws current. This current will flow through thyratron A and charge up condenser C_2. It will also flow through R_2, keeping the grid on thyratron B negative so that it cannot draw current. When condenser C_2 becomes fully

[1] HULL, DR. A. W., "Hot Cathode Thyratrons," *Gen. Elec. Rev.*, Vol. 32, p. 399, (1929).

charged, the plate current in thyratron A ceases, thereby opening the low impedance shunt around R_1C_1. The negative voltage on the grid of thyratron B is removed and this tube will then draw plate current, causing C_2 to discharge and C_1 to charge. The discharge current of C_2 flows through R_1 keeping the grid of tube A negative so that no plate current can flow through this tube. When C_2 is fully discharged and C_1 fully charged, the plate current to tube B ceases, the negative voltage on the grid of tube

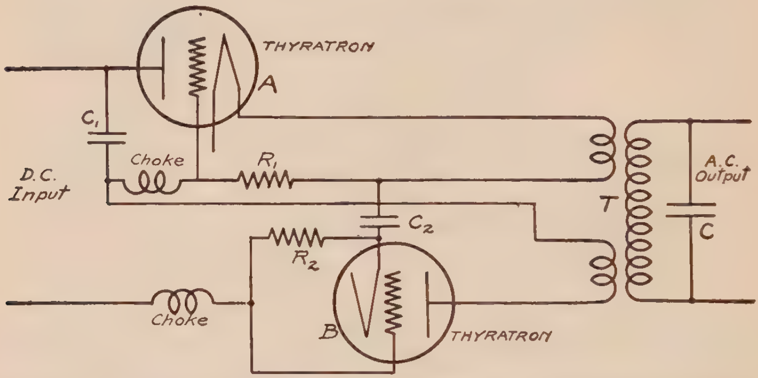


FIG. 18.19.—Self-controlled series-type inverter. The frequency is determined by the characteristics of the grid transformers.

A is removed and tube A again draws current. This cycle is repeated at a frequency determined by the characteristics of the grid transformer.

Due to the low internal drop in the tube, the efficiency of thyratron oscillators, or *inverters* as they are usually called, is quite high, being higher than that of inverted rotary converters. They have the advantages of quiet operation, no wear and quick starting with reasonably low first cost, low maintenance and long life.

Vacuum-tube oscillators find their greatest application in radio communication, as practically the only means for efficiently producing high-frequency alternating currents. Oscillators have been used to produce frequencies of any desired value up to 300,000,000 cycles per sec.

The thyratron inverter is more satisfactory than the regular vacuum-tube oscillator where large currents are involved, due to its low internal drop. The standard vacuum tube has an internal impedance of several thousand ohms and requires up to several

thousand volts for operation. The thyratron operates with an internal drop of approximately 12 volts for all values of the load current. At present thyratron inverters find their chief application in the conversion of direct currents to alternating-current power in small units.

IV. AMPLIFIERS

In Fig. 19.19 is shown the circuit diagram of a vacuum-tube amplifier. The alternating-current input is applied to the grid of the tube through the input transformer. As the grid voltage increases, first, in a positive direction and, then, in a negative

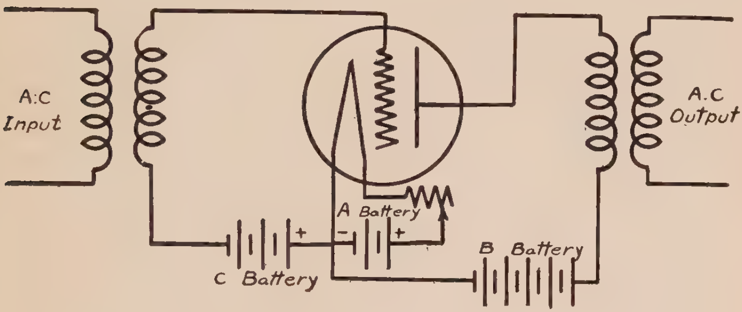


FIG. 19.19.—Circuit diagram of a vacuum-tube amplifier.

direction, the valve action of the tube, explained above under Oscillators, causes the plate current to rise and fall. This pulsating plate current in turn flows through the output transformer and induces an alternating-current voltage in the output circuit. If the potentials of the B and C batteries shown in Fig. 19.19 have the proper values, the alternating-current output will have exactly the same wave shape as the alternating-current input, but its magnitude will be greater as a result of the amplification of the vacuum tube.

It should be noted that all of the energy in the alternating-current input circuit is consumed in supplying losses in the grid circuit; none of it appears in the plate circuit. All of the power output of the plate circuit and the losses in this circuit are supplied by the B battery under control of the grid. The potential of the C battery is made at least equal to the maximum value of the alternating-current input voltage to the grid, in order to keep the potential of the grid always negative with respect to the

filament. If this is not done, the grid current will flow when the grid voltage becomes positive, increasing the losses in the input circuit and causing distortion of the output wave shape.

Vacuum-tube amplifiers are ideal for amplifying the complex wave shapes found in communication work, and are used largely in the radio- and telephone-communication fields. They are also used to a limited extent in controlling relays and protective devices, and in the moving-picture field, to produce and project sound-moving pictures.

CHAPTER XX

INSULATION. THE DIELECTRIC CIRCUIT

I. INSULATION. LEAKAGE CURRENT

The chief function of the dielectric is to provide insulation between conductors so as to confine as much of the electric current as possible to narrow paths, and thus establish definite electric circuits. In this respect a good dielectric is simply a very poor conductor, and hence the possibility of producing well-defined electric circuits depends directly upon the relative conductivity of the materials used in the conductor and the surrounding dielectric. In most dielectrics the resistance is very high, or the conductivity is extremely low as compared to metals. Thus the resistance of glass, rubber, porcelain and similar good insulators is of the order 10^{21} times that of copper, an enormously large ratio. However, in general, experimental data tend to prove that although the resistance may be very large, whenever a difference of potential exists, a current flows through the dielectric and thus forms an electric circuit. This is the *true leakage current.*

In constant-potential systems the dielectric forms an infinite series of parallel circuits, each with a very high resistance. In large systems the cross-section of the dielectric through which the leakage current flows is very large, and hence while at any part the resistance is very high, still over the total section a considerable leakage current may flow. The dimensions of length and cross-section of the dielectric are generally so irregular that it becomes impracticable to determine the average resistance by means of space measurements and the specific resistance of the material. As an illustration, consider the comparatively simple case of the insulation between the several turns in a transformer winding. Any attempt to measure accurately the length and cross-section of the dielectric for determining the leakage conductance would be futile, since the leakage current tends to flow between any two points, at a difference of potential, and not merely between adjacent layers. Hence, while the law

for the leakage current is simple, the practical determination of the average length and cross-section of the irregularly shaped dielectric, by space measurements, makes a direct application impossible. The total equivalent conductance of any machine or system is usually determined by measuring the leakage current for any applied voltage and then applying Ohm's law.

$$ {}_{l}I = {}_{l}gE \tag{1.20} $$

The measurements are made with direct currents. In alternating currents the leakage current is in phase with the voltage, and the power lost is, therefore,

$$ {}_{l}P = {}_{l}IE = {}_{l}rE^2 = {}_{l}r_{l}I^2 \tag{2.20} $$

In all cases the leakage current represents a transformation of electric energy into heat, light or chemical reactions. Since the process is not reversible, the energy is not returned to the circuit when the voltage decreases; hence the loss represents a drain of energy from the electric circuit in the same manner as the resistance losses in the metallic conductor.

Leakage through the dielectric is always undesirable. The energy loss may be small and in most cases the heat generated gives no troublesome rise in temperature, but very often the leakage current causes changes in the chemical composition of the dielectric that rapidly destroy its insulating properties.

Three factors of special importance in affecting both the leakage conductance and the rate of deterioration should be mentioned, namely:

1. Moisture.
2. Temperature.
3. Voltage gradient.

Even very small amounts of moisture affect the leakage conductance to a marked degree. The leakage current is greatly increased and chemical reactions follow that produce deterioration at a more or less rapid rate. In many cases, electrical apparatus cannot be protected from moisture and the selection of the most serviceable insulation for the given conditions must be made. In other cases, like the oil insulation of transformers, extreme care is exercised in removing all the water, and in keeping the apparatus well protected against moisture.

In a paper describing an extended investigation on press-board insulation, the effect of moisture is summarized:

The weakening effect in press-board, and very likely other water-absorbent insulations, may increase as great as the fifth and sixth power of the per cent absorbed moisture. When the free moisture is above 3 per cent, the weakening effects due to its presence are quite pronounced.

The temperature limit for various kinds of electrical apparatus is in almost all cases based directly on the characteristics of the dielectric used. The temperature range for commercial operation is seldom above 100°C. and in most cases considerably less, because at higher temperatures the rate of chemical change becomes too great, causing a rapid deterioration of the insulating properties of the dielectrics. It is therefore, the rate of chemical change in the dielectrics that limits the permissible temperature and the rating of most electrical machinery, as is readily seen from the Standardization Rules of the A.I.E.E. "The weakening effects in insulation, as shown by the dielectric losses, power factors and currents, may increase as great as the fifth or sixth power of the temperature."

The voltage gradient at any point in the dielectric is of greater importance than the total voltage difference between two conductors, in determining the stresses and resultant strains and possible ruptures in the dielectric. Dielectrics are *electrically elastic* and have a more or less definite *elastic limit*, above which the insulation breaks down or ruptures. In the irregularly shaped dielectric between conductors, the stresses are not uniform, and the elastic limit may be reached in the layers near the conductors at a much lower total voltage than at points farther away. Thus the rupture may begin at the surface of the conductor long before the average stress exceeds the elastic limit of the particular dielectric, as illustrated by corona, spark-over, etc. It is therefore necessary to design electrical apparatus so that at no point the voltage gradient exceeds the elastic limit of the dielectric, or to provide safety appliances that will protect the apparatus against excessive voltage gradients.

In solid dielectrics the rupture, produced by a stress in excess of the elastic limit, is a permanent deformation or break, while in liquids and gases the insulating properties of the dielectric are immediately restored as soon as the electric stress is sufficiently reduced. In solids the puncture causes a short-circuit, generally followed by a "burn-out," due to the excessive leakage current flowing through the ruptured dielectric. In gases and

liquids the insulation is automatically restored after the short-circuit arc is broken.

Briefly, the existence of a leakage current premises imperfect insulation. Under ideal conditions with perfect insulation, no leakage current could exist, and all of the electric circuit would be confined to the conductors.

II. THE DIELECTRIC CIRCUIT

When the conductors are at a difference of potential, an electric stress is exerted upon the materials in the intervening space. This stress produces what is equivalent to a strain in the electrically elastic dielectrics. The product of stress and strain represents energy. At potentials less than the rupturing voltage the strain is proportional to the stress and hence the energy stored in the dielectric is proportional to the square of the impressed voltage.

$$\text{Dielectrically stored energy} = \frac{Ce^2}{2} \text{ (see Chap. I)} \qquad (3.20)$$

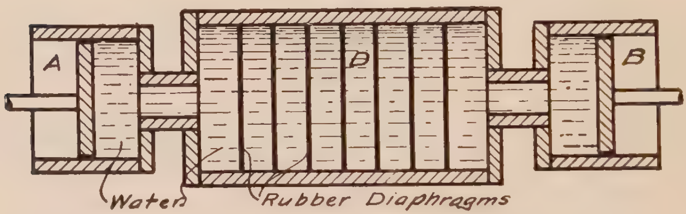


FIG. 1.20.

For voltages within the electrical elastic limit of the dielectric, the process is reversible. Hence the energy flows from the electric circuit into the dielectric while the voltage increases, and returns to the electric circuit while the voltage decreases. This energy flow to and from the dielectric is separate and apart from the leakage current and follows independent laws. The leakage may interfere with the dielectric circuit, as both phenomena occur in the same space, but the basic processes are entirely different. A mechanical analogue will illustrate both the independence and the interference of the two sets of phenomena. Let three cylinders be arranged as shown in Fig. 1.20 with the pistons at the center position in cylinders A and B. Cylinder D is divided into a number of compartments by elastic rubber diaphragms and the space between the pistons is filled with water. Move the two pistons in A and B to the right as

shown in Fig. 2.20. A stress and consequent strain will appear in the rubber diaphragms, and energy is transferred from the pistons to the elastic rubber. If the rubber is perfectly elastic and frictionless, the energy stored in the diaphragm returns to the pistons upon removing the pressure applied by the pistons. Moving the pistons to the left, as in Fig. 3.20, energy is stored in the elastic diaphragms, and again returns to the piston during

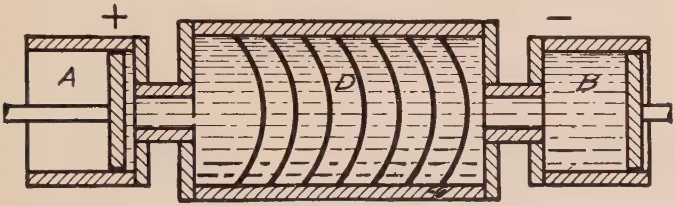


FIG. 2.20.

the return half cycle. This process is analogous to the energy flow in the dielectric circuit. If there are minute holes in the diaphragms allowing a little water to leak between the several compartments, this is analogous to the leakage current. With some water leaking through the diaphragms, the energy stored in the rubber in Figs. 2.20 and 3.20 would soon escape and the strain be removed. The energy thus transferred from the elastic rubber to the leaking water would not be returned to the pistons

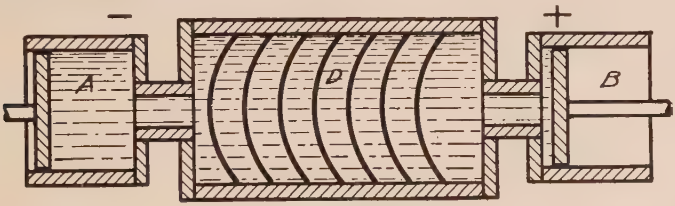


FIG. 3.20.

upon the removal of the applied force. The transfer and the storing of energy in the mechanical model are analogous to the energy changes in the dielectric circuit and the leakage current. The one is due to the elastic properties of the dielectric and the other to imperfections in the insulation. With perfect insulation (no leakage) the dielectric circuit is free from interference; and, conversely, in conductors having little or no electric elasticity and no energy storage there is no dielectric circuit.

The strain in the dielectric is represented both in magnitude and in direction by dielectric lines of force, in precisely the same

way as the intensity and direction of the magnetic field are represented by magnetic lines of force.

The elastance of the dielectric circuit corresponds to the reluctance of the magnetic circuit or the resistance of the electric circuit. Condensance (capacity) is the reciprocal of the elastance and hence corresponds to the permeance of the magnetic circuit or the conductance of the electric circuit. The unit of condensance is the farad (see Chap. III) and of the elastance the daraf.

$$\text{Elastance} = \frac{1}{\text{condensance}};\ S\ (\text{darafs}) = \frac{1}{C(\text{farads})} \qquad (4.20)$$

The specific condensance or permittivity κ is the ratio of the condensance of the given dielectric to that of air. The elastance of any given circuit also depends upon the dimensions of the dielectric and follows the same laws as the resistance of an electric circuit; that is, it varies directly as the length l and inversely as the cross-section A.

$$S = \frac{4\pi l(\text{cm.})}{\kappa A(\text{cm.}^2)};\ \text{or } C = \frac{\kappa A}{4\pi l}\ (\text{abstatfarads}) \qquad (5.20)$$

The factor 4π is introduced by the definition of the unit dielectric line of force (one line per cm.^{2} on the surface of a sphere of unit radius).

To change from the c.g.s. electrostatic (abstat) to the c.g.s. electromagnetic absolute (ab) system of units, the constant v^2 must be introduced in the equations.

$$S = \frac{4\pi v^2 l}{\kappa A}\ (\text{abdarafs})$$

or

$$C = \frac{\kappa A}{4\pi v^2 l}\ (\text{abfarads}) \qquad (6.20)$$

The constant, $v = 3 \times 10^{10}$ cm. per sec., is the velocity of propagation of electric field in space, which is the same as the velocity of light.

The absolute electromagnetic units are reduced to darafs and farads by the factor 10^9.

$$S(\text{darafs}) = \frac{4\pi v^2 l(\text{cm.})}{\kappa A(\text{cm.}^2)10^9} = 113.1\frac{l(\text{cm.})}{\kappa A(\text{cm.}^2)}10^{11} \qquad (7.20)$$

$$C(\text{farads}) = \frac{\kappa A(\text{cm.}^2)10^9}{4\pi v^2 l(\text{cm.})} = 0.8842\frac{\kappa A(\text{cm.}^2)}{l\ (\text{cm.})}10^{-13} \qquad (8.20)$$

$$C(\text{microfarads}) = 0.8842\frac{\kappa A}{l}10^{-7} \qquad (9.20)$$

The relation between the voltage, flux and elastance or condensance in the dielectric circuit is the same as for the voltage, current and resistance or conductance of the electric circuit, and hence may be stated in the same form as Ohm's law.

Dielectric flux $\alpha \frac{\text{voltage}}{\text{elastance}}$, or condensance $\times$ voltage.

$$\psi \, \alpha \, \frac{e}{S} \text{ or } Ce \tag{10.20}$$

Expressed in international or practical c.g.s. units (Chap. III):

$$\frac{\psi(\text{lines of force})}{3.77 \cdot 10^{10}} = \frac{e(\text{volts})}{S(\text{darafs})} = C(\text{farads})e(\text{volts}) \tag{11.20}$$

$$= \frac{\kappa A(\text{cm.}^2)e(\text{volts})10^9}{4\pi v^2(\text{cm.}^2)l(\text{cm.})} \tag{12.20}$$

Hence,

$$\psi = \frac{\kappa A e}{300l} \text{ lines of dielectric force} \tag{13.20}$$

or

$$e = \frac{300l\psi}{\kappa A} \text{ volts} \tag{14.20}$$

The potential or voltage gradient G in a dielectric circuit of constant cross-section A and of homogeneous material is the voltage per unit of length,

$$G = \frac{e}{l} \text{ volts per cm.} \tag{15.20}$$

Let

K be the field intensity under the above given conditions.

$$K = \frac{\psi}{\kappa A} = \frac{e}{300l} \tag{16.20}$$

The flux density D may be defined either, as the product of the permittivity and the field intensity, or as the flux per unit cross-sectional area.

$$D = \kappa K = \frac{\psi}{A} = \frac{\kappa e}{300l} \tag{17.20}$$

The difficulty in applying the above equations to any dielectric circuit comes chiefly from the irregular shapes of the dielectric in electrical appliances. The dimensions are seldom simple geometrical forms. Often it is impracticable to determine either the cross-section or the length. Another factor is the small range in the permittivity of insulating materials, thus making it

impossible to confine the dielectric flux to a definite path as may be done by both the electric and magnetic circuits.

TABLE XV

Material	Permittivity	Material	Permittivity
Glass (easily fusible)	2.0 to 5.0	Air and other gases	1.0
Glass (difficult to fuse)	5.0 to 10.0	Alcohol, amyl	15.0
Gutta-percha	3.0 to 5.0	Alcohol, ethyl	24.3 to 27.4
Ice	3.0	Alcohol, methyl	32.7
Marble	6.0	Bakelite	6.6 to 16.0
Mica	5.0 to 7.0	Benzine	1.9
Paper with turpentine	2.4	Benzol	2.2 to 2.4
Paper or jute impregnated	4.3	Micarta	4.1
Porcelain	5.3	Olive oil	3.0 to 3.2
Rubber	2.4	Paraffin	2.3
Rubber vulcanized	2.5 to 3.5	Paraffin oil	1.9
Shellac	2.7 to 4.1	Petroleum	2.0
Silk	1.6	Turpentine	2.2
Sulphur	4.0		

Three important differences in the constants of the magnetic and dielectric circuits should be noted:

1. The permeability of soft steel or iron may reach several thousand while the permittivity of most dielectrics is less than ten. Hence, while very great changes can be made in the strength of the magnetic field by using iron-clad circuits, only comparatively small changes can be secured in the dielectric field by using different materials (Table XV).

2. The permeability of iron and other magnetic materials varies widely for different magnetic densities, while the permittivity of most dielectrics is constant for all potentials up to the rupturing voltage. The dielectric field has no saturation point, and the strength of a magnetic field is not limited by a breakdown of the magnetic material.

3. The magnetic flux density may be increased indefinitely by increasing the m.m.f., for although the permeability, even in iron-clad circuits, is small above the saturation point, it is always positive. The practical limit for the magnetic density in iron-clad circuits is the saturation point of the steel or iron used. The dielectric field intensity increases directly with the impressed voltage until the insulation is pierced or breaks down. The rupturing voltage, therefore, limits the dielectric flux density.

(*a*) **Condensers in Series and in Parallel.**—Since the relation between the dielectric flux, voltage and elastance or condensance

may be expressed by Ohm's law (equations [10.20] and [11.20]), it follows that the total elastance or condensance may be found for series and parallel arrangements in the same manner as the total resistance or conductance is found for similar electric circuits.

Thus in Fig. 4.20, the total elastance is the sum of the several elastances in series.

For series circuits:

$i = \frac{dq}{dt}$

$Q = CE$

$$S_t = S_1 + S_2 + S_3 + S_4 + \cdots + S_n \qquad (18.20)$$

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \cdots + \frac{1}{C_n} \qquad (19.20)$$

Likewise for parallel circuits, as in Fig. 5.20, the sum of the several condensances is the total condensance.

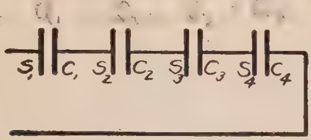


FIG. 4.20.

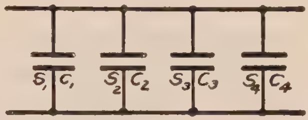


FIG. 5.20.

For parallel circuits: $E_0 = E_1 + E_2 + E_3 + E_4$

$$C_t = C_1 + C_2 + C_3 + C_4 + \cdots + C_n \qquad (20.20)$$

$$\frac{1}{S_t} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \frac{1}{S_4} + \cdots + \frac{1}{S_n} \qquad (21.20)$$

The distribution of the voltage in series circuits and the current in parallel circuits is therefore the same as for the corresponding electric circuits.

(*b*) **Charging Current.**—The current I flowing in the electric circuit, supplying the energy to the dielectric circuit (corresponding to the magnetizing current for the magnetic circuit), is called the *charging*, condensance or capacity current. As energy is stored in and returned from the dielectric, the term *charging* current is appropriate; *but it should be kept clearly in mind that the energy supplied by the charging current is held within the dielectric and not located on the surface of the conductors.* As stated in Chap. I, the charging current is proportional to the rate of change in the voltage; and hence for a sine-voltage wave the current is of the same form and leads by 90°.

$$E = \frac{-j_c I}{2\pi f C} = -j x_c I \qquad (22.20)$$

Since the dielectric circuit is in parallel with the electric circuit in the conductors, it is customary to express the relation between

the voltage and the charging current by a susceptance instead of a reactance.

$$ {}_cI = j\,{}_cbE \qquad (23.20) $$

If there are no losses in the dielectric circuit, the susceptance represents the reciprocal of the reactance, but with energy losses (series resistance, dielectric hysteresis, etc.) the equivalent resistance also enters the equations.

The reactive power, represented by the product of the voltage and the charging current, surges to and fro with double the frequency of the impressed voltage.

$$ {}_xP = {}_cbE^2 = {}_cx\,{}_cI^2 \qquad (24.20) $$

(*c*) **Energy Losses.**—In a constant dielectric field the energy loss in the dielectric is represented by the leakage. When an alternating voltage is impressed, the energy loss is increased. This additional loss is in the dielectric circuit proper and is probably partly due to a molecular friction, similar to magnetic hysteresis, and partly to a viscous give in the elastic medium. From a series of tests in 1901, on commercial condensers of the paraffin-tinfoil type, Steinmetz[1] drew the following conclusions:

1. With a sine wave of impressed e.m.f. the current in the condenser is directly proportional to the impressed e.m.f. and to the frequency; that is, the capacity of the condenser on an alternating current is constant, within the range of the tests (220 to 980 volts; 57 to 133 cycles).
2. The loss of the condenser is, within the range of the tests, proportional to the square of the impressed e.m.f.; that is, the power factor of the condenser at constant frequency is constant and independent of the impressed voltage.
3. The power factor of the condenser does not seem to vary with the frequency; that is, the efficiency of the condenser appears to be independent of the frequency; or, in other words, the loss of energy per cycle is proportional to the square of the electrostatic field strength, but independent of the frequency.
4. The power factor in well-made condensers is extremely low, averaging about 0.005, or the efficiency is about 99.5 per cent.

While the losses have been represented by a variety of empirical equations, no simple form seems to apply to all cases. Expressed

[1] Steinmetz, *Elec. World*, Vol. 37, p. 1065.

in the form of a complex equation, with the conductance and susceptance constants determined for each case, the dielectric circuit admittance is:

$$Y_d = g_d + j\,{}_cb_d \tag{25.20}$$

In combination with the leakage the admittance of the circuits in the dielectric is:

$$Y_d = {}_lg + g_d + j\,{}_cb_d \tag{26.20}$$

The current flowing in the conductor supplying both the energy losses and the reactive power in the dielectric:

$${}_c\dot{I} = Y_d\dot{E} = ({}_lg + g_d + j\,{}_cb_d)\dot{E} \tag{27.20}$$

(*d*) **Residual Charge.**—Due to the heterogeneous nature of insulation materials and compositions, combined with a slow leakage, all the energy stored dielectrically is not released immediately upon short-circuiting the conductors. This is particularly the case with the use of direct currents when the voltage is applied for long periods in one direction. The energy stored in the dielectric at some distance from the conductor must be transmitted through the intervening layers. This transmission is not all made instantaneously and is often complicated by leakage of various amounts in the several layers, and hence an appreciable length of time is required both to energize and to discharge the dielectric. On this account a single discharge of a cable or other apparatus having considerable condensance may not be sufficient. The two sides should be short-circuited for some time or several "groundings" may be necessary to remove the stored energy, and thus prevent any dangerous shock to the workmen.

(*e*) **Dielectric Induction (Electrostatic Influence).**—The e.m.f. in an alternating circuit induces in all nearby conductors, by dielectric induction, voltage opposite in direction but proportional in magnitude. The induced voltage reacts upon the impressed e.m.f. in somewhat the same manner as the two currents in mutual magnetic induction. The induced voltages cause currents to flow in the conductors, thereby changing the distribution of the energy content in the system. As the impressed voltage is continually changing in value, the induced voltage is likewise either increasing or decreasing. This necessitates a flow of current in all nearby conductors with attendant conversion of some electric

energy into heat by the rI^2 losses. The source for this heat energy is in the primary circuit. Therefore, the phenomenon of dielectric induction (electrostatic influence) increases both the condensance and the energy loss through the dielectric. That part of the charging current due to mutual dielectric induction may also be represented by components in phase and in quadrature with the impressed voltage, and the equivalent conductance and susceptance may be included in the notation used in equation (27.20).

(*f*) **Disruptive Strength. Piercing Pressure.**—When the voltage between two conductors is gradually increased the stress on the intervening dielectric likewise increases until the pressure is suddenly equalized by a discharge or formation of a short-circuit through the dielectric. If sufficient energy is supplied to the conductors, the current continues to flow in the form of an arc at a comparatively low voltage, depending upon the resistance of the breakdown path. Instead of stating the disruptive strength of an insulating material in terms of the density of dielectric lines of force, it is customary to give the voltage at which a given thickness of the specified material will be punctured. This is called the *disruptive strength* or *piercing pressure*, and its value depends upon many factors. These may be grouped in two divisions: (*g*) factors affecting the specific dielectric strength of any given material; (*h*) factors affecting the voltage gradient at various points in the dielectric circuits.

Table XVI.—Approximate Piercing Pressure for 1 mm. Thickness

	Volts
Boiled linseed oil	8,000
Dry wood fiber	13,000
Insulating varnish	50,000
Melted paraffin	8,000
Mica	58,000
Micanite	33,000
Oiled linen	12,500
Paraffined paper	30,000
Porcelain	13,000
Transformer oil	9,000
Turpentine	6,500
Vulcanized rubber	10,000

(*g*) **Specific Dielectric Strength.**—The more important factors are: (1) material; (2) moisture; (3) temperature; (4) thickness; (5) mechanical stresses; (6) length of time the voltage is applied.

1. The specific piercing pressure varies widely for different materials, and bears no direct relation to the insulation resistance. Solids like rubber, glass, mica, porcelain or fiber have much higher disruptive strengths than liquids and gases. But in a solid, puncture is not readily repaired, while in liquids and gases the path of the disruptive spark is quickly filled and the dielectric automatically regains its normal strength. For definite chemical compounds, the piercing pressure for a given thickness

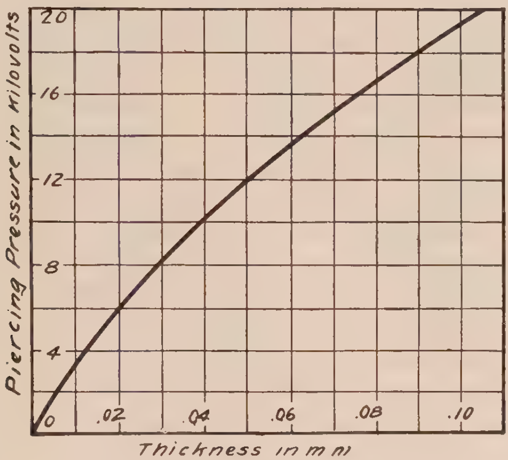


FIG. 6.20.

is fairly constant, but many insulating compositions are mere physical mixtures and a lack of uniformity necessarily produces great variation in the disruptive strength.

2. The presence of moisture, especially in solids and liquids, decreases the disruptive strength. As most materials are more or less hydroscopic, the moisture content must be carefully determined.

3. The temperature within ordinary limits has little effect upon the disruptive strength, provided no chemical changes are produced in the material. For commercial operation the permissible rise in temperature is definitely limited so as to prevent chemical changes in the material.

4. The disruptive strength is not directly proportional to the thickness of the dielectric. Usually the thickness increases faster than the piercing pressure. This is illustrated for mica in

Fig. 6.20. For very thin layers the variation is generally the reverse.

5. In the presence of mechanical stresses the piercing pressure is decreased.

6. The length of time during which the pressure is applied has considerable effect upon the piercing pressure. There seems to be something similar to fatigue or a slightly viscous give in the material, requiring a few seconds or minutes to reach equilibrium. In testing insulation on commercial machines it is customary to apply the pressure for 1 min. when they are warm, as this provides the necessary time duration and also conforms with working temperature conditions.

In some materials like oiled press-boards it requires several minutes for the fatigue to reach a permanent value at any given voltage. For practical application of the factors affecting the dielectric strength a number of empirical formulas have been devised. In commercial work the data given in handbooks and the Standardization Rules of the A.I.E.E. are the main guides in determining the permanent limits for electrical designs.

(*h*) **The Voltage Gradient.**—The voltage gradients in the space surrounding conductors at a difference of potential depend directly on the relative density of the dielectric lines of force. The factors that determine the distribution of the dielectric flux may be discussed under four divisions.

1. One homogeneous dielectric.
2. Two or more insulating materials forming parallel dielectric circuits.
3. Two or more insulating materials forming series dielectric circuits.
4. Combinations of series and parallel circuits.

1. *One Dielectric.*—In the space surrounding two conductors, at a difference of potential, the distribution of the lines of force depends directly upon the geometric form or the dimensions of the dielectric. In dielectric circuits of simple form the flux density and voltage gradient may be readily calculated.

First Example. Lead-covered Cable.—Given a single-conductor, lead-covered cable with an insulating dielectric of one homogeneous material. When a difference of voltage exists between the conductor and the lead sheath, the direction of the stress, Fig. 7.20, and hence the flux, is radial. Since all the lines connect the conductor and the lead sheath, the density is greatest at

the surface of the conductor and least at the lead sheath. Concentric cylinders form equipotential surfaces. From equation (14.20), Fig. 8.20, the voltage required to pass the flux ψ through the elemental cylinder 1 cm. long, of dx thickness and at x distance from the center is:

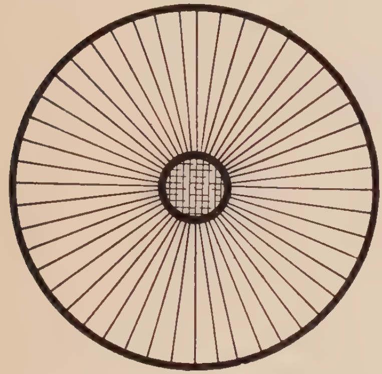


FIG. 7.20.

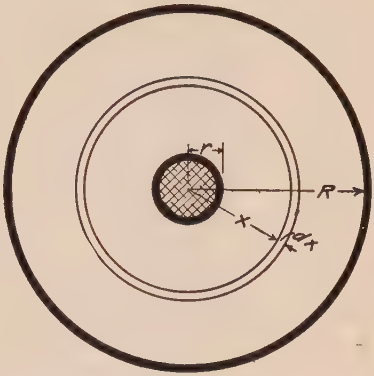


FIG. 8.20.

$$de = \frac{300\psi dx}{2\pi\kappa x} \text{ volts} \tag{28.20}$$

E KA

$dE = \frac{\psi 300 dx}{KA}$

Let

r_1 = radius of conductor.
r_2 = radius (inside) of lead sheath.

The voltage between the conductor and sheath is, therefore:

$$e = \frac{300\psi}{2\pi\kappa}\int_{r_1}^{r_2}\frac{dx}{x} = \frac{300\psi}{2\pi\kappa}\log_\epsilon\frac{r_2}{r_1} \text{ volts} \tag{29.20}$$

$$\psi = \frac{2\pi\kappa e}{300\log_\epsilon\frac{r_2}{r_1}} \text{ lines per cm. length of cable} \tag{30.20}$$

$$G = \frac{de}{dx} = \frac{300\psi}{2\pi\kappa x} = \frac{e}{x\log_\epsilon\frac{r_2}{r_1}} = \frac{0.4343e}{x\log_{10}\frac{r_2}{r_1}} \tag{31.20}$$

For constant voltage applied to any given cable the curve for equation (31.20) is that of a hyperbola referred to its asymptotes. In Fig. 9.20 $r_1 = 5$, $r_2 = 30$ and the ordinates represent either the voltage gradient or the dielectric flux density, for values of x between 5 and 30.

The maximum voltage gradient is at the surface of the conductor:

$$^{M}G = \frac{e}{r_1 \log_\epsilon \frac{r_2}{r_1}} = \frac{0.4343e}{r_1 \log_{10} \frac{r_2}{r_1}} \tag{32.20}$$

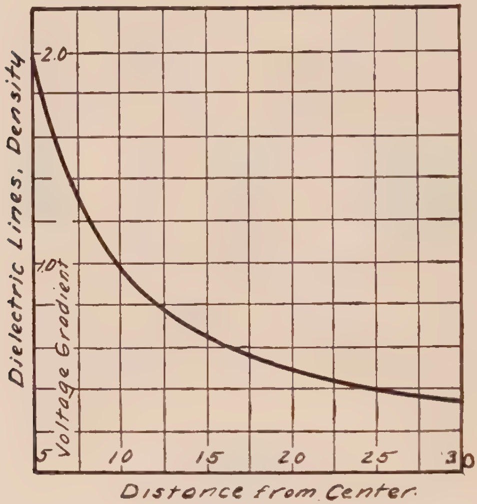


FIG. 9.20.

Second Example. Two Parallel Plates.—If the plates are of considerable size, the distribution of the flux, in the area between the plates, is uniform except at or near the edges. Near the

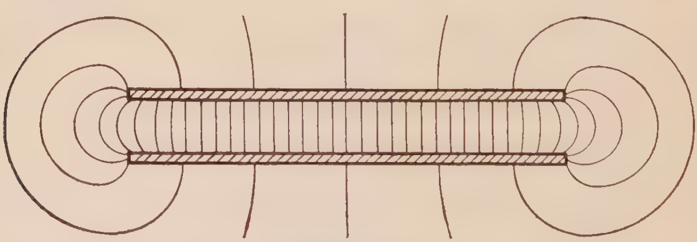


FIG. 10.20.

edges the density is greater, and at points outside, less than in the space between the plates, Fig. 10.20.

Let

x = the distance between the plates.

A = the area over which the flux is uniformly distributed.

$$\text{The elastance, } S = \frac{4\pi v^2 x}{\kappa A 10^9} = (\text{a constant})\,\frac{x}{A}, \text{ darafs} \tag{33.20}$$

$$\text{The condensance, } C = \frac{\kappa A 10^9}{4\pi v^2 x} = (\text{a constant})\,\frac{A}{x}, \text{ farads} \tag{34.20}$$

$$\text{Dielectric flux, } \psi = \frac{3.77 \cdot 10^{10} e}{S} = 3.77 \cdot 10^{10} Ce = \frac{\kappa A e}{300x} = (\text{a constant})\,\frac{Ae}{x} \text{ lines} \tag{35.20}$$

$$\text{Flux density, } D = \frac{\psi}{A} = \frac{\kappa e}{300x} = (\text{a constant})\,\frac{e}{x}, \text{ lines per } (\overline{\text{cm.}})^2 \tag{36.20}$$

$$\text{Voltage, } e = \frac{300x\psi}{\kappa A} = (\text{a constant})\,\frac{x\psi}{A} \text{ volts} \tag{37.20}$$

$$\text{Voltage gradient, } G = \frac{de}{dx} = \frac{300\psi}{\kappa A} = (\text{a constant})\,\frac{\psi}{A}, \text{ volts per cm.} \tag{38.20}$$

Third Example. Two Parallel Round Conductors in Air.—The most important practical example is the transmission line and the discussion deals with the problem under the restrictions of high-tension transmission-line conditions. It is therefore assumed that:

a. The diameters of the two conductors are equal.

b. The distance between the conductors is large as compared with the diameter of each wire.

Since the two conductors have constant diameters and are parallel in position, equipotential surfaces must be cylinders and hence the distribution of the lines of force is the same in any normal plane. All phases of the problem may therefore be shown on a cross-section perpendicular to the conductors. Consider a neutral plane midway between the conductors and perpendicular to the plane joining their centers, as indicated by the line NN' in Fig. 11.20. The two wires are equally positive and negative referred to the line NN'. When taken independently, the fields for each are radial straight lines. The resultant field is the superposition of the separate fields, and the flux lines assume curved paths as shown in Fig. 12.20. These curved paths are arcs of circles passing through the points A and B and hence have their centers along the line NN'. This may be shown as in Fig. 13.20. Take any point P in a normal

plane. Let $AP = s$ and $BP = u$. The dielectric stress or field intensity at the point P due to the negative voltage of A is

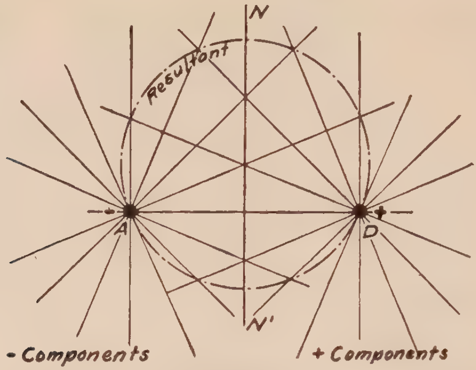


Fig. 11.20.

along PA and inversely proportional to the distance s. Likewise the stress due to the positive voltage of B is in the direction

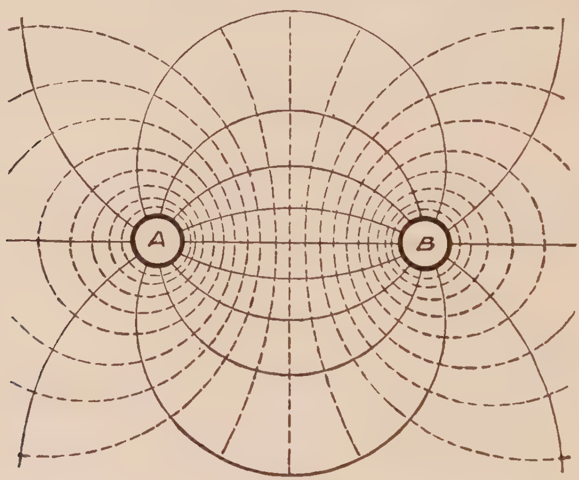


Fig. 12.20.

BP and inversely proportional to the distance u. Lay off the lines PN and PH in Fig. 13.20, so that

$$PN:PH::\frac{1}{u}:\frac{1}{s} \tag{39.20}$$

Complete the parallelogram $PHKN$. Then the line PK represents, in direction and relative magnitude, the dielectric field

intensity at the point P. In the two triangles APB and PHK two sides are proportional and the included angles equal; hence the triangles are similar. Therefore, the angles ABP and HPK are equal. Hence the line PK must be tangent to a circle passing through the three points A, P and B. Since P is any point in the normal plane, the direction of the dielectric lines of force between the points A and B is along arcs of circles passing through A and B.

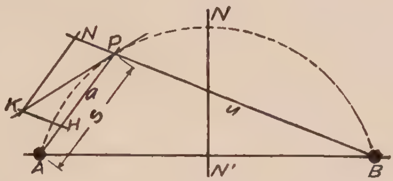


FIG. 13.20.

The equipotential surfaces are perpendicular at every point to the direction of the flux lines and therefore must be cylinders with their axes in the plane passing through both conductors, as indicated by the equipotential circles, dotted lines in Fig. 12.20.

From the diagram it is evident that the maximum flux density and voltage gradient must be in the plane passing through the centers of the two conductors, or along the line XX in Figs. 12.20 and 14.20. Since the maximum values are of special impor-

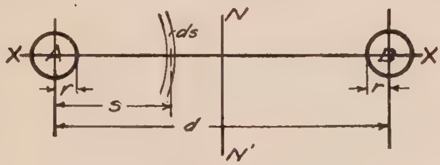


FIG. 14.20.

tance in commercial problems, and in order to simplify the expressions, the discussion will be limited to the plane represented by the line XX.

Let

d = the distance between the centers of the conductors.
r = the radius of each conductor.
s = distance of any point a from the center of conductor A.
K_a = dielectric field intensity at point a.
D_a = dielectric flux density at point a.
ψ = total flux.

κ = (permittivity of air) = 1.0.
e_n = voltage to neutral.

At the point a the dielectric flux density due to the voltage difference from A to NN' is proportional to $\frac{1}{s}$; and for the voltage difference from NN' to B it is similarly proportional to $\frac{1}{d - s}$: at any point in the plane of two parallel conductors the total flux density is the scalar sum of the flux density from each conductor. Hence, at any point a on a line joining the centers of conductors A and B in Fig. 14.20 and at a distance s from conductor A, the flux density is given by equation (40.20).

$+ \frac{\psi}{2\pi(d-s)}$

$$D_a = \frac{1}{2\pi}\psi\left(\frac{1}{s} + \frac{1}{d - s}\right) \text{ lines per cm.}^2 \qquad (40.20)$$

The voltage absorbed in any elemental zone ds is:

$$de_n = 300 D_a ds$$

$$= \frac{300}{2\pi}\psi\left(\frac{1}{s} + \frac{1}{d - s}\right) ds \text{ volts} \qquad (41.20)$$

The total voltage from A to neutral NN' is, therefore,

$= D \frac{300 l}{\kappa}$

$$e_n = \frac{300}{2\pi}\psi \int_r^{\frac{d}{2}} \left(\frac{1}{s} + \frac{1}{d - s}\right) ds \text{ volts} \qquad (42.20)$$

$$= \frac{300}{2\pi}\psi \log_\epsilon\left(\frac{d - r}{r}\right) \text{ volts} \qquad (43.20)$$

Therefore,

$$\psi = \frac{2\pi e_n}{300 \log_\epsilon\left(\frac{d - r}{r}\right)} \text{ lines} \qquad (44.20)$$

Voltage gradient:

$$G = \frac{de_n}{ds} = \frac{300}{2\pi}\psi\left(\frac{1}{s} + \frac{1}{d - s}\right) \qquad (45.20)$$

$$= \frac{e_n}{\log_\epsilon\left(\frac{d - r}{r}\right)}\left(\frac{1}{s} + \frac{1}{d - s}\right) \qquad (46.20)$$

$$= \frac{e_n d}{(sd - s^2)\log_\epsilon\left(\frac{d - r}{r}\right)} \text{ volts per cm.} \qquad (47.20)$$

Maximum voltage gradient:

$$^mG = \frac{e_n d}{(rd - r^2)\log_\epsilon\left(\frac{d - r}{r}\right)} \text{ volts per cm.} \qquad (48.20)$$

Under the assumption that r is small as compared to d, which is the case in transmission-line problems, the maximum voltage gradient becomes:

$$^{M}G = \frac{e_n}{r \log_\epsilon \left(\frac{d}{r}\right)} = \frac{0.4343 e_n}{r \log_{10} \left(\frac{d}{r}\right)} \tag{49.20}$$

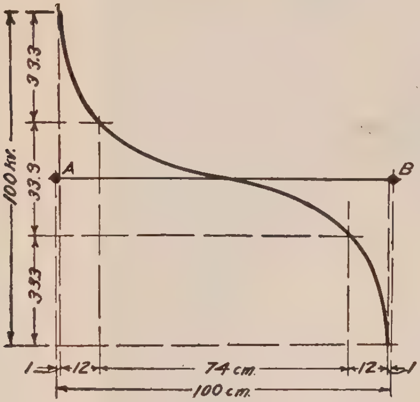


FIG. 15.20.

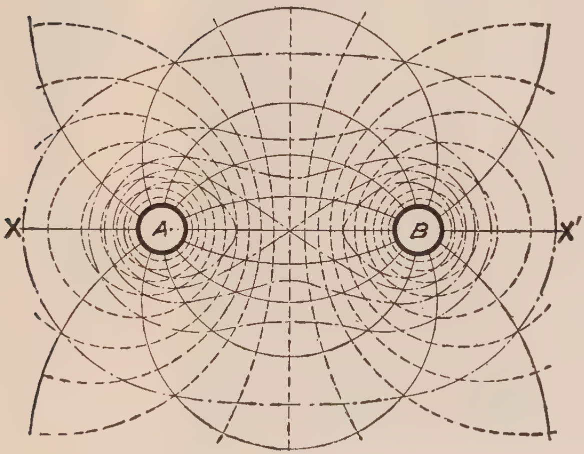


FIG. 16.20.

For e, the total voltage between conductors, the maximum gradient:

$$^{M}G = \frac{e}{2r \log_\epsilon \left(\frac{d}{r}\right)} = \frac{0.4343 e}{2r \log_{10} \left(\frac{d}{r}\right)} \tag{50.20}$$

In Fig. 15.20 the voltages are plotted as ordinates with the distances from the wires as abscissæ in the plane through the wires. The broken lines in Fig. 16.20 show the equigradient lines in the field around the conductors.

FIG. 17.20.—448,000-volt flashover test on one of the early condenser bushings. (*Westinghouse Electric and Manufacturing Company.*)

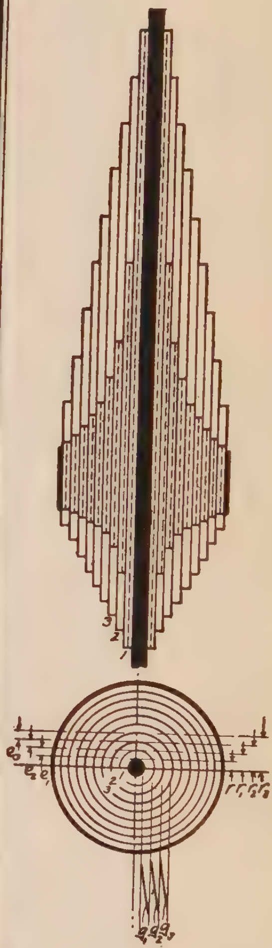


FIG. 18.20.

Fourth Example. Condenser Bushings for High-tension Apparatus.—By means of concentric cylindrical layers differing in length as well as in diameter it is possible to obtain practically

the same voltage gradient in all parts of the insulating layers. The total cylindrical insulation is divided into several concentric cylinders around the transformer lead with the layers separated by metal sheaths (tinfoil). The cylinder near the central conductor is longest and the outside layer in contact with the transformer casing shortest as indicated by a cross-sectional view in Fig. 18.20. The heavy line at the outer surface shows the metal holder or ring by which the condenser bushing is attached to the transformer case.

Let the radius of the conducting lead be r, and the radii of the successive tinfoil cylinders be $r_1, r_2, r_3, \ldots$ Let $e_1, e_2, e_3, e_4, \ldots$ be the corresponding voltages between $r - r_1$, $r_1 - r_2$, $r_2 - r_3$, $r_3 - r_4, \ldots$ as indicated in Fig. 18.20.

The potential gradients at the conductor surface and at the outside surfaces of the successive tinfoil cylinders (that is, on the inside surface of the insulating cylinders) are:

$$g_\circ = \frac{e_1}{r \log_\epsilon \frac{r_1}{r}};\ g_1 = \frac{e_2}{r_1 \log_\epsilon \frac{r_2}{r_1}};\ g_2 = \frac{e_3}{r_2 \log_\epsilon \frac{r_3}{r_2}};\ \cdots \tag{51.20}$$

With the same material in the several insulation cylinders, the flux density and hence the dielectric gradient should be the same for the most desirable distribution of the flux, hence:

$$g_\circ = g_1 = g_2 = g_3 = \cdots \tag{52.20}$$

The numerical value of the gradient depends upon the material used and the assumed factor of safety, usually about 25 per cent less than the rupturing voltage. The gradient through each insulating cylinder is greatest at the inner and least at the outer surface, as was shown under the first example. The permissible difference must be assumed for any design in much the same manner as the selection of a suitable factor of safety. This assumed ratio determines the approximate thickness of each insulating cylinder, since the density at the outer and inner surfaces necessarily must be inversely as the radii. Hence the values of $e_1, e_2, e_3, e_4, \ldots$ may be found from equation (51.20).

Let it be assumed that all the flux between the transformer lead and casing passes through the insulating cylinders and hence that there is no leakage through the surrounding air. The error introduced by this assumption will be noted in the next para-

graph. The flux ψ passing through the successive insulating cylinders is constant for any given total voltage e, and since:

$$e = e_1 + e_2 + e_3 + e_4 + \cdots \tag{53.20}$$

$$\psi = 3.77 \cdot 10^{10} Ce, \text{ and, therefore}$$

$$C_1 e_1 = C_2 e_2 = C_3 e_3 = C_4 e_4 = \cdots \tag{54.20}$$

or the condensances of the several cylinders are inversely proportional to the corresponding voltages. From equations (11.20) and (30.20), the condensance of a cylindrical insulator between two conductors, as the above tinfoil layers, is:

$$C = \frac{\kappa l}{1.8 \cdot 10^{12} \log_\epsilon \frac{r_2}{r_1}} \text{ farads} \tag{55.20}$$

l = the length in cm.
r_2 = the outside radius of insulating cylinder.
r_1 = the inside radius of insulating cylinder.

Hence,

$$\frac{l_1 e_1}{\log_\epsilon \frac{r_1}{r}} = \frac{l_2 e_2}{\log_\epsilon \frac{r_2}{r_1}} = \frac{l_3 e_3}{\log_\epsilon \frac{r_3}{r_2}} = \cdots \tag{56.20}$$

By assuming a value for the length of either the inner or the outer layer, the length of all the other cylinders may be calculated. The shaded portion in Fig. 18.20 gives the shape of the bushing as determined from the above equations. It is, however, more convenient to manufacture a form tapering uniformly as shown in Fig. 22.20. This modifies the voltage distribution, increasing the voltage gradient near the casing and the lead and proportionately decreasing it in the middle layers.

To prevent hydroscopic action it has been found advantageous to give the part of the bushing above the oil a smooth cylindrical form.

Fifth Example. Oil-filled Bushings for High-tension Systems.—The oil-filled bushing has a combination of liquid and solid insulation arranged so as to produce an effective puncture structure against both sustained and transient voltages. Oil has negligible dielectric losses and can withstand a high percentage of its puncture voltage a long time. The solid insulation employed (paper cylinders), while subject to dielectric losses, is adequately cooled by the automatic circulation of the oil. The

combination of the solid and oil insulation provides ample *time lag* against high-voltage impulses, so that the bushing will flash over at lower voltage impulses than would be required to puncture the insulation. Moreover, the oil impregnates the paper cylinders and thus prevents the progressive damage that repeated impulses have on dry and brittle insulations.

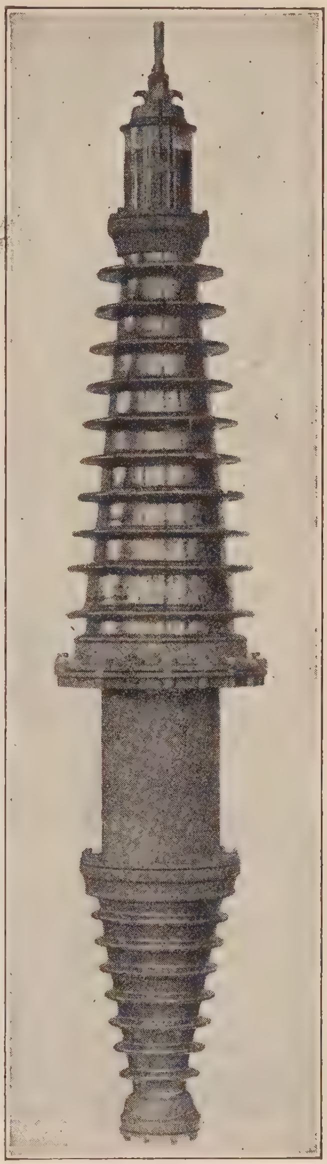


FIG. 19.20.—High-voltage bushing oil filled. Type CF1. (*General Electric Company.*)

2. *Two or More Dielectrics in Parallel.*—Let the space between two metallic plates be divided into three equal parts, filled with rubber, air and glass as indicated in Fig. 21.20. With a difference of voltage existing between the plates, the density of the dielectric flux in the three insulating materials will be directly proportional to their permittivities, or approximately as 3:1:5. By varying the impressed voltage, the flux density also changes, but the relative flux densities are still in the same ratio. The voltage distribution in the space may be represented by a straight line, Fig. 21.20; and the voltage gradient $G = \frac{de}{dx}$ is a constant. In order to illustrate the elemental principles it was assumed that there is no leakage, *i.e.*, the dielectric flux is confined to a definite path, as is the case with electric circuits. Under actual conditions there are no sharp divisions between good and poor conductors of dielectric flux. A good dielectric like glass has a permittivity only about five times that of air or of a vacuum. Hence, in the above illustration there would be no material

FIG. 20.20.—Dry arc-over at 395 kv. of standard 135 kv., type CF high-voltage bushing. (*General Electric Company.*)

available for separating the rubber and the air, or the glass and the air. Likewise, the flux cannot be confined to the direct path between the two plates but also passes through the air on the sides. In the discussion of the condenser bushings in the last paragraph, it was assumed that all the flux between the casing and

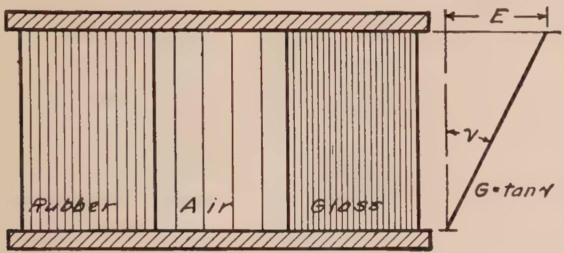


FIG. 21.20.

the transformer lead passed through the insulating cylinders. However, since the permittivity of the solid material is probably at best only four or five times as great as that of the surrounding air, it is evident that a considerable share of the flux leaks around the solid dielectric. The flux density for any path is in all cases proportional to the permittivity of the material, and inversely proportional to the length. The flux distribution may be represented as in Fig. 22.20; and the equivalent dielectric circuits by Fig. 23.20.

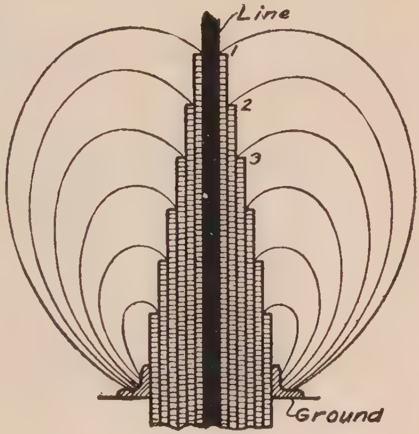


FIG. 22.20.

3. *Two or More Dielectrics in Series.*—Let three equal thicknesses of rubber, air and glass fill the space between two parallel metallic plates, as indicated in Fig. 24.20; and consider only that portion having uniform distribution of the field flux. The dielectric flux passes through the three materials in series and hence the flux densities in the glass, air and rubber are the same. The voltages required to force the flux through equal distances of the three materials are inversely proportional to the permittivities.

This may be indicated graphically by plotting the relation between volts and length of flux path as in Fig. 24.20. For each material the curve is a straight line, but the slope differs in the three cases. The voltage gradients are therefore in the ratio of $\frac{1}{3}:\frac{1}{1}:\frac{1}{5}$ or as 5:15:3. Hence while the air occupies only one-third of the space it takes $\frac{15}{23}$ of the total voltage. The voltage gradient in the air is therefore almost twice as great as would be the case if the rubber and glass were removed. If the voltage gradient is near the break-down strength of air alone between the plates, the insertion of the rubber and glass would so increase the gradient in the air that corona and sparks would appear. For example, let the voltage between the plates be 20 kv. times the distance in cm. apart. With air alone as the dielectric the voltage gradient would be 20 kv., which is below the rupturing voltage. After inserting the rubber and glass

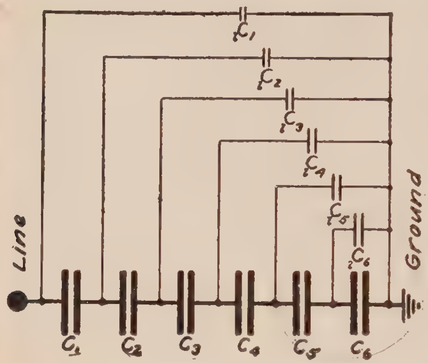


Fig. 23.20.

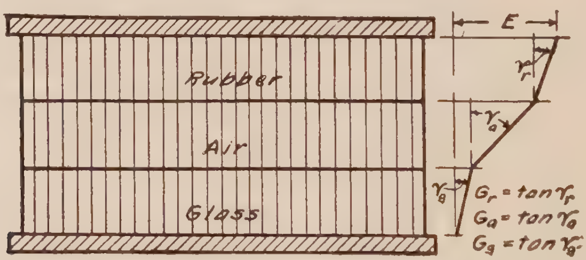


Fig. 24.20.

plates the voltage gradient in the air is 39.1 kv., far above its dielectric strength, and the air space is aglow with corona and spark discharges.

4. *Combination of Series and Parallel Circuits.*—From the discussion of the simple series and parallel dielectric circuits it is seen that the laws are similar to those of the electric circuits. In principle, therefore, the combination of series and parallel

circuits offers no difficulties, and the laws may be easily formulated. The calculation of the numerical values for electrical appliances, however, is in most cases very difficult because of the irregular shape of the dielectric. Electrical conductors are usually of simple geometric forms. Wires, bars, ribbons, cables, etc., whose dimensions are readily determined, form the bulk of resistance problems in electric circuits. In the dielectric circuit the same simple law applies: namely, the elastance varies directly as the length and the specific constant of the material used and inversely as the cross-section, but the shape of the dielectric makes it very difficult to determine the dimensions. In most cases it is manifestly impossible to measure either the length or the cross-section of the dielectric circuit, and as a consequence the exact distribution of the dielectric flux cannot be calculated.

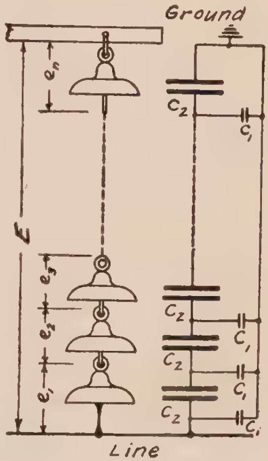


FIG. 25.20.

(*i*) **Suspension Insulator.**—As an example of a simple case consider the dielectric circuits in a suspension insulator. Let the several units be of the same size and type, and hence the condensances of the several insulators are equal as indicated in Fig. 25.20. The dielectric flux is, however, not entirely confined to the path through the string of insulators, since the condensance between the metal parts and the ground is comparable with the condensance across the insulator. Hence, the actual dielectric circuit between the transmission line and the supporting arm (ground) of the tower consists of a combination of series and parallel condensances, c_1 and c_2, as indicated in Fig. 25.20. The total condensance of the string of n insulators is, therefore, not equal to $\frac{c_2}{n}$ but depends upon the number, arrangement and relative values of c_1 and c_2.

$$\text{Let } c_2 = kc_1. \tag{57.20}$$

The total condensance of the suspension insulator may be found as follows:

For one insulator:

$$C_1 = c_1 + c_2 = c_1(1 + k) \tag{58.20}$$

For two insulators:

$$C_2 = c_1 + \cfrac{1}{\cfrac{1}{c_1 + c_2} + \cfrac{1}{c_2}} = c_1 + c_2 - \frac{c_2^2}{2c_2 + c_1} \tag{59.20}$$

For three insulators

$$C_3 = c_1 + \cfrac{1}{\cfrac{1}{C_2} + \cfrac{1}{c_2}} = c_1 + c_2 - \frac{c_2^2}{c_2 + C_2}$$

$$= c_1 + c_2 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{2c_2 + c_1}} \tag{60.20}$$

For a string of n insulators:

$$C_n = c_1 + c_2 - \frac{c_2^2}{c_2 + C_{n-1}}$$

$$= c_1 + c_2 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{\cfrac{\cdots}{2c_2 + c_1}}}}} \tag{61.20}$$

The fraction is continued to $n - 1$ of the $2c_2 + c_1$ terms.

The voltage distribution is therefore not uniform but greatest across the insulator nearest the line and least across the insulator next to the cross-arm. Let E = the total voltage across a suspension insulator of n units, and e_1 the voltage across the first unit nearest the line, then:

$$\psi = \psi_2 + \psi_1 \tag{62.20}$$

or

$$C_n E = c_2 e_1 + c_1 E \tag{63.20}$$

Hence,

$$e_1 = \frac{E}{k}\left(\frac{C_n}{c_1} - 1\right) \tag{64.20}$$

Voltage across the second insulator,

$$e_2 = e_1 - \frac{E - e_1}{k} \tag{65.20}$$

Voltage across the third insulator,

$$e_3 = e_2 - \frac{E - (e_1 + e_2)}{k} \tag{66.20}$$

Voltage across the nth insulator,

$$e_n = e_{n-1} - \frac{E - (e_1 + e_2 + e_3 + \cdot \cdot \cdot + e_{n-1})}{k} \tag{67.20}$$

$C_1 +$ $2C_2 + C_1$ $2C_2 + C_1$ $2C_2 + C_1$ $2C_2 +$

or

$$E = \frac{e_1 k}{\dfrac{C_n}{c_1} - 1} \qquad (68.20)$$

Hence, if ${}_ae_1$ is the arc-over voltage for one unit, the arc-over strength of the string of n units is:

$$ {}_aE = \frac{{}_ae_1 k}{\dfrac{C_n}{c_1} - 1} \qquad (69.20)$$

The chief factor that makes it difficult to calculate dielectric flux distribution is the small range in specific condensance or permittivity of insulating materials including the vacuum. The electric circuits may be confined to narrow definite paths because of the enormous difference in the resistance of good conductors, like copper, and poor conductors, like rubber, air or a vacuum. On account of this difference in range of specific resistance and of specific elastance the electric circuit can be confined to the wires in a transmission line with very small leakage; while the dielectric circuit embraces all the space surrounding the conductors. Combinations of several insulating materials may, to a certain extent, modify the distribution of the dielectric flux, but cannot confine it entirely to any definite path. Fortunately, the relative distribution of the dielectric flux is of little importance unless the voltage gradient at any point approaches or exceeds the dielectric strength of the material. For all low-voltage apparatus no calculations are necessary. For high-voltage systems the dielectric circuit and the flux distribution are of great importance and apparatus for continuous operation must be so designed that at no point shall the maximum voltage gradient exceed the dielectric strength of the insulation.

PROBLEMS

1.20. Given a transmission line having suspension insulators with nine disks in each string. $E = 150{,}000$ volts between mains. Star connected with neutral grounded. Let $k = 9.5$.

Find the voltage across each disk.

Plot curve with the number of disks on the axis of abscissæ and the voltage across the disks as ordinates.

2.20. A three-phase transmission line of 4/0 hard-drawn copper wires has 60,000 volts between the mains. Spacing of wires: 6 ft., equilateral triangle. Find the maximum voltage gradient at surface of conductors.

CHAPTER XXI

CORONA, SPARK-OVER

Under ordinary conditions and for low voltages, air is very nearly a perfect insulator and its dielectric properties are almost the same as for a vacuum. In other words, the conductivity of air is practically zero, the permittivity unity and the energy loss extremely small. For higher voltages the air insulation may break down, either partially or completely, causing important changes in the electric and dielectric circuits.

For illustration, consider two parallel, smooth, bare wires, spaced as in a commercial high-tension transmission line, with dry, clean air as the dielectric. Let the room be dark, or if the wires are strung in the open, let the observations be taken at night. If the voltage be gradually increased until the gradient near the wires reaches 30 kv. per cm. (^{M}E) a faint purplish light may be seen at or near the surface of the wires, and a hissing sound may be heard. By still further increasing the voltage both the luminous glow and the hissing sound increase in volume and intensity. The phenomenon, of which the light and sound are indications, is called the *corona.* A full understanding of the laws of the corona is of great importance in designing long-distance transmission lines, and all other high-voltage apparatus.

The lowest voltage gradient at which corona appears, a fairly definite value, is called the *critical visual-corona voltage.* If the wires are far apart in comparison to their diameters, as is the case in transmission lines, the voltage may be increased far beyond the critical visual-corona value before a disruptive spark passes between the wires. The voltage at which the spark breaks through the intervening air is called the *disruptive* or *spark-over voltage,* and this is also a fairly definite value, which may be calculated from the dimensions of the system. As soon as spark-over occurs, the corona disappears. If the impressed voltage is only a little above the disruptive strength of the air, the spark-overs will be intermittent. After each spark passes, the corona reappears and persists until the next spark disrupts the insulating

air. The corona may be considered as a partial, and the spark-over as the complete breakdown of the insulation; and, although differing widely in their characteristics, they are fundamentally two stages of the same group of phenomena. Since corona depends directly upon changes in the insulation of the air, the phenomena relate primarily to the electric circuit (leakage) between the wires. Changes in the insulation, however, with the resulting variations in the leakage current, modify the condensance, and therefore the corona indirectly also affects the dielectric circuit.

(*a*) **Theory.**—Extended discussions of various physical theories, some of which are largely "metaphysical fairy tales," by which groups of physical phenomena may be coördinated or "explained," are outside the scope of this book. The laws of energy transmission and transformation form the natural foundation upon which the engineer may safely build. Terms like *magnetic and dielectric fields or lines of force, current, potential,* etc., are merely pictorial illustrations which enable the student to use the *visual memory method.* In discussing dielectric phenomena in the preceding chapters the mechanical analogue of stress and strain in an electrically elastic medium has been used for the same purpose. It is essential that the student should fully realize that dielectric phenomena consist of *energy changes in time and space,* and follow just as simple laws as do the better-known phenomena of the electric and magnetic circuits. The stress and strain analogue may be used to advantage when studying the corona and spark-over phenomena because the attention is kept focused on the energy changes causing the phenomena, which are of fundamental importance.

Following the epoch-making experiments on cathode rays by J. J. Thomson (1897), a large number of physicists, chemists and engineers have conducted extensive researches on the electric conductivity of gases in general, and of air in particular. Thomson showed that the particles moving in the cathode stream were much smaller than the ordinary atom or molecule; in fact, only about $\frac{1}{1{,}848}$ of the mass of a hydrogen atom. Moreover, it was found that these *corpuscles* or *electrons* invariably carried a charge of negative electricity of 4.774×10^{-10} c.g.s. electrostatic units, which is approximately the quantity carried in electrolysis by a hydrogen ion. That these quantities are not

merely averages but that the individual units are of equal magnitude has been shown experimentally by Millikan[1] in an elegant and convincing manner.

From experimental observations it is known that the conductivity of air and other gases is greatly modified by the presence of radioactive substances, by the action of the cathode rays and ultraviolet light, by electric stress causing corona and by other factors. According to the electron theory, the changes in the conductivity of the air are due to similar variations in the number of free electrons, and the conduction of the electric energy through the air is accomplished by the space movements of electrons and ions. Under normal conditions the electron is combined with, or connected to, an ordinary molecule. The rapid motion and frequent collisions of the molecules in a gas often knock off an electron carrying a negative charge and leaving a positively charged ion of practically the same mass as the original molecule. Various combinations are possible between the free electrons, the positive ions and the neutral molecules. An electron combining with a positive ion forms a neutral molecule; positive ions may combine with neutral molecules forming larger aggregate masses, etc. It is evident that so many combinations, and combinations of combinations, are possibile that almost any range of experimental data may be accounted for or explained by the electron theory.

Since it is always advantageous to observe, as directly as possible, the energy changes that cause any physical phenomenon, a minimum of theoretical scaffolding is desirable. For this reason the simple stress-and-strain principle has an advantage over the more elaborate electron theory. The quantitative relations between the several factors, that determine the appearance and extent of the corona and spark-over on transmission lines or other electrical apparatus, have been derived from experimental data and are now available in the form of empirical equations. The researches of Ryan,[2] Mershon[2] and Steinmetz[2] and especially the extensive investigations of Peek[2] and Whitehead[2] are of great importance, since they deal with the problem from the engineer's point of view and give summaries of the results in forms that may readily be applied to engineering problems.

[1] Millikan, R. A., *Phil. Mag.*, Vol. 19, p. 209, February, 1910.
[2] *Trans. Am. Inst. Elec. Eng.* Vols. 28–34.

(*b*) **Visual Corona.**—The more important factors that enter into the equations for determining the visual-corona voltage on parallel wires are:

1. The diameter of the wire.
2. The spacing of the wires.
3. The distance from the wire.
4. The condition of the conductor surface.
5. The air density.
6. The shape of the voltage wave.

Other factors, like the conductor material, the amount of current flowing in the wire, the frequency within ordinary commercial range and the humidity of the air, have little if any effect on the starting voltage of visual corona.

Factors 1, 2, *and* 3.—The first three factors may be discussed together. Let the conductors be wires with smooth surfaces, like those used in transmission lines; the air temperature and pressure normal (25°C., 76 cm.); and the voltage wave of a simple sine form.

From equation (49.20) the maximum voltage gradient between parallel wires is at the surface of the conductor;

$$ {}^{M}G = \frac{e_n}{r \log_\epsilon \frac{d}{r}} \tag{1.21} $$

It has, however, been found experimentally that the corona appears on wires of different sizes for a constant gradient at a distance of $0.301\sqrt{r}$ from the surface of the conductor, and not for a constant gradient at the surface itself. The value of the voltage gradient at the distance $0.301\sqrt{r}$ at which visual corona is produced is 29.8 kv.-cm., or approximately 30 kv.-cm. For sine waves this is the maximum value and hence the corresponding effective values are 21.1 and 21.2 kv.-cm.

Let

${}_ve$ = visual-corona voltage.

${}_0G$ = visual-corona gradient (at a distance $0.301\sqrt{r}$ from the surface of the conductor), a constant.

$x = r + 0.301\sqrt{r}$ in cm.

d = distance between centers in cm.

${}_vG$ = visual-corona gradient at the surface of the wire.

$$ {}_gG = \frac{{}_ve}{x \log_\epsilon \frac{d}{r}} = \frac{{}_ve}{(r + 0.301\sqrt{r}) \log_\epsilon \frac{d}{r}} = \frac{0.4343\,{}_ve}{(r + 0.301\sqrt{r}) \log_{10} \frac{d}{r}} \tag{2.21} $$

$$ {}_gG = \frac{{}_ve}{r \log_\epsilon \frac{d}{r}} = {}_0G\left(1 + \frac{0.301}{\sqrt{r}}\right) \text{ kv. per cm.} \tag{3.21} $$

Hence,

$$ {}_ve = {}_0G\left(1 + \frac{0.301}{\sqrt{r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \tag{4.21} $$

For sine voltage waves:

$$ {}^m_vE = 29.8\left(1 + \frac{0.301}{\sqrt{r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \tag{5.21} $$

$$ {}_vE = 21.1\left(1 + \frac{0.301}{\sqrt{r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \tag{6.21} $$

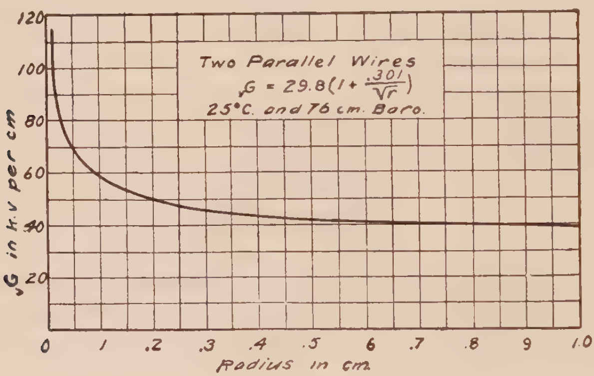


FIG. 1.21.

The relation between the surface gradient ${}_gG$ and the radius of the wire as given by equation (3.21) is shown graphically in Fig. 1.21. This curve agrees very closely with smooth wires at 25°C. temperature and 76-cm. pressure. If it be assumed that a definite amount of work is required to rupture the electrically elastic air, it follows that a stress in excess of the elastic limit must exist over a certain distance before the breakdown occurs in order that the product of stress and strain may supply the required energy. Hence the smaller the wire, the less is the distance, and therefore the higher the voltage gradient at the surface of the wire.

Factor 4.—For conductors with rough surfaces or for cables the voltage gradient for visual corona is less than for smooth wires. A correction may be introduced by using an *irregularity factor* m as in equation (7.21). The value of the irregularity factor varies greatly, and, in general, must be determined by direct measurements. Dirt, oil and water on the surface of the conductor also affect the gradient for visual corona and the necessary corrections may be included in the irregularity factor m.

$$ {}_vG = {}_oGm\left(1 + \frac{0.301}{\sqrt{r}}\right) \text{ kv. per cm.} \tag{7.21} $$

Factor 5.—Changes in air density, whether produced by changes in temperature or barometric pressure, affect the voltage gradient of visual corona. Normal conditions are given by 25°C. temperature and 76-cm. pressure. The *density factor* for any temperature t, in centigrade degrees, and any barometric pressure b, in cm. of mercury, is given by equation (8.21).

K = 3.92

$$ \delta = \frac{3.92b}{273 + t} = \frac{K(76)}{273+25} \tag{8.21} $$

Including both the density and irregularity factors, the expressions for the surface gradient and voltage to neutral required to produce visual corona are:

(m) is less than 1

$$ {}_vG = 29.8m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) \text{ kv. per cm.} \tag{9.21} $$

$$ {}_ve = 29.8m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \tag{10.21} $$

For sine voltage waves:

$$ {}^m_vE = 29.8m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \tag{11.21} $$

$$ {}_vE = 21.1m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \tag{12.21} $$

For smooth wires $m = 1$ and for normal temperature (25°C.) and pressure (76 cm.) $\delta = 1$.

Factor 6.—For peaked voltage waves the effective voltage is less and for flat-topped waves greater than in equation (12.21). The variation depends upon the distortion and no general equation has been derived.

Concentric Cylinders.—For smooth concentric cylinders the voltage gradient required to give visual corona near the inside

cylinder is slightly larger than for parallel wires, as shown in equation (13.21).

$$_vG = 31.0\delta\left(1 + \frac{0.308}{\sqrt{\delta r}}\right) \text{ kv. per cm.} \qquad (13.21)$$

Spheres.—For spheres of equal diameters and spaced like the parallel wires, far enough apart to give corona at a lower voltage than required for the disruptive spark, the voltage gradient at the surface is:

$$_vG = 27.2\delta\left(1 + \frac{0.54}{\sqrt{\delta r}}\right) \text{ kv. per cm.} \qquad (14.21)$$

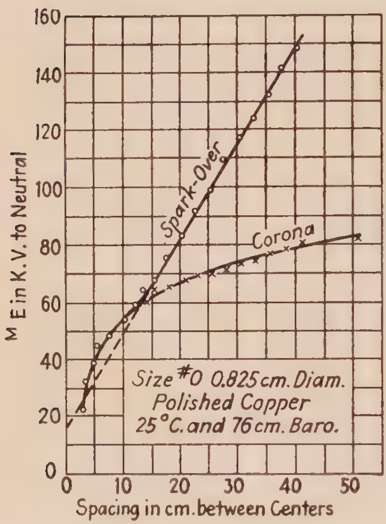


FIG. 2.21.

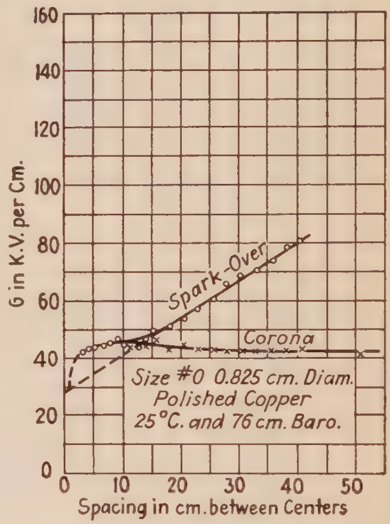


FIG. 3.21.

(*c*) **Spark-over.**—In visual corona the air breaks down near the wires in the space filled with the luminous glow. If the voltage is increased, the corona increases in volume and intensity. By increasing the voltage still further until the stress becomes sufficient to pierce all of the intervening dielectric, a disruptive spark passes between the wires. At small spacing the spark-over may occur before the corona. A certain critical spacing, depending upon the size of the wire, must be used in order to have a decided difference in voltage for visual corona and spark-over. This may be seen from the experimental curves shown in Figs. 2.21 and 3.21. The lower curve shows the voltage required to produce corona at the different spacings, while the upper curve

shows the corresponding voltages required to produce spark-over. Below the point where the two curves intersect the spark-over occurs before the voltage gradient is sufficient to produce corona. The corona and spark-over curves (extended) intersect at a voltage gradient of approximately 30 kv.-cm.

The ratio between the spacing d and the radius of the wires for the point where the corona and spark-over curves intersect is found to be $\frac{d}{r} = 30$. Hence, if it be assumed that the spark-

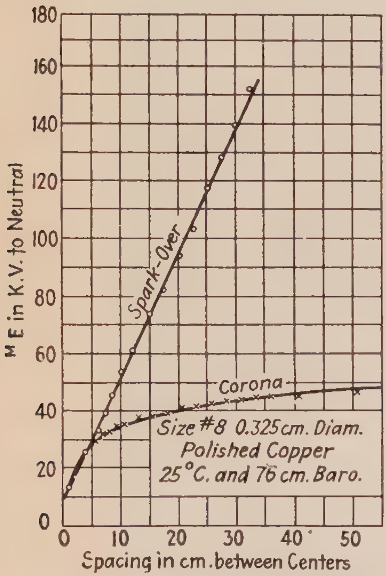


FIG. 4.21.

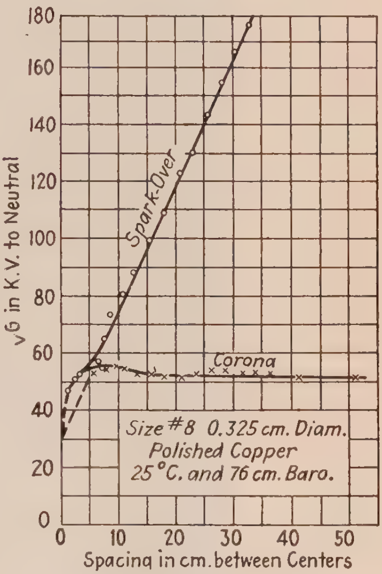


FIG. 5.21.

over gradient is a straight line, it is determined by these two points. Therefore, the spark-over gradient ${}_sG$, under normal conditions, is:

$$ {}_sG = {}_oG\left(1 + \frac{0.301}{\sqrt{r}} \cdot \frac{d}{r} \cdot \frac{1}{30}\right) \text{ kv. per cm.} \qquad (15.21) $$

$$ = 30\left(1 + \frac{0.01d}{r\sqrt{r}}\right) \text{ kv. per cm.} \qquad (16.21) $$

At the point of intersection of the two curves the ratio $\frac{d}{r} = 30$, and the spark-over gradient equals the visual-corona gradient in equation (13.21).

Corrections for changes in air density may be made by means of the density factor δ.

$$_sG = 30\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) \text{ kv. per cm.} \qquad (17.21)$$

The corresponding spark-over voltage,

$$_se = {_sG}r \log_\epsilon \frac{d}{r} = 30\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \qquad (18.21)$$

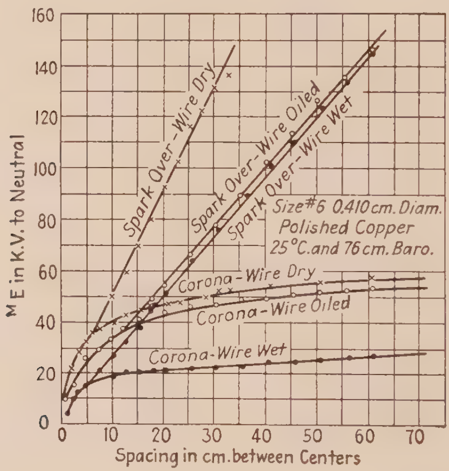


FIG. 6.21.

For sine voltage waves:

$$^M_sE = 30\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \qquad (19.21)$$

$$_sE = 21.2\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) r \log_\epsilon \frac{d}{r} \text{ kv. to neutral} \qquad (20.21)$$

The effect of water or oil on the surface of the wires upon the spark-over voltage is shown in Fig. 6.21.

A simple apparatus for experimental work on the corona and spark-over between parallel wires is shown in Fig. 7.21. Each wire is supported by two insulators having shields made from the sheath of a lead-covered cable. The wires must be kept taut and carefully spaced. The voltage gradient is less on the shields than on the intervening wires, and visual corona will form along the whole length of wire between the shields.

The Sphere Spark Gap.—The general solution of the voltage gradient between spheres of any radius R, and of any spacing D, leads to complicated expressions that cannot be readily applied to practical problems. The voltage gradient is at a

FIG. 7.21.—Apparatus for measuring voltage for visual corona. The rims of the insulators are covered by lead-sheath shields.

maximum for the points of intersection of the surfaces with the line joining the centers of the two spheres. For spheres of equal radii, R cm., and spaced a distance, D cm., between the surfaces, and with a difference of e kv. potential, the maximum voltage gradient is:

$$^{m}G = \frac{e}{D}(f) \text{ kv. per cm.} \tag{21.21}$$

The factor f is a function of R and D and for ungrounded spheres may be expressed as in equation (22.21).[1]

$$(f) = \frac{1}{4}\left[\frac{D}{R} + 1 + \sqrt{\left(\frac{D}{R} + 1\right)^2 + 8}\right] \qquad (22.21)$$

The values of f for different ratios of D and R are given in Table XVII.

TABLE XVII

$\frac{D}{R}$	f	$\frac{D}{R}$	f
0.0	1.0000	0.8	1.2881
0.1	1.0337	0.9	1.3268
0.2	1.0681	1.0	1.3660
0.3	1.1032	1.5	1.5687
0.4	1.1390	2.0	1.7808
0.5	1.1754	3.0	2.2247
0.6	1.2124	4.0	2.6861
0.7	1.2500	5.0	3.1583

With one sphere grounded, different values must be used, since the dielectric field is not symmetrical. The distance from the ground also affects the flux distribution.

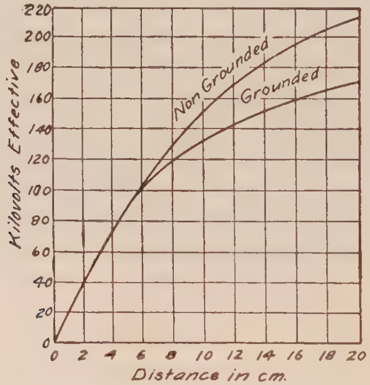


FIG. 8.21.—Spark-over curves for spheres 12.5 cm. in diameter.

In commercial work the sphere spark gap is important as a convenient instrument for measuring high voltages. For spheres of diameters greater than the spacing, spark-over occurs at a lower voltage than that required to form corona. For spacings

[1] DEAN, *Gen. Elec. Rev.*, Vol. 16, p. 148.

within the limits $0.5\sqrt{R}$ and $2R$ the voltage gradient is practically constant.

$$_{s}G = 27.2\left(1 + \frac{0.54}{\sqrt{\delta R}}\right)\text{kv. per cm.} \qquad (23.21)$$

The spark-over voltage for different spacings may be found directly from the curves in Figs. 8.21 and 9.21. It is essential, when using the sphere spark gap for measuring high voltages, that a high resistance be inserted in the leads to prevent an excessive current flowing across the arc and to eliminate abnormal voltages. For this purpose glass tubes filled with water are well adapted. The dimensions of the tubes and the quality of the water should be adjusted so as to limit the current in the spark-over arc from ¼ to 1 amp.

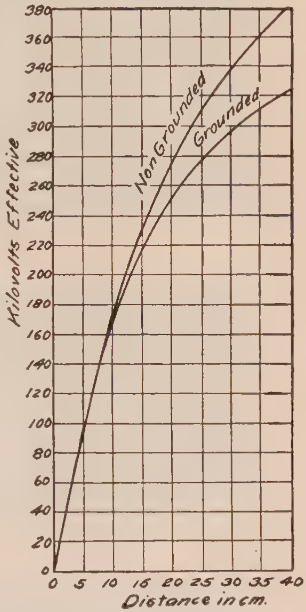


FIG. 9.21.—Spark-over curves for spheres 25 cm. in diameter.

(*d*) **Corona Energy Losses.**—If the voltage between two parallel wires is raised beyond the point required for the visual corona the energy losses rapidly increase. Under normal atmospheric conditions and for smooth wires and constant frequency the energy loss varies as the square of the voltage in excess of the critical disruptive voltage.

Let

$_{a}W$ = energy loss in the air.
E = voltage to neutral.
$_{d}E$ = critical disruptive voltage.

Then,

$$_{a}W \propto (E - {}_{d}E)^2 \qquad (24.21)$$

The critical disruptive voltage is obtained when the gradient at the surface of the wire equals $_{o}G$.

The values for $_{d}E$ for different spacings and sizes of wire are obtained from the relation between voltage gradient, spacing and size of wire already derived. $_{d}E = {}_{o}Gm\delta\, r \log_\epsilon \frac{d}{r}\cdot$ For trans-

mission lines ${}_oG = 21.1$ (effective values). For cables between 0.583- and 1.18-cm. diameter and various spacings, ${}_oG = 18.1$ kv.-cm.

For higher frequencies the losses increase but not in direct proportion. There seems to be a loss proportional to the frequency superimposed upon a constant loss, and the relation between the energy loss and the frequency may be expressed as in equation (25.21).

$$ {}_aW \propto (f + 25)(E - {}_aE)^2 \qquad (25.21) $$

Correction for variations in air density and surface roughness may be made by introducing the density factor δ and irregularity factor m into the equation for the critical disruptive voltage ${}_aE$.

$$ {}_aE = \delta m \, {}_oGr \log_\epsilon \frac{d}{r} \qquad (26.21) $$

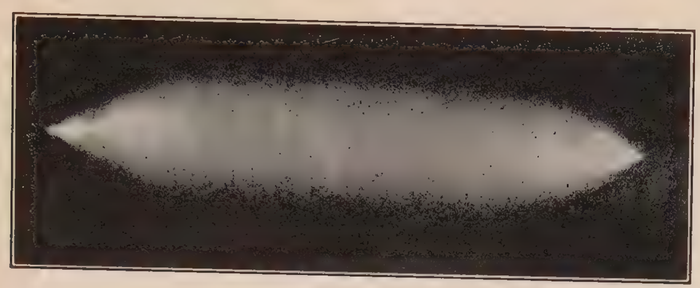
The equation for the energy loss above the critical voltage for visual corona as derived from experimental data in kilowatts per kilometer is:

$$ {}_aW = 241 \frac{\sqrt{r}}{\delta\sqrt{d}}(f + 25)(E - {}_aE)^2 10^{-5} \text{ kw./km.} \qquad (27.21) $$

(*e*) **The Alternating-current Corona Cycle.**—Let a sufficiently high voltage, of sine-wave shape and of any commercial frequency, be impressed on two parallel, smooth wires, properly spaced, so as to give corona without spark-over. The appearance of the corona is a continuous uniform glow or luminous brush surrounding the wires. However, if observations be taken through a stroboscope, it is found that the corona pulsates in synchronism with the impressed voltage. By turning the stroboscope through 360 electrical degrees it may be seen that the corona disappears twice each cycle (that is, during the portions of the cycle when the instantaneous voltage is less than the required value to produce visual corona); and that the appearance of the corona for the positive and negative half waves is decidedly different, Fig. 10.21. The luminous brush extending a distance from the wires is generated by the positive half wave, while the negative half produces a bright glow in close contact with the wire. With the neutral grounded, the luminous brush or positive corona appears alternately on the two wires, but with ordinary frequencies the pulsations overlap so rapidly that when viewed

without the aid of a stroboscope the corona appears to be continuous.

(*f*) **The Dielectric Circuit.**—From the preceding discussion it is seen that the corona primarily affects the electric circuit (leakage) between the conductors. The corona results from a partial



(*a*) Without stroboscope—72,000 volts.

− (*b*) With stroboscope—72,000 volts. +

+ (*c*) Stroboscope rotated 180°—72,000-volts. −

Fig 10.21.

breakdown of the insulation with a resultant leakage current and corresponding energy loss. In the space filled by the corona the resistivity is less than for air under normal conditions. As a consequence of this change in the resistance of the layer near the conductors, the condensance of the dielectric circuit is increased,

because the insertion of a layer of comparatively low resistance, as compared to the normal air, around the wires, is in effect an increase in the conductor diameter, and this increases the condensance of the dielectric circuit. The specific resistance of the corona, while less than for normal air, is still very much greater than for metallic conductors. On this assumption the corona introduces a fairly large resistance into the dielectric circuit in series with the condensance, through which the reactive energy flows when passing to and from the electrically elastic air outside the corona zone. The increase in the condensance may be only apparent as the increase in condensance current may be due to harmonics produced by the pulsating corona. For alternating currents the corona and the resistance of the electric circuit (leakage) through the air, and likewise the condensance and series (corona) resistance of the dielectric circuit, pulsate in synchronism with the impressed voltage, with alternate values for the successive half waves.

PROBLEMS

1.21. Calculate the required voltage to neutral to give visual corona on two smooth parallel wires of No. 2 A.W.G. copper, for 5-, 10-, 15-, 20-, 25- and 30-cm. spacing between centers. Barometer = 74 cm. Temperature = 20°C. Plot curves with spacing as abscissæ and visual-corona voltages as ordinates.

2.21. Find the spark-over voltage for No. 2 copper wires spaced as in problem 1.21. Plot curve with spark-over voltage as ordinates and distance between centers as abscissæ.

3.21. Calculate the voltage gradients at the surface of the wire corresponding to the spacings and size of wire given in problem 1.21 with 50,000 volts alternating current, one-phase between wires. Plot the voltage gradients as ordinates and distance between centers as abscissæ.

4.21. Same as problems 1.21, 2.21 and 3.21 but with No. 12 wire.

5.21. An alternating-current voltage of 24,000 volts, 60-cycle, is impressed on two No. 00 A.W.G. copper conductors placed at spacings of 2, 4, 6, 8, 12, 16, 20, 24, 36 and 48 in. Find the maximum surface gradients for the several spacings. Plot curves with voltage gradients as ordinates and spacings as abscissæ.

6.21. In problem 5.21 let the wire be No. 16 A.W.G. Find the voltage gradients and plot curve as in problem 5.21.

7.21. What is the corona loss at 150,000 volts on 10 miles of single-phase line consisting of No. 0000 wire with 10-ft. spacing? What size of wire is required to eliminate the corona with the same voltage? Elevation 3,000 ft.

60 ~ degrees C

$\log_e \frac{d}{r} = \frac{3.97}{r}$

$\frac{d}{r} = (\varepsilon)^{\frac{3.97}{r}}$

$d = r(\varepsilon)^{\frac{3.97}{r}}$

CHAPTER XXII

SHORT TRANSMISSION LINES

In direct-current systems the voltage drop on the transmission line is equal to the product of the resistance and the current. The voltage at the generator is the arithmetical sum of the voltage at the receiver and the line drop, and hence the voltage at the load is always less than at the generator. For alternating currents the transmission-line problem is much more complex. The magnetic field surrounding each wire and the dielectric field between the wires as well as the ohmic resistance of the wires are distributed along the transmission line, thus forming an interwoven network of series and parallel circuits over the whole length of the line. Through this complex circuit, power is transmitted to a load that may vary both in magnitude and in power factor. The voltage and current values along the transmission line depend not only upon the magnitude of the component parts but also upon their time-phase and space-phase relations; and the problem of voltage regulation becomes radically different from the simple conditions existing in direct-current systems. The voltage at the load may be less, equal to or even greater than the voltage at the generator. The voltage along the line may be greater than at either end, and the conditions may be so adjusted as to keep automatically constant voltage at three points on the system for wide variations of load. The basic factors in the problem are the resistance, inductance and condensance of the transmission line, the so-called *line constants;* and it will be necessary first to derive expressions by which these may be found quantitatively for any given line.

(*a*) **Line Resistance.**—In commercial calculations the ohmic resistance is generally used, and only in special cases are corrections for obtaining the *effective resistance* introduced.

R = resistance.
l = length of wire.
r = radius of wire.
ρ_0 = resistivity at 0° temperature.

ρ_t = resistivity at $t°$ temperature.

α = temperature coefficient, usually taken at 0.0042 between 0 and 50°C. for an initial temperature of 0°.

$\rho_t = \rho_0(1 + \alpha t)$.

The resistance varies directly as the length and inversely as the cross-section of the conductor.

$$R_t = \frac{\rho_t l}{\pi r^2} \tag{1.22}$$

(*b*) **Line Inductance.**—As the inductance in a circuit is a measure of the magnetic field produced by unit current, it depends upon the size and spacing of the conductors and the permeability of the conductors and the surrounding medium. Let the con-

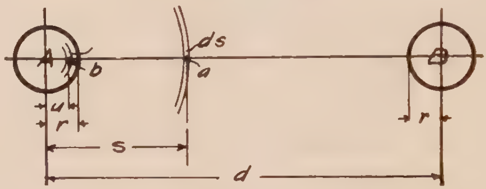


FIG. 1.22.

ductor be a cylindrical wire in a single-phase circuit, Fig. 1.22.

$\mathcal{H}_a$ = strength of the magnetic field, at any point *a* outside the wire.

$\mathcal{H}_b$ = strength of the magnetic field, at any point *b* inside the wire.

s = the distance from *a* to the center of the wire in cm.

u = the distance from *b* to the center of the wire in cm.

$\mathcal{B}_a$ = flux density at *a*.

$\mathcal{B}_b$ = flux density at *b*.

r = radius of the wire in cm.

d = distance between centers of the conductors in cm.

μ' = permeability outside the wire.

μ = permeability inside the wire.

I = current in amperes.

Inside the conductor with uniform current density the m.m.f. for any cylindrical shell a centimeter in length and of u radius equals

$0.4\pi I \frac{u^2}{r^2}$ and the reluctance is proportional to $\frac{2\pi u}{\mu du}$.

The number of lines in a cylindrical shell du of radius u and 1 cm. long is $d\phi_b$.

$$d\phi_b = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{0.2\mu I u du}{r^2} \tag{2.22}$$

As these lines interlink only with that part of the current lying inside of the corresponding cylindrical shell it is desirable to find the equivalent flux, which, multiplied by the total current, equals the integral of the products of the separate lines into their respective parts of the current.

Let ϕ_{0-r} be the equivalent flux surrounding the whole current.

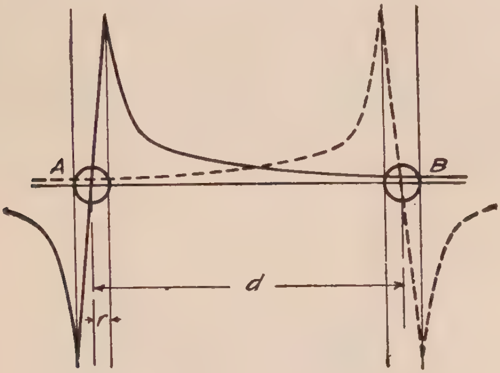


Fig. 2.22.

$$\phi_{0-r} = \int_0^r \frac{\pi u^2}{\pi r^2} d\phi_b = \int_0^r \frac{0.2\mu I u^3 du}{r^4} = \frac{\mu I}{20} \text{ lines of force} \tag{3.22}$$

Hence the flux inside the wire is equivalent to $\frac{\mu I}{20}$ lines interlinking the total current I in the wire, and the equivalent inductance $= \frac{\mu}{20}$.

$$\mathcal{H}_b = \frac{\text{m.m.f.}}{\text{length of path}} = \frac{0.2Iu}{r^2} \tag{4.22}$$

$$\mathcal{B}_b = \mu\mathcal{H}_b = \frac{0.2\mu I u}{r^2} \tag{5.22}$$

Outside the conductor, Fig. 1.22, at a distance s from the center of the wire, and for 1-cm. length of wire,

$$\mathcal{H}_s = \frac{0.4\pi I}{2\pi s} = \frac{0.2I}{s} \tag{6.22}$$

$$\mathcal{B}_a = \frac{0.2\mu' I}{s} \tag{7.22}$$

$$d\phi_a = \mathcal{B}_a ds = \frac{0.2\mu' I ds}{s} \tag{8.22}$$

$$\phi_{r-d} = \int_r^{d-r} \frac{0.2\mu' I ds}{s} = 0.2\mu' I \log_\epsilon \left(\frac{d-r}{r}\right) \tag{9.22}$$

Hence total flux per centimeter length of conductor:

$$\phi = \phi_{r-d} + \phi_{o-r} = 0.2I\mu' \log_\epsilon \left(\frac{d-r}{r}\right) + \frac{\mu I}{20} \tag{10.22}$$

$$L = \frac{\phi}{I \cdot 10^8} = \left[0.2\mu' \log_\epsilon \left(\frac{d-r}{r}\right) + \frac{\mu}{20}\right] 10^{-8} \text{ henrys} \tag{11.22}$$

The flux distribution for each wire in a single-phase line is shown diagrammatically in Fig. 2.22 and the total flux for both wires in Fig. 3.22.

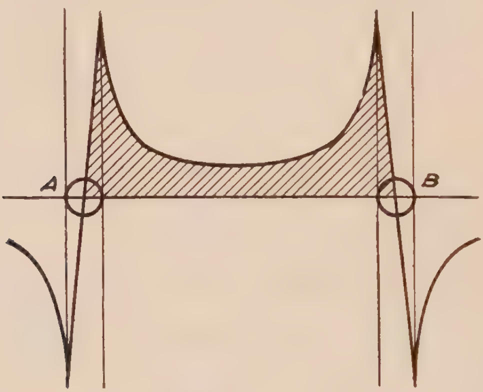


Fig. 3.22.

For copper, aluminum and all non-magnetic materials the permeability is unity; hence for most transmission lines

$$\mu = \mu' = 1. \tag{12.22}$$

Therefore, in copper or aluminum aerial single-phase transmission lines,

$$L = \left[2 \log_\epsilon \left(\frac{d-r}{r}\right) + 0.5\right] 10^{-9} \text{ henrys per cm. of conductor} \tag{13.22}$$

Transforming from the Napierian to the common system of logarithms,

$$L = \left[4.61 \log\left(\frac{d-r}{r}\right) + 0.5\right]10^{-9} \text{ henrys per cm. of conductor}$$

$$= 3.048 \left[4.61 \log\left(\frac{d-r}{r}\right) + 0.5\right]10^{-5} \text{ henrys per 1,000 ft. of conductor} \quad (14.22)$$

$$= \left[74.1 \log\left(\frac{d-r}{r}\right) + 8.05\right]10^{-5} \text{ henrys per mile of conductor} \quad (15.22)$$

In electrical handbooks, the corresponding reactances for 25, 60 and 100 cycles are given in tabular form, for the more common spacings and sizes of wire. For iron wire μ is not unity and the inductance inside the wire varies with the permeability.

(*c*) **Line Condensance.**—When a difference of potential exists between two conductors a stress is exerted upon the dielectric between the conductors. This stress produces the equivalent of

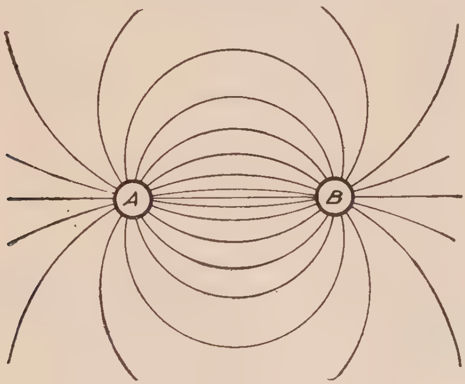


FIG. 4.22.

a strain and the product of the stress and strain measures the energy stored in the dielectric. Thus in Fig. 4.22 let A and B represent the two conductors (wires) of a single-phase transmission line. The voltage applied to the line exerts a stress and produces a strain in the dielectric (air) between the wires. With alternating currents the stress and the accompanying strain increase until the maximum point of the wave is reached and then decrease to zero, reverse, reach a maximum in the other direction and again decrease to zero. Thus electric energy is stored in the dielectric and returned to the electric circuit twice during each voltage cycle. The strain is usually

represented by dielectric lines of force or dielectric flux, its relative magnitude by the flux density, and the total strain by the total number of lines of force. The distribution of the stress and strain in the dielectric circuit follows simple laws similar to those of the magnetic and electric circuits. In Fig. 5.22, let A and B represent the two wires in a single-phase line.

d = distance between centers of the conductors.
r = radius of each wire.
s = distance of a point a from center of conductor A.
κ = permittivity. For air $\kappa = 1$.
K_a = dielectric-field intensity at a point a.
$D_a = \kappa K_a$ = dielectric-flux density at a point a
$v = 3 \cdot 10^{10}$ cm. per sec. (velocity of light).
NN' = neutral plane.
E_n = the voltage to neutral.

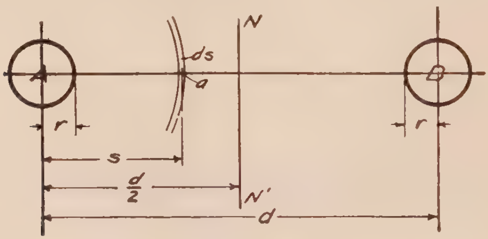


FIG. 5.22

The dielectric stresses are symmetrical with respect to the neutral plane, and the voltage from the neutral plane is positive to one wire and negative to the other.

As the total stress in the dielectric is the same for any path between the two conductors, the discussion may be confined to the condition along the straight line connecting the centers. The flux density along this line is proportional to the total dielectric flux between the wires for different values of d and r, and the proportionality factor is determined by the system of units employed. Since the dielectric stress varies inversely as the distance from the center of each wire and directly as the voltage, the dielectric-flux density at the point a is expressed by equation (16.22).

$$D_a = \frac{\Psi}{2\pi}\left[\frac{1}{s} + \frac{1}{d - s}\right] \text{ lines per } \overline{\text{cm.}}^2 \qquad (16.22)$$

The voltage absorbed in any elemental zone is equal to the product of the flux density and the distance.

$$de = 300\, D_a ds \text{ volts.} \tag{17.22}$$

The total voltage from conductor A to neutral is therefore the integral of equation (17.22) between the limits of r and $\frac{d}{2}$.

$$E_n = 300 \int_r^{\frac{d}{2}} D_a ds \text{ volts} \tag{18.22}$$

Hence: $$\frac{E_n}{\Psi} = \frac{300}{2\pi} \int_r^{\frac{d}{2}} \left[\frac{1}{s} + \frac{1}{d - s} \right] ds = \frac{300}{2\pi} \log_\epsilon \left(\frac{d - r}{r} \right) \tag{19.22}$$

From Table IV:

$$C = \frac{\Psi}{3.77 \cdot 10^{10} E_n} = \frac{\Psi}{3 \cdot 4\pi \cdot 10^{11} E_n} \text{ farads} \tag{20.22}$$

Combining equations (19.22) and (20.22):

$$C = \frac{1}{18 \cdot 10^{11} \log_\epsilon \left(\frac{d - r}{r} \right)} \text{ farads} \tag{21.22}$$

$$= \frac{10^9}{2v^2 \log_\epsilon \left(\frac{d - r}{r} \right)} \text{ farads} \tag{22.22}$$

For circuits in which r is small, as compared to d, equations (21.22) and (22.22) may be simplified:

$$C = \frac{10^9}{2v^2 \log_\epsilon \left(\frac{d}{r} \right)} = \frac{1}{18 \cdot 10^{11} \log_\epsilon \left(\frac{d}{r} \right)} \text{ farads} \tag{23.22}$$

For microfarads and common system of logarithms,

$$C = \frac{0.241}{10^6 \log \frac{d}{r}} \text{ mf. per cm. of wire to neutral} \tag{24.22}$$

$$= \frac{7{,}360}{10^6 \log \frac{d}{r}} \text{ mf. per 1,000 ft. of wire to neutral} \tag{25.22}$$

$$= \frac{3.883}{100 \log \frac{d}{r}} \text{ mf. per mile of wire to neutral} \tag{26.22}$$

In electrical handbooks the *charging currents* for a wide range of wire sizes and spacings and for standard frequencies are given in tabular form. It is generally more convenient to use the charging current in commercial problems than to derive the microfarads by equation (26.22).

$$ {}_c\dot{I} = 2\pi f C\dot{E} = {}_c b\dot{E} \tag{27.22} $$

(*d*) **Velocity of Propagation of an Electric Field.**—The speed at which the electric field travels through space may be expressed in terms of the inductance and condensance of the transmission line.

From equation (9.22):

$$ L' = 2\log_\epsilon\left(\frac{d-r}{r}\right), \text{ the inductance outside the conductor in air per cm. of length of line} \tag{28.22} $$

From equation (21.22):

$$ C = \frac{1}{2v^2\log_\epsilon\left(\frac{d-r}{r}\right)}, \text{ the condensance outside the conductor in air per cm. of length of line} \tag{29.22} $$

Therefore,

$$ CL' = \frac{1}{v^2};\ \text{or},\ v = \frac{1}{\sqrt{CL'}} \tag{30.22} $$

Due to retardation produced by the losses the actual speed is somewhat less. Since v, the velocity of propagation of an electromagnetic field (same as the speed of light), is known, it is possible to calculate either the inductance or the condensance when the other is given. This method is sometimes used in commercial problems when it is difficult to measure both quantities.

(*e*) **Line Leakage.**—In addition to the resistance, inductance and condensance which, as already shown, may be calculated with a fair degree of accuracy, or obtained from tables in electrical handbooks, another factor, more or less constant, enters into the transmission-line problem. The dielectric between the conductors is not a perfect insulator and hence a current in phase with the voltage leaks through the dielectric throughout the whole length of the line. As a result, power is dissipated along the transmission line in proportion to the product of the leakage current and the line voltage. The amount of this leakage and

the consequent power loss are not so readily predetermined as the resistance and reactance factors. Even in aerial high-tension transmission lines the leakage depends on many factors that are difficult to define, or whose quantitative values are either not known, or continually vary within wide limits. The temperature, humidity and barometric pressure of the air, the amount of dust in the air and the velocity of the wind, the condition of the insulators and line, the wave shape and maximum voltage, corona and other factors enter into the problem, and it is evident that exact calculations are impossible. Fortunately, the leakage loss along commercial lines in good condition is relatively small and may, in most cases, be neglected.

(*f*) **Transmission-line Constants.**—The transmission line consists of a series circuit having resistance and inductance interwoven with parallel circuits which have conductance and susceptance between the conductors. The properties of the transmission line may be stated in terms of four constants, called the *transmission-line constants.*

(1) r = effective resistance.
rI = voltage consumed in phase with the current.
rI^2 = power consumed by the ohmic resistance, mutual inductance, magnetic hysteresis. Series circuit.

(2) x = effective reactance.
jxI = voltage consumed in quadrature with the current.
jxI^2 = reactive power from self- and mutual inductance. Series circuit.

(3) g = effective conductance.
gE = current consumed in phase with the voltage.
gE^2 = power consumed by leakage, dielectric induction, dielectric hysteresis, etc. Parallel circuits between the line wires.

(4) b = effective susceptance.
jbE = current consumed in quadrature with the voltage.
jbE^2 = reactive power due to line condensance and dielectric induction. Parallel circuits between the line wires.

These constants are uniformly distributed over the whole length of the transmission line and hence the elemental unit of the network is the differential length dl, and the corresponding line constants dr, dx, dg and db. The complete solution of the

problem for transmission lines with uniformly distributed resistance, inductance, condensance and leakage is given in Chap. XXVIII. The derived equations are complicated and their application to commercial lines involves long and tedious calculations.

For short transmission lines certain assumptions may be made that greatly reduce the labor involved in making the calculations. The results obtained by short-cut methods are in most cases sufficiently close approximations to the actual values for ordinary commercial work.

(*g*) **Equations for Short Lines under Special Assumptions.** *Case I.—When both line condensance and line leakage are omitted from the calculations.*

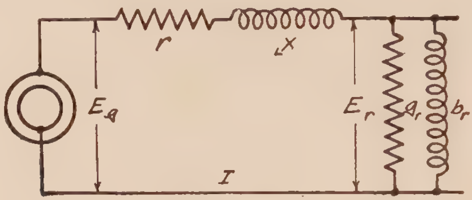


FIG. 6.22.

The corresponding circuit and vector diagrams are shown in Figs. 6.22 and 7.22. With the condensance and leakage omitted the problem becomes simply the line resistance and inductance in series with the receiver load.

E_r = voltage at the receiver end.
E_g = voltage at the generator end.
r = line resistance.
$_Lx$ = line inductive reactance.
$\cos\theta_r$ = power factor at receiver end.
$\cos\theta_g$ = power factor at generator end.
g_r = conductance of receiver.
b_r = susceptance of receiver.

From vector diagram in Fig. 7.22 we have:

$$E_g^2 = (E_r \cos\theta_r + rI)^2 + (E_r \sin\theta_r + {}_Lx I)^2 \qquad (31.22)$$

$$\cos\theta_g = \frac{E_r \cos\theta_r + rI}{E_g} \qquad (32.22)$$

Mershon's Diagram.—For any given commercial transmission line operating at constant frequency the impedance is constant

and hence the voltage triangle ABD, Fig. 7.22, is directly proportional to the current. The line regulation is dependent upon both the impedance drop and the phase relations of the voltages or upon the power factor. With a variable power factor it is desirable to have some convenient way in which to calculate the

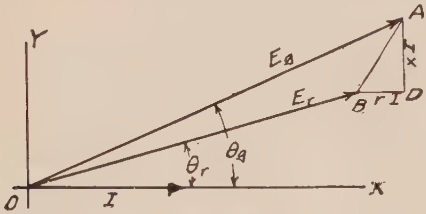


FIG. 7.22.

line regulation for commercial loads of any power factor. A number of graphic charts have been proposed and several are in commercial use. Mershon's diagram is convenient and frequently used. Let OB in Fig. 7.22 be rotated around O as a center as shown in Fig. 8.22. Let the system operate with a constant voltage at the receiver end and express the voltages in per cent with OB the receiver voltage as 100 per cent. Draw

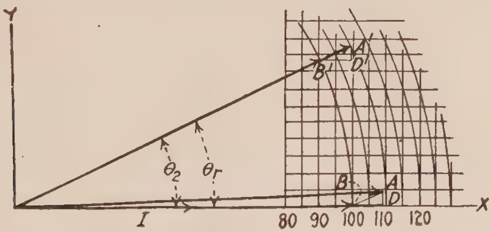


FIG. 8.22.

concentric circles at convenient differences, as 5 per cent in Fig. 8.22. A section of Fig. 8.22 drawn to a larger scale with circles 1 per cent apart is shown in Fig. 9.22. Let the distance OB along the X-axis be the power-factor scale for the receiver load. To find the regulation proceed as follows:

Given: The size, length and spacing of the line conductors, the load and power factor at the receiver end and the receiver voltage.

1. Find the resistance and reactance drops in per cent of the receiver voltage from the given data.

2. Starting on the X-axis at the given power factor, follow the ordinate until it intersects the OB circle. From the point of intersection lay off along the coördinate parallel to the X-axis

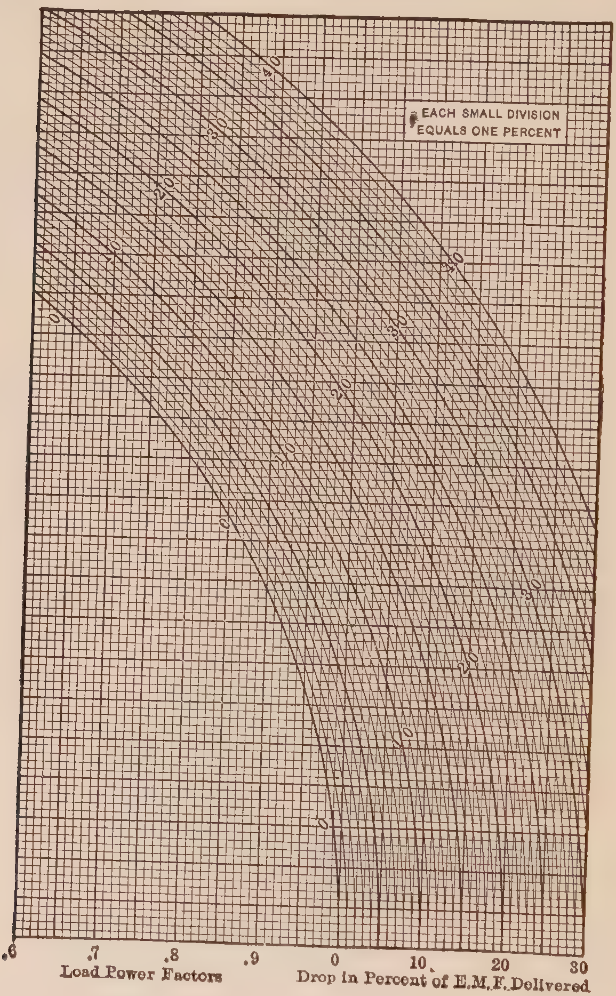


FIG. 9.22.

the resistance drop in per cent as found under 1. From this point lay off on the ordinate upward the reactance drop in per cent. Follow the arc of the circle passing through the last point down to the X-axis and obtain the per cent regulation for the given load and power factor.

An example will make the process plain.

Given: Single-phase transmission line, 2 miles long, of No. 0 copper wire, spaced 24 in. Receiver voltage 2,400, receiver load 220 kw. at 85 per cent power factor, f = 25 cycles.

1. Find line resistance and reactance drops.

$$\text{Line current} = \frac{220 \times 1{,}000 \times 100}{2{,}400 \times 85} = 107.8 \text{ amp.}$$

Line resistance (handbook tables, per mile of No. 0 = 0.534) = 2.14 ohms.

Line reactance (handbook tables, per mile of No. 0 = 0.265) = 1.06 ohms.

$$\text{Resistance drop} = \frac{107.8 \times 2.14}{2{,}400} = 9.6 \text{ per cent.}$$

$$\text{Reactance drop} = \frac{107.8 \times 1.06}{2{,}400} = 4.8 \text{ per cent.} \quad 4.50$$

2. From the 85 per cent point on the X-axis, Fig. 8.22, follow the ordinate until it intersects the OB circle, at B'. Lay off the resistance drop of 9.6 per cent parallel to the X-axis $B'D'$. From D' lay off the reactance drop, 4.8 per cent parallel to the Y-axis to A'. Follow the circle passing through A' to the X-axis and the regulation is found to be 11 per cent of the receiver voltage. Expressed in terms of the generator voltage, the regulation

$$= \frac{11}{100 + 11} = 9.9 \text{ per cent.} \quad 9.09$$

In order to reverse the process and find the size of wire for a given load, power factor and line drop one must solve by *trial and error*. As commercial wires differ in diameter by definite steps the nearest size is readily found.

Case II.—Let the additional assumption be made that the power factor at the generator is the same as at the receiver end of the line, a condition that approximately obtains in short lines. This simplifies the equations, as may be seen from Fig. 10.22. If $\cos\theta_g - \cos\theta_r$ is small, the projection of vector E_g along E_r, produced, is almost equal to E_g, or FH may be neglected in the calculations.

Hence (approximately):

$$E_g - E_r = BF = I(r\cos\theta_r + {}_Lx\sin\theta_r) \qquad (33.22)$$

This gives the line drop per wire to neutral. Between lines the voltage drops for single-phase and three-phase lines are:

$$e_{1\phi} = 2I(r \cos \theta_r + {}_Lx \sin \theta_r) \text{ for single-phase} \tag{34.22}$$

$$e_{3\phi} = \sqrt{3}I(r \cos \theta_r + {}_Lx \sin \theta_r) \text{ for three-phase} \tag{35.22}$$

In order to facilitate the line-drop calculations, graphic methods are often used. The chart in Fig. 12.22 is convenient for rapidly calculating line drop under the conditions assumed. When using the chart proceed as follows:

Find the given size of wire on the positive X-axis. Follow the ordinate to the proper power-factor line; from there move along the abscissa to scale I or II on the right-hand edge. This gives the factor $r \cos \theta_r$. Similarly, in order to find $x \sin \theta_r$, find the given size of wire along the Y-axis. Follow the line to the left until it intersects the desired spacing curve. From the

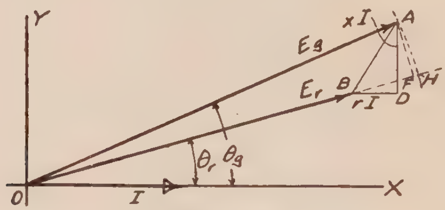


FIG. 10.22.

point of intersection follow the ordinate to the X-axis and obtain the reactance for 100 cycles. Then follow the arc of the circle to the intersection of the desired frequency line. From this intersection move horizontally to the right and intersect with the desired power-factor line. The ordinate passing through this intersection gives, on the upper scale, the desired value for $x \sin \theta_r$. An example will illustrate the use of the chart:

Three-phase line, distance..................	25 miles.
Spacing of wires..........................	30 in.
Full load to be delivered..................	4,000 kw.
Voltage at receiver end....................	30,000 volts.
Frequency..................................	60 cycles.
Power factor of load.......................	85 per cent.
Size of wire, copper.......................	No. 0 A.W.G.

In the right-hand quadrant follow the line for No. 0 wire and find the intersection with the 85 per cent power-factor line. $r \cos \theta_r = 0.440$. In the left-hand quadrant find the intersection of No. 0 wire and the 30-in. spacing curve, move along the ordinate to the X-axis, follow the circular arc to the 60-cycle line, then

36.15 −j

move horizontally to the 85 per cent power-factor line, and from the intersection vertically to the upper scale. $x \sin \theta_r = 0.335$. Total drop per mile of wire per amp. $= 0.440 + 0.335 = 0.775$ volt.

$$I = \frac{4{,}000 \times 1{,}000}{30{,}000 \times \sqrt{3} \times 0.85} = 90.7 \text{ amp.}$$

$$\text{Total drop} = \sqrt{3} \times 90.7 \times 25 \times 0.775 = 3{,}440 \text{ volts}$$

$$\text{Regulation} = \left(\frac{30{,}000 + 3{,}440}{30{,}000}\right)100 - 100 = 11.4 \text{ per cent.}$$

Case III.—Let the total line condensance be represented by a condenser across the middle of the line as indicated in Fig. 11.22. *No leakage.*

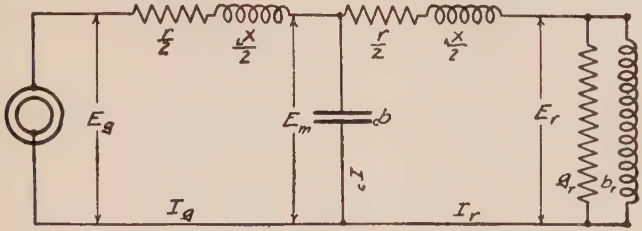


FIG. 11.22.

In addition to the notation used under case I, let

${}_cb$ = line susceptance.
E_m = voltage at middle of line.
${}_cI$ = condensance or charging current.
I_r = current at receiver end.
I_g = current at generator end.

Beginning at the receiver end of the line we have:

$$\dot{I}_r = \dot{E}_r(g_r - jb_r) \tag{36.22}$$

$$\dot{E}_m = \dot{E}_r + \left(\frac{r + j\,{}_Lx}{2}\right)\dot{I}_r = \dot{E}_r\left[1 + \frac{(r + j\,{}_Lx)(g_r - jb_r)}{2}\right] \tag{37.22}$$

$$\dot{I}_g = \dot{I}_r + j\,{}_cb\dot{E}_m = \dot{E}_r\left\{g_r - jb_r + j\,{}_cb\left[1 + \frac{(r + j\,{}_Lx)(g_r - jb_r)}{2}\right]\right\}$$

$$= \dot{E}_r\left\{\left[g_r + \frac{{}_cb}{2}(rb_r - {}_Lxg_r)\right] + j\left[{}_cb - b_r + \frac{{}_cb}{2}(rg_r + {}_Lxb_r)\right]\right\} \tag{38.22}$$

$$\dot{E}_0 = \dot{E}_m + \frac{(r + j\,{}_Lx)}{2}\dot{I}_0 = \dot{E}_r\left[1 + \frac{(r + j\,{}_Lx)(g_r - jb_r)}{2}\right.$$
$$\left. + \frac{(r + j\,{}_Lx)(g_r - jb_r)}{2} + \frac{j\,{}_cb(r + j\,{}_Lx)}{2} + \frac{j\,{}_cb(r + j\,{}_Lx)^2(g_r - jb_r)}{4}\right]$$
$$= \dot{E}_r\left[1 + (r + j\,{}_Lx)\left(g_r - jb_r + \frac{j\,{}_cb}{2}\right) + \frac{j\,{}_cb}{4}(r + j\,{}_Lx)^2(g_r - jb_r)\right] \tag{39.22}$$

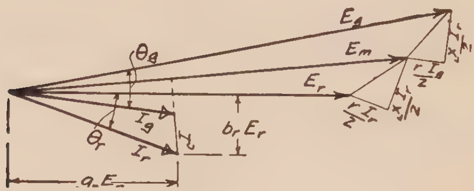


Fig. 13.22.

The current and voltage relations are shown graphically by the vector diagram in Fig. 13.22.

Case IV.—Let the total condensance of the line be represented by three condensers, one-sixth at each end, and two-thirds at the middle. No leakage. The circuit diagram under the above assumptions is shown in Fig. 14.22. In addition to the notation used in case II, we have, as indicated in the diagram:

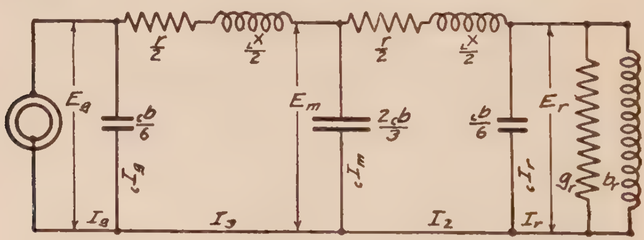


Fig. 14.22.

${}_cI_r$ = condensance or charging current at receiver end.
${}_cI_m$ = condensance or charging current at middle of line.
${}_cI_0$ = condensance or charging current at generator end.

Starting at the receiver end and proceeding toward the generator, we have:

$$\dot{I}_r = \dot{E}_r(g_r - jb_r) \tag{40.22}$$
$$\dot{I}_2 = \dot{I}_r + {}_c\dot{I}_r = \dot{I}_r + j_c\frac{b\dot{E}_r}{6} = \dot{E}_r\left(g_r - jb_r + \frac{j\,{}_cb}{6}\right) \tag{41.22}$$

$$\dot{E}_m = \dot{E}_r + \left(\frac{r + j\,{}_Lx}{2}\right)\dot{I}_2 = \dot{E}_r\left[1 + \left(\frac{r + j\,{}_Lx}{2}\right)\left(g_r - jb_r + \frac{j\,{}_cb}{6}\right)\right] \tag{42.22}$$

$$\dot{I}_3 = \dot{I}_2 + {}_c\dot{I}_m = \dot{I}_2 + \left(\frac{j2\,{}_cb}{3}\right)\dot{E}_m$$

$$= \dot{E}_r\left[g_r - jb_r + j\frac{5}{6}{}_cb + \frac{j\,{}_cb}{3}(r + j\,{}_Lx)\left(g_r - jb_r + \frac{j\,{}_cb}{6}\right)\right] \tag{43.22}$$

$$\dot{E}_g = \dot{E}_m + \left(\frac{r + j\,{}_Lx}{2}\right)\dot{I}_3 = \dot{E}_r\left[1 + (r + j\,{}_Lx)\left(g_r - jb_r + \frac{j\,{}_cb}{2}\right) + \frac{j\,{}_cb}{6}\left(g_r - jb_r + \frac{j\,{}_cb}{6}\right)(r + j\,{}_Lx)^2\right] \tag{44.22}$$

$$\dot{I}_g = \dot{I}_3 + {}_c\dot{I}_g = \dot{I}_3 + \frac{j\,{}_cb}{6}\dot{E}_g$$

$$= \dot{E}_r\left[(g_r - jb_r + j\,{}_cb) + \frac{j\,{}_cb}{6}(r + j\,{}_Lx)\left(3g_r - 3jb_r + \frac{5}{6}j\,{}_cb\right) - \frac{{}_cb^2}{36}(r + j\,{}_Lx)^2\left(g_r - jb_r + \frac{j\,{}_cb}{6}\right)\right] \tag{45.22}$$

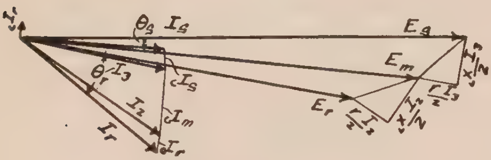


FIG. 15.22.

The current and voltage relations at the receiver, generator and at the middle of the line are shown graphically by the vector diagram in Fig. 15.22.

Case V.—For moderately long transmission lines a closer approximation to the actual condition of uniformly distributed inductance, condensance and resistance is necessary. The complete solution gives complicated equations that are not readily applied but which must be used for very long lines, as shown in Chap. XXVIII. These equations may be expanded into rapidly converging series, and the first few terms give results sufficiently accurate for commercial purposes for lines up to 150 miles long. These approximate equations are derived in Chap. XXVIII page 640.

In transmission problems in polyphase circuits it is most convenient to calculate the line characteristics of one line to neutral.

The solution of any given problem is also made on the basis of one line to neutral. Thus in a three-phase circuit, by taking one-third of the load and the voltage to neutral the solution is the same as for a single wire to neutral in a single-phase system.

(*h*) **Line Transformers.**—In the above discussion the transformers at both ends of the transmission line are not included. Usually it is desirable to calculate the voltage relations on the low side at both the generator and receiver ends. This introduces both the step-up and the step-down transformers between the generators and the receiver distribution system. The resistance, inductance and condensance of the transformers may be considered as part of the line constants, and a complete solution obtained from one set of equations. This is often done in commercial problems. The resistance and reactance of the transformers at both ends of the line are added to the constants of the line proper and all calculations made in terms of the high-tension circuit. For preliminary calculations, or in the absence of definite information, it is customary to assume the transformer resistance drop to be 1 per cent and the inductive reactance drop 6 per cent of the impressed voltage. The actual values may vary from 0.5 to 2.0 per cent for the resistance, and from 3.5 to 9.0 per cent for the reactance. The lower limit for the resistance and the higher values for the inductance refer to transformers on long-distance, high-voltage transmission lines. At the receiver end the equivalent value for the step-down transformers may be found as follows:

Let

P_r = power delivered to receiver.
${}_xd_{rt}$ = per cent reactance drop assumed for the transformer.
${}_rd_{rt}$ = per cent resistance drop assumed for the transformer.
I_r = line current at receiver.
E_r = receiver voltage between mains.
${}_nE_r = \dfrac{E_r}{\sqrt{3}}$ = receiver voltage to neutral.

$$ {}_xd_{rt} = \frac{100 I_r x_{rt}}{{}_nE_r} \tag{46.22} $$

$$ {}_rd_{rt} = \frac{100 I_r r_{rt}}{{}_nE_r} \tag{47.22} $$

$$ I_r = \frac{P_r}{\sqrt{3} E_r \cos \theta_r} \tag{48.22} $$

100,000 volts .2% = 200

200 = IR

$$x_{rt} = {}_x d_{rt} \frac{E_r^2 \cos \theta_r}{100P_r} \text{ ohms} \tag{49.22}$$

$$r_{rt} = {}_r d_{rt} \frac{E_r^2 \cos \theta_r}{100P_r} \text{ ohms} \tag{50.22}$$

Similar equations give the constants for the transformers at the generating station. The transformers are considered as connected from line to neutral in making the above calculations, whether the actual connections are in delta or star.

(*i*) **Line Regulation.**—From the previous discussion in this chapter, it is evident that the transmission-line drop depends upon the line constants and also upon both the magnitude and power factor of the load. This interaction of the reactive component of the receiver circuit with the impedance of the line forms the basis for voltage regulation on the whole transmission system and merits careful consideration. For long transmission lines it is necessary to consider the distribution of the inductance and condensance on the line, as discussed in Chap. XXVIII, in determining the effects of variations in the power factor of the receiver circuit. For short lines this is not necessary, and the equations and computations may be greatly simplified by assuming that the transmission line consists merely of a resistance and an inductance in series with the receiver. A closer approximation for somewhat longer lines may be made by considering a condenser equivalent to the line condensance connected across the line at the receiver end. This condenser may be included in, and considered as part of, the receiver circuit, thus requiring no changes in the equations derived under the condition that the line condensance is omitted. Let the discussion be for a single-phase circuit. The same equations may be applied to balanced, three-phase circuits by using either *equivalent single-phase* values or solving *for one-third of the load with one line wire and voltage to neutral.*

$z_l = r_l + jx_l$ impedance of transmission line.
$y_r = g_r - jb_r$ admittance of receiver circuit.
E_g = voltage at generator end of line.
E_r = voltage at receiver end of line.
I_l = current in transmission line.

For any given transmission line the resistance and reactance are constants while the load in the receiver circuit may vary in

both magnitude and power factor. In the discussion it will be assumed that the generator voltage, the line resistance and reactance are constants, while the receiver conductance and susceptance are variable, as indicated in Fig. 16.22. It is necessary to derive the equations for the receiver voltage and line current in terms of the given constants and for any desired load.

$$\dot{I}_l = \dot{E}_r y_r \tag{51.22}$$

$$\dot{E}_g = \dot{E}_r + \dot{I}_l z_l = \dot{E}_r(1 + y_r z_l) \tag{52.22}$$

$$\dot{E}_r = \frac{\dot{E}_g}{1 + y_r z_l} = \frac{\dot{E}_g}{(1 + r_l g_r \pm x_l b_r) + j(x_l g_r \mp r_l b_r)} \tag{53.22}$$

$$\dot{I}_l = \frac{\dot{E}_g y_r}{1 + y_r z_l} = \frac{\dot{E}_g(g_r - jb_r)}{(1 + r_l g_r \pm x_l b_r) + j(x_l g_r \mp r_l b_r)} \tag{54.22}$$

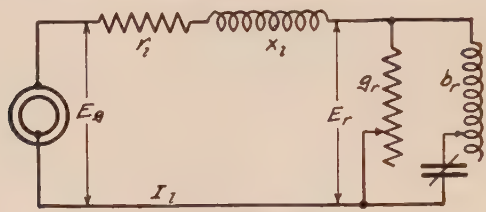


FIG. 16.22.

For absolute values:

$$E_r = \frac{E_g}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{55.22}$$

$$I_l = \frac{E_g\sqrt{g_r^2 + b_r^2}}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{56.22}$$

For convenience, let α represent the ratio of the receiver and generator voltages.

$$\alpha = \frac{E_r}{E_g} = \frac{1}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{57.22}$$

$$\text{The regulation} = \frac{E_g - E_r}{E_r} = \frac{1 - \alpha}{\alpha} \tag{58.22}$$

The power delivered to the receiver circuit:

$$P_r = E_r^2 g_r = E_g^2 \alpha^2 g_r \tag{59.22}$$

Efficiency of transmission

$$= \frac{E_r^2 g_r}{E_r^2 g_r + r_l I_l^2} = \frac{g_r}{g_r + r_l y_r^2} \tag{60.22}$$

With a constant voltage at the generator and a given transmission line, the power transmitted, current in the line, voltage at the receiver, efficiency of transmission and voltage regulation all depend upon both the magnitude and relative values of the conductance and the susceptance of the receiver circuit. In order to determine the effects produced by either variable it is convenient to discuss the problem under the following assumptions:

Case I. g_r variable, $b_r = 0$.
Case II. b_r variable, $g_r = 0$.
Case III. b_r variable, g_r constant.
Case IV. g_r variable, b_r constant.
Case V. Maximum efficiency of transmission.
Case VI. Maximum voltage at receiver circuit.

Case I. g_r Variable, $b_r = 0$.—In this case the receiver voltage, line current, ratio of voltages, power delivered and efficiency of transmission are given in equations (61.22) to (67.22) inclusive.

$$\dot{E}_r = \frac{\dot{E}_g}{(1 + r_l g_r) + jx_l g_r} \tag{61.22}$$

$$\dot{I}_l = \frac{g_r \dot{E}_g}{(1 + r_l g_r) + jx_l g_r} \tag{62.22}$$

In absolute values:

$$E_r = \frac{E_g}{\sqrt{(1 + r_l g_r)^2 + (x_l g_r)^2}} \tag{63.22}$$

$$I_l = \frac{g_r E_g}{\sqrt{(1 + r_l g_r)^2 + (x_l g_r)^2}} \tag{64.22}$$

$$\alpha = \frac{1}{\sqrt{(1 + r_l g_r)^2 + (x_l g_r)^2}} \tag{65.22}$$

$$P_r = E_r^2 g_r = E_g^2 \alpha^2 g_r \tag{66.22}$$

Efficiency of transmission

$$= \frac{1}{1 + r_l g_r} \tag{67.22}$$

Case II. b_r Variable, $g_r = 0$.—The equations are readily derived in the same manner as under case I. Since no power is delivered to the receiver circuit, the assumed conditions are seldom realized in commercial systems.

Case III. b_r Variable, g_r Constant.—From equation (59.22) the power delivered to the receiver circuit is:

$$P_r = E_g^2 \alpha^2 g_r \tag{68.22}$$

For constant conductance g_r, in the receiver, the power will be a maximum when α is a maximum, or when $\dfrac{d\alpha}{db_r}$ equals zero.

Hence:

$$x_l(1 + r_l g_r + x_l b_r) - r_l(x_l g_r - r_l b_r) = 0 \tag{69.22}$$

or

$$\frac{x_l}{r_l^2 + x_l^2} + b_r = b_l + b_r = 0 \tag{70.22}$$

Hence for maximum power the line susceptance must be equal in magnitude to the receiver susceptance but of opposite sign. That is, if the line has inductive reactance the receiver must have condensive reactance of an equal amount. Substituting this value for b_r in equations (53.22) to (60.22) inclusive, we have:

$$\dot{E}_r = \frac{\dot{E}_g}{1 + y_r z_l} = \frac{\dot{E}_g}{(r_l + jx_l)(g_r + g_l)} \tag{71.22}$$

$$E_r = \frac{E_g}{Z_l(g_r + g_l)} \tag{72.22}$$

$$\dot{I}_l = \frac{\dot{E}_g y_r}{1 + Z_l y_r} = \frac{\dot{E}_g(g_r + jb_l)}{(1 + r_l g_r - x_l b_l) + j(r_l b_l + x_l g_r)} \tag{73.22}$$

$$\dot{I}_l = \dot{E}_g \sqrt{\frac{g_r^2 + b_l^2}{(1 + r_l g_r - x_l b_l)^2 + (r_l b_l + x_l g_r)^2}} = \dot{E}_g \frac{\sqrt{g_r^2 + b_r^2}}{Z_l(g_r + g_l)} \tag{74.22}$$

$$\alpha = \frac{1}{(r_l + jx_l)(g_r + g_l)} \tag{75.22}$$

Maximum power,

$$P_r = E_g^2 \alpha^2 g_r \tag{76.22}$$

Power factor at receiver $= \cos \theta_r$; where

$$\theta_r = \tan^{-1}\frac{b_r}{g_r} = \tan^{-1}\frac{(-b_l)}{g_r} \tag{77.22}$$

Power factor at generator $= \cos \theta_g$; where

$$\theta_g = \tan^{-1}\left(\frac{x_r + x_l}{r_r + r_l}\right) \tag{78.22}$$

Case IV. g_r Variable, b_r Constant.—The power delivered to the receiver, $P_r = E_g^2 \alpha^2 g_r$, will be a maximum when $\alpha^2 g_r$ is maximum, or when

$$\frac{d(\alpha^2 g_r)}{dg_r} = 0$$

$$\frac{d(\alpha^2 g_r)}{dg_r} = \frac{d}{dg_r}\left[\frac{g_r}{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}\right] = 0 \tag{79.22}$$

$$\therefore (1 + r_l g_r + x_l b_r) - 2g_r(r_l + r_l g_r + x_l b_r) = 0 \qquad (80.22)$$

Hence the conductance for maximum power in the receiver circuit is:

$$g_r = \sqrt{g_l^2 + (b_r + b_l)^2} \qquad (81.22)$$

Substituting this value in the equations for receiver voltage and power delivered to the receiver, the following expressions are obtained:

$$\dot{E}_r = \frac{\dot{E}_g(g_l - jb_l)}{\sqrt{2g_r(g_r + g_l)}} \qquad (82.22)$$

$$\dot{E}_r = \frac{\dot{E}_g Y_l}{\sqrt{2g_r(g_r + g_l)}} \qquad (83.22)$$

$$P_r = \frac{E_g^2 Y_l^2}{2(g_r + g_l)} = \frac{E_g^2}{2\left[r_l + \sqrt{r_l^2 + \left(x_l + x_r\frac{z_l}{z_r}\right)^2}\right]} \qquad (84.22)$$

$$\alpha = \frac{Y_l}{\sqrt{2g_r(g_r + g_l)}} \qquad (85.22)$$

If in the above expressions the receiver susceptance be taken equal to minus the line susceptance, $b_r = -b_l$, as derived in case III for the condition of maximum power as a function of the receiver susceptance, we have:

$$b_r = -b_l;\ g_r = g_l;\ y_r = y_l;\ x_r = -x_l;\ r_r = r_l;\ z_r = z_l \qquad (86.22)$$

$$Y_l = \sqrt{g_l^2 + b_l^2} = \sqrt{g_r^2 + b_r^2} = Y_r \qquad (87.22)$$

$$Z_l = \sqrt{r_l^2 + x_l^2} = \sqrt{r_r^2 + x_r^2} = Z_r \qquad (88.22)$$

With the inductive reactance in the line balanced by an equal condensive reactance in the receiver, the maximum power delivered becomes the same as for direct currents. For, substituting the values given in equation (86.22) in equations (84.22) and (85.22):

$$^{M}P_r = \frac{E_g^2}{4r_l} \qquad (89.22)$$

$$^{M}\alpha = \frac{Y_l}{2g_l} = \frac{Z_l}{2r_l} \qquad (90.22)$$

Case V. Maximum Efficiency of Transmission.—Since the line loss varies as the square of the current and for any given load

the current is a minimum when the power factor is unity, the maximum efficiency for any given load is obtained when the current is in phase with the voltage.

Hence:

$$\cos \theta_g = 1, \text{ or } x_r = -x_l \tag{91.22}$$

Substituting in equations (90.22) and (92.22):

$$\alpha = \frac{Z_r}{(r_r + r_l)} \tag{92.22}$$

$$P_r = \frac{E_g^2 r_r}{(r_r + r_l)^2} \tag{93.22}$$

The power at maximum efficiency will be a maximum when $g_r = g_l$ as explained under case IV.

Hence:

$$\text{Maximum } P_r \text{ at maximum efficiency} = \frac{E_g^2}{4r_l}$$

and

$$\alpha = \frac{Z_l}{2r_l} \tag{94.22}$$

Case VI. Maximum Voltage at Receiver Circuit.—The ratio of the receiver and generator voltages α contains both the constants of the line and the receiver conductance and susceptance. For any given line and constant generator voltage the receiver voltage depends upon both g_r and b_r.

$$\alpha = \frac{E_r}{E_g} = \frac{1}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{95.22}$$

Hence for maximum receiver voltage,

$$\frac{d\alpha}{dg_r} = 0 \text{ and } \frac{d\alpha}{db_r} = 0 \tag{96.22}$$

These conditions are fulfilled when $g_r = 0$, and $b_r = -b_l$. Hence, for maximum receiver voltage a condition of resonance exists and the line resistance alone opposes the flow of the current. Substituting in equations (57.22) and (59.22):

$$\alpha = \frac{E_r}{E_g} = \frac{Z_l}{r_l} = \frac{Y_l}{g_l} \tag{97.22}$$

$$I_l = \frac{E_g}{r_l} \tag{98.22}$$

PROBLEMS

1.22. Show that in a three-phase transmission line with equilateral spacing of conductors the condensance to neutral is twice the condensance between two conductors. Show that the inductance per wire is one-half the inductance of two wires.

2.22. Two parallel conductors carry the same current.

Let the conductors be spaced 2, 4, 6, 8, 10, 12, 14, 16, 18, etc., in. (or cm.) in successive setting, center to center. Assume the current flow in the two conductors to be opposite in direction. Plot curve, using magnetic force per in. length as ordinates and spacing as abscissæ.

(*a*) Derive an expression for the magnetic force per unit length acting on each conductor.

(*b*) Calculate numerical values of the maximum force per unit length when the current is:

(1) 50,000 amp. direct current.

(2) 50,000 amp., r.m.s., sine wave.

CHAPTER XXIII

PHASE CONTROL OR REGULATION BY POWER FACTOR

The interaction of the line reactance and the receiver susceptance provides a basis for voltage regulation in alternating-current systems that is not available in direct-current systems. The receiver voltage may be varied, within limits, independently of the power delivered by adjusting the receiver susceptance while the generator voltage is held constant.

In direct currents the receiver voltage is equal to the generated voltage minus the line drop and is always numerically less than the generator voltage. In alternating currents the receiver voltage is the vector difference of the generated voltage and the line drop but may be less or greater than the generated voltage depending on the load power factor. A simple transmission circuit is shown in Fig. 1.23 in which r_l represents the line resist-

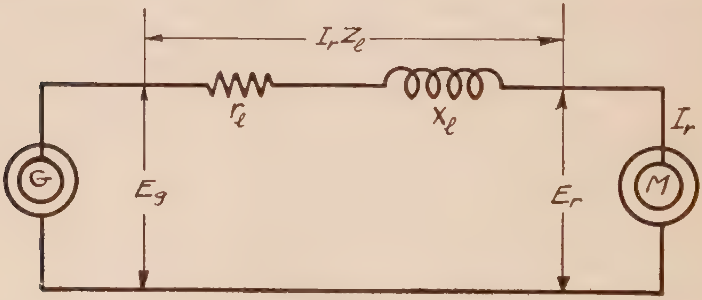


FIG. 1.23.

ance and x_l the total (constant) line reactance, consisting of the normal line inductive reactance plus any reactance purposely introduced into the line circuit to permit phase control of receiver voltage. If the load is lagging as indicated in Fig. 2.23 the line drop is represented by the vector $I_r Z_l$. If E_g is known, E_r may be found by vectorially subtracting the line drop as indicated. E_r is obviously less in value than E_g. For a leading load, the line drop is shown in Fig. 3.23. E_r is again found by vectorially

subtracting the line drop from E_g. In this case it is evident that the receiver voltage is greater than the line voltage. Obviously if the generator voltage is held constant, the magnitude of the receiver voltage may be controlled by varying the load reactive power.

This method of voltage control in a circuit delivering power over an inductive line by varying the receiver power factor is

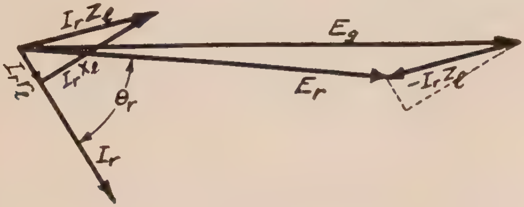


FIG. 2.23.

known as *phase control* or *regulation by power factor.* It is used to a large extent in transmission lines supplying power to electric railways through synchronous motors or rotary converters, and is the accepted standard in long-distance transmission systems. For short lines the required variation in the

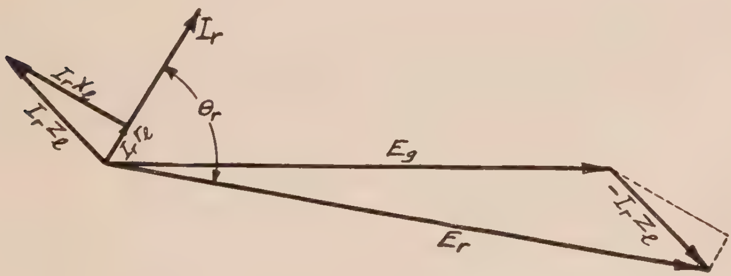


FIG. 3.23.

receiver susceptance is supplied by adjusting the field excitation of the synchronous motors or rotary converters. The required variation is provided automatically by a series winding on the field coils. At no load or light loads the fields are underexcited and therefore the current is lagging; at overload the compound field winding overexcites the synchronous machine, which then takes a leading current from the line. The receiver sus-

ceptance is in this manner automatically adjusted from inductive at no load to condensive at overload and provides the means for voltage regulation. In some cases shunt-wound converters or synchronous motors are used and the field excitation is controlled by Tirrill regulators in much the same manner as was explained for voltage regulation of alternators in Chap. XIV.

In long-distance transmission lines synchronous condensers are installed at the receiver end, with their field excitation controlled by Tirrill regulators. These machines carry no load but automatically supply the leading or lagging reactive component of the current and thereby keep the voltage at the receiver constant, at some predetermined value, for all loads. The line

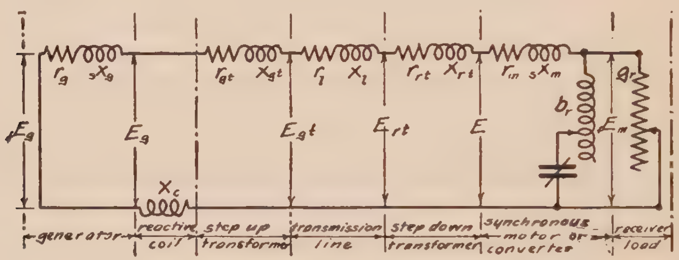


FIG. 4.23.

condensance forms an important factor in the calculations, as is shown in Chap. XXVIII.

Imp.

For short lines, only the inductive reactance need be considered. In many systems the inductive reactance in the line does not give a sufficient range in voltage regulation for the changes in load. The additional reactance required for phase control is obtained by inserting reactive coils in the line or by a magnetic bridge in the transformers, methods often used in transmission lines supplying power to electric railways. Hence, while the resistance in the system is kept at a minimum, the reactance is often greatly increased by inserting reactive coils in order to secure the advantage of automatic voltage regulation by phase control.

A diagrammatic representation of the complete circuit, omitting line and transformer condensance, in a system consisting of generators, step-up transformers, transmission lines, step-down transformers and synchronous motors or rotary converters is shown in Fig. 1.23.

g_r = receiver conductance, from the direct-current load.

b_r = receiver susceptance, from the synchronous motor or converter, may be either inductive or condensive, and is obtained by under- or overexciting the motor field.

r_m, ${}_sx_m$ = resistance and synchronous reactance of synchronous motor or converter.

r_{rt}, x_{rt} = resistance and reactance of step-down transformers.

r_l, x_l = resistance and reactance of transmission line.

r_{gt}, x_{gt} = resistance and reactance of step-up transformers.

x_c = reactance of inserted reactive coil.

r_g, ${}_sx_g$ = resistance and synchronous reactance of generator.

${}_fE_g$ = nominal induced voltage in generator, proportional to the field excitation.

E_g = voltage at generator terminals.

E_r = voltage at terminals of synchronous motor.

${}_fE_m$ = nominal induced voltage in synchronous motor, proportional to the field excitation.

If the calculations are made for the nominal induced voltages, the synchronous impedance of both the ~~generator and the~~ synchronous motor must be included. If the terminal voltages of the generator and synchronous motor are used in the equations, only the impedances between the two points enter into the calculations. Since the resistance and the reactance in the line, step-up and step-down transformers and reactive coil form one series circuit the notation may be abbreviated.

Let

$R = r_{rt} + r_l + r_{gt}$ = total resistance between generator and motor terminals.

$X = x_{rt} + x_l + x_c + x_{gt}$ = total reactance between generator and motor terminals.

$Z = R + jX$ = total impedance.

Selecting E_r as reference vector:

$$\dot{I} = g_r E_r - jb_r E_r = {}_gI + j\,{}_bI \tag{1.23}$$

$$\dot{E}_g = \dot{E}_r + Z\dot{I} = (E_r + R\,{}_gI - X\,{}_bI) + j(X\,{}_gI + R\,{}_bI) \tag{2.23}$$

$$E_g^2 = (E_r + R\,{}_gI - X\,{}_bI)^2 + (X\,{}_gI + R\,{}_bI)^2 \tag{3.23}$$

This is the fundamental equation for phase control.

From equation (3.23):

$$E_r = \sqrt{E_g^2 - (X\,_gI + R\,_bI)^2} - (R\,_gI - X\,_bI) \tag{4.23}$$

$$_bI = \frac{E_rX}{Z^2} - \sqrt{\frac{E_g^2}{Z^2} - \left(\frac{E_rR}{Z^2} + {}_gI\right)^2}$$

$$= \frac{E_rX - \sqrt{E_g^2Z^2 - (E_rR + {}_gIZ^2)^2}}{Z^2} \tag{5.23}$$

The maximum load, or power component of current at which unity power factor can be maintained with the supply voltage, E_g, constant, is found by equating the quantity under the radical to zero.

$$_gI_{max} = \frac{E_gZ - E_rR}{Z^2} \tag{6.23}$$

Phase control is in most cases used for regulating the voltage so as to hold automatically a predetermined ratio between the generator and receiver voltages for all loads. This ratio may be equal to, less than, or greater than unity.

First, $E_r = E_g$; second, $E_r < E_g$; third, $E_r > E_g$.
Let

E_g and E_r be constant and the load variable.

In the problem four current values are of special importance.

(1) $_bI_o$ = the reactive component at no load.
(2) $_gI_{max}$ = the power component at maximum load.
(3) $_gI_u$ = the power component at which the reactive component is zero.
(4) $_gI_f$ = the power component at full load.

At no load $_gI = 0$, and from equation (5.23):

$$_bI_o = \frac{E_rX - \sqrt{E_g^2Z^2 - E_r^2R^2}}{Z^2}$$

$$= \frac{E_rX - \sqrt{(E_g^2 - E_r^2)R^2 + E_g^2X^2}}{Z^2} \tag{7.23}$$

For

$$E_r = E_g,\ _bI = 0 \tag{8.23}$$

$$E_r < E_g,\ _bI < 0;\ \text{hence current lagging} \tag{9.23}$$

$$E_r > E_g,\ _bI > 0;\ \text{hence current leading} \tag{10.23}$$

When power is delivered to the receiver circuit the field excitation of the synchronous motors or converters must be increased if the voltages shall remain constant.
Hence with load, ${}_{g}\mathrm{I} > 0$.
For

$$E_r = E_g,\ {}_bI > 0, \text{ hence current leading} \qquad (11.23)$$

$$E_r < E_b,\ {}_bI \lesseqgtr 0, \text{ hence current lagging, in phase or leading} \qquad (12.23)$$

$$E_r > E_g,\ {}_bI > 0, \text{ hence current leading} \qquad (13.23)$$

When E_r is less than E_g the current is lagging at no load and for light loads, in phase at some loads, depending upon the ratio selected, and leading for larger loads. The average quadrature component for a variable load is therefore less when $E_r < E_g$ than for either $E_r = E_g$ or $E_r > E_g$. If the choice of the ratio is made on the basis of a minimum copper loss the shunt field of the synchronous motor or converter must be adjusted to give unity power factor at such load as to require a minimum quadrature component over the given range. With a continually varying load the actual minimum may be difficult to determine. Generally, the reduction in efficiency due to copper losses over the possible minimum is of little importance. However, these losses affect the temperature rise in the machines and thereby affect the permissible output. On this account it is customary so to adjust the shunt field as to give unity power factor at the generator at full load. The possible maximum load and the no-load reactive current depend on the amount of reactance in the line; the larger the required overload capacity the larger the reactance in the line, and consequently the larger the no-load current. As the line reactance may be adjusted by inserting reactive coils, a compromise is usually made in a more or less arbitrary manner, between a desirable large overload capacity and a likewise desirable minimum no-load current. It is generally assumed that ${}_bI_o = k_gI_f$, in which the factor k depends on the conditions of the system and may vary widely, probably from 20 to 50 per cent. Hence, in problems of phase control seven quantities are involved: E_r, E_g, X, ${}_gI_f$, ${}_gI_u$, ${}_bI_o$, I_{max}; of these, two are given by the load requirements (E_r, ${}_gI_f$), two are assumed (${}_gI_u$, ${}_bI_o$) and the remaining three are calculated. Assuming ${}_gI_u = {}_gI_f$, ${}_bI_o = k_gI_u$ and letting the reactive component produced by the compound field winding vary as a linear func-

tion of the load, ${}_bI = k({}_gI_u - {}_gI)$, then by substituting in equation (3.23) we have:

At no load:

$$ {}_gI = 0, \; {}_bI = k\,{}_gI_u \tag{14.23} $$

$$ E_g^2 = (E_r - kX\,{}_gI_u)^2 + (kR\,{}_gI_u)^2 \tag{15.23} $$

At full load:

$$ {}_gI = {}_gI_u, \; {}_bI = 0 \tag{16.23} $$

$$ E_g^2 = (E_r + R\,{}_gI_u)^2 + (X\,{}_gI_u)^2 \tag{17.23} $$

From equations (15.23) and (17.23):

$$ X = \frac{\dfrac{kE_r}{{}_gI_u} \pm \sqrt{\dfrac{E_r^2}{{}_gI_u^2}(k^2+1) - \left[\dfrac{E_r}{{}_gI_u} - R(k^2-1)\right]^2}}{(k^2-1)} \tag{18.23} $$

E_g may be found by substituting the value of X from equation (14.23) in either equation (15.23) or equation (16.23), and ${}_gI_{max}$ by substituting the above derived values in equation (6.23). Similar equations may be derived if the maximum load current ${}_gI_{max}$ is assumed instead of the no-load current ${}_bI_o$, as above.

The double sign before the radical in equation (18.23) shows that two values for the reactance, X_1 and X_2, satisfy the given conditions. Hence two values of the generator voltage are found to produce the required constant voltage at the receiver; or by dividing the reactance in two parts, X_1 and $X_2 - X_1$, three points on the system may be kept at constant voltage for wide variations in load. It is therefore possible automatically to keep the voltages constant at the receiver E_r, at the generator terminals E_g and also the nominal induced voltage in the generator ${}_fE_g$, which is proportional to the field excitation. Since the generator armature has a resistance r_g, in addition to the reactance ${}_sx_g$, the equations for the two values of X that satisfy the required conditions of constant voltage at the receiver and generator terminals with constant field excitation of the generator are given by equations (19.23) and (20.23):

$$ X_1 = \frac{\dfrac{kE_r}{{}_gI_u} - \sqrt{\dfrac{E_r^2}{{}_gI_u^2}(k^2+1) - \left[\dfrac{E_r}{{}_gI_u} - R(k^2-1)\right]^2}}{(k^2-1)} \tag{19.23} $$

$$ X_2 = \frac{\dfrac{kE_r}{{}_gI_u} + \sqrt{\dfrac{E_r^2}{{}_gI_u^2}(k_2+1) - \left[\dfrac{E_r}{{}_gI_u} - (R+r_g)(k_2-1)\right]^2}}{(k^2-1)} \tag{20.23} $$

Referring to Fig. 4.23, for constant E_r, E_g and ${}_fE_g$,

$$X_1 = X_{gt} + X_l + X_{rt} + X_c \tag{21.23}$$

$$X_2 = X_1 + {}_sX_g \tag{22.23}$$

The value of ${}_sX_g$ may be adjusted by means of a reactive coil in addition to the normal synchronous reactance of the generator.

$\sqrt{A_1^2+B_1^2}\left(\sin x \frac{A_1}{\sqrt{A_1^2+B_1^2}} + \cos x \frac{B_1}{\sqrt{A_1^2+B_1^2}}\right)$

$\sin(x+r_1)$

CHAPTER XXIV

COMMERCIAL WAVE FORMS—HARMONICS

The preceding chapters have dealt almost exclusively with alternating current and voltage waves of the simple harmonic form. The fundamental relations of alternating-current circuits have been discussed under the assumption that the instantaneous values of the voltage and the current could be expressed by $e = {}^ME \sin \omega t$ and $i = {}^MI \sin \omega t$. Both in theoretical discussions and in practical operation the sine wave is taken as the standard and any deviation is termed a distortion. Although the sine wave is the ideal form sought by both the operating and designing engineer, many factors in both the generators and distribution system cause distortion in the current and voltage wave shapes. As mentioned in Chap. II and illustrated by the oscillograms in Figs. 13.2 and 14.2, the distortions may be so large that even the semblance of a fundamental sine wave is lost.

(*a*) **Fourier's Series.**—Before attempting an analysis of the several factors that produce distortions in the shape of the voltage and current waves, it is desirable to note two fundamental characteristics:

1. The waves are periodic, that is, the successive cycles are alike.

2. The function is single valued. At any point in the system, at any instant, the voltage or current has only one value.

In 1822, long before alternating currents came into commercial use, Fourier published his researches and proved that any single-valued, periodic function can be completely expressed by a simple trigonometric series, equation (1.24), now known as Fourier's series.

$$y = A_1 \sin x + A_2 \sin 2x + A_3 \sin 3x + \cdots + A_n \sin nx + B_1 \cos x + B_2 \cos 2x + B_3 \cos 3x + \cdots + B_n \cos nx \quad (1.24)$$

The same series is often written in another form, as shown in equation (2.24), and using the notation applicable to any distorted voltage wave.

drop out.

$$e = {}^ME_1 \sin(\omega t + \gamma_1) + \boxed{{}^ME_2 \sin(2\omega t + \gamma_2)} + {}^ME_3 \sin(3\omega t + \gamma_3) + {}^ME_4 \sin(4\omega t + \gamma_4) + \cdots {}^ME_n \sin(n\omega t + \gamma_n) \cdots \quad (2.24)$$

The first term in equation (2.24) is called the *fundamental* and the other terms the *harmonics*. The transformation factors between equations (2.24) and (1.24) are:

$$y = e;\ x = \omega t$$

$${}^ME_1 = \sqrt{A_1^2 + B_1^2};\ {}^ME_2 = \sqrt{A_2^2 + B_2^2};\ {}^ME_3 = \sqrt{A_3^2 + B_3^2}, \text{ etc.}$$

$$\gamma_1 = \tan^{-1}\frac{B_1}{A_1};\ \gamma_2 = \tan^{-1}\frac{B_2}{A_2};\ \gamma_3 = \tan^{-1}\frac{B_3}{A_3};\ \text{etc.}$$

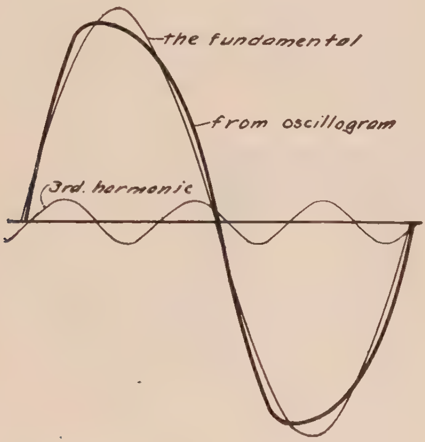


FIG. 1.24.

Any voltage or current wave may in this manner be expressed by a series of sine waves of multiple frequency and of different maximum values and phase positions. While the series has an infinite number of terms, only a few are required to express the elemental characteristics of waves in commercial power systems. In most cases a few of the harmonics are present and three or four terms fully express the distorted voltage or current wave.

(*b*) **Distorted Voltage Waves.**—In commercial power systems the alternating voltage is produced by rotating machinery and hence the positive and negative halves of the wave are equal in magnitude and similar in shape. It is readily seen that waves with equal positive and negative halves can have no even harmonics. Hence for voltage waves in power circuits,

$$e = {}^ME_1 \sin(\omega t + \gamma_1) + {}^ME_3 \sin(3\omega t + \gamma_3) + {}^ME_5 \sin(5\omega t + \gamma_5) + \cdots {}^ME_{2n-1} \sin[(2n-1)\omega t + \gamma_{2n-1}] \quad (3.24)$$

In most power circuits the voltage wave consists of a fundamental combined with the third and fifth harmonics, while sometimes the seventh and ninth harmonics are of importance. Except in special cases harmonics above the eleventh may be neglected. Thus, in Fig. 1.24, is shown a wave consisting of the fundamental and a third harmonic. The expression for the

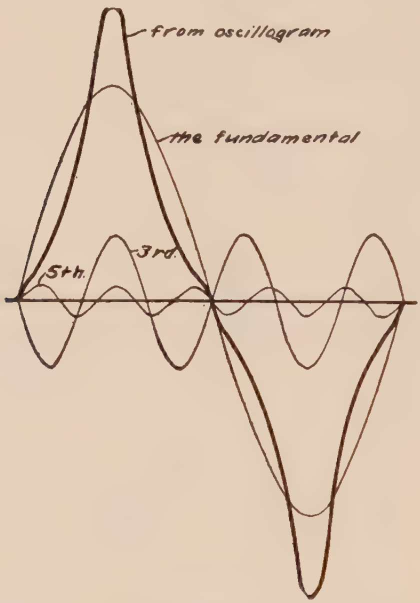


FIG. 2.24.

instantaneous voltage, equation (4.24), has two terms only. The coefficients ${}^M E_5$, ${}^M E_7$, ${}^M E_9$ . . . ${}^M E_{2n-1}$ are equal to zero and hence all but the first two terms of the series vanish.

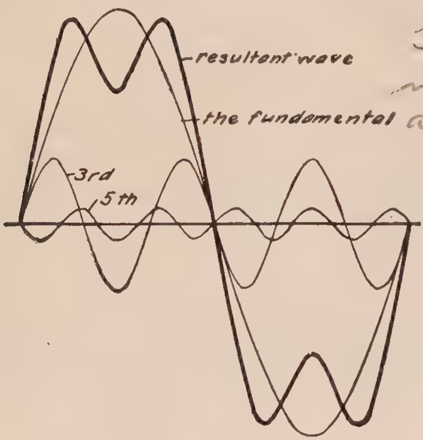
$$e = 100 \sin (\omega t + 2°) + 9.7 \sin (3\omega t - 27°) \qquad (4.24)$$

In this case the maximum of the fundamental is greater than the actual voltage in the circuit.

In Fig. 2.24 and in equation (5.24) is shown a wave consisting of a fundamental and the third and fifth harmonics. The maximum of the fundamental is considerably less than the maximum of the actual voltage wave.

$$e = 100 \sin (\omega t) + 28.2 \sin (3\omega t + 180°) + 5.4 \sin (5\omega t) \qquad (5.24)$$

The three component waves reach a positive maximum at the same time, producing a peaked voltage wave whose maximum is equal to the sum of the three maxima or 133.6 per cent of the fundamental. The waves shown in Figs. 1.24 and 2.24 were taken by an oscillograph from two machines in the laboratory. The fundamental and component waves were found by analysis as will be explained in a later paragraph. It is, however, evident that not only the relative magnitude of the fundamental and the harmonics but also their phase position affect the shape of the resultant wave. By keeping the same magnitude of the three component waves as in Fig. 2.24, but changing the phase positions of the harmonics by 180° the resulting complex wave is changed



Two peaks wave indicates arc in a system.

FIG. 3.24.

into the form shown in Fig. 3.24 and the corresponding equation is given in equation (6.24):

$$e = 100 \sin(\omega t) + 28.2 \sin(3\omega t) + 5.4 \sin(5\omega t + 180°) \quad (6.24)$$

The effect of phase position is more fully illustrated in Figs. 4.24 and 5.24. In Fig. 4.24 is shown a series of waves formed by combining the same fundamental with a 20 per cent, third harmonic but differing by 30° in phase angle. Similar effects due to changes of phase angle of 30° between a fundamental and a 20 per cent, fifth harmonic are shown in Fig. 5.24. The component curves and corresponding equations are given in each case. From these illustrations it is apparent that the combination of a fundamental with two or three harmonics may produce

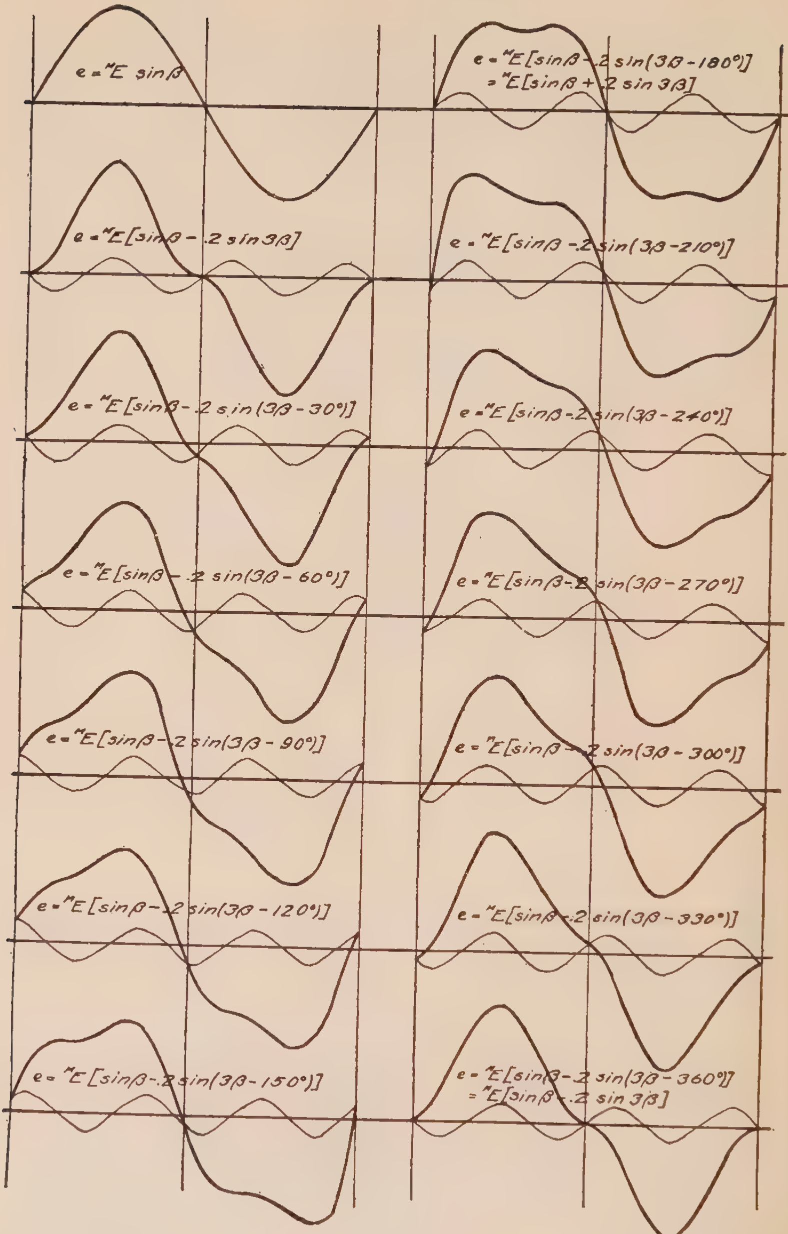

e = ${}^mE$ sin β
e = ${}^mE$[sin β − .2 sin(3β − 180°)]
= ${}^mE$[sin β + .2 sin 3β]
e = ${}^mE$[sin β − .2 sin 3β]
e = ${}^mE$[sin β − .2 sin(3β − 210°)]
e = ${}^mE$[sin β − .2 sin(3β − 30°)]
e = ${}^mE$[sin β − .2 sin(3β − 240°)]
e = ${}^mE$[sin β − .2 sin(3β − 60°)]
e = ${}^mE$[sin β − .2 sin(3β − 270°)]
e = ${}^mE$[sin β − .2 sin(3β − 90°)]
e = ${}^mE$[sin β − .2 sin(3β − 300°)]
e = ${}^mE$[sin β − .2 sin(3β − 120°)]
e = ${}^mE$[sin β − .2 sin(3β − 330°)]
e = ${}^mE$[sin β − .2 sin(3β − 150°)]
e = ${}^mE$[sin β − .2 sin(3β − 360°)]
= ${}^mE$[sin β − .2 sin 3β]

FIG. 4.24.

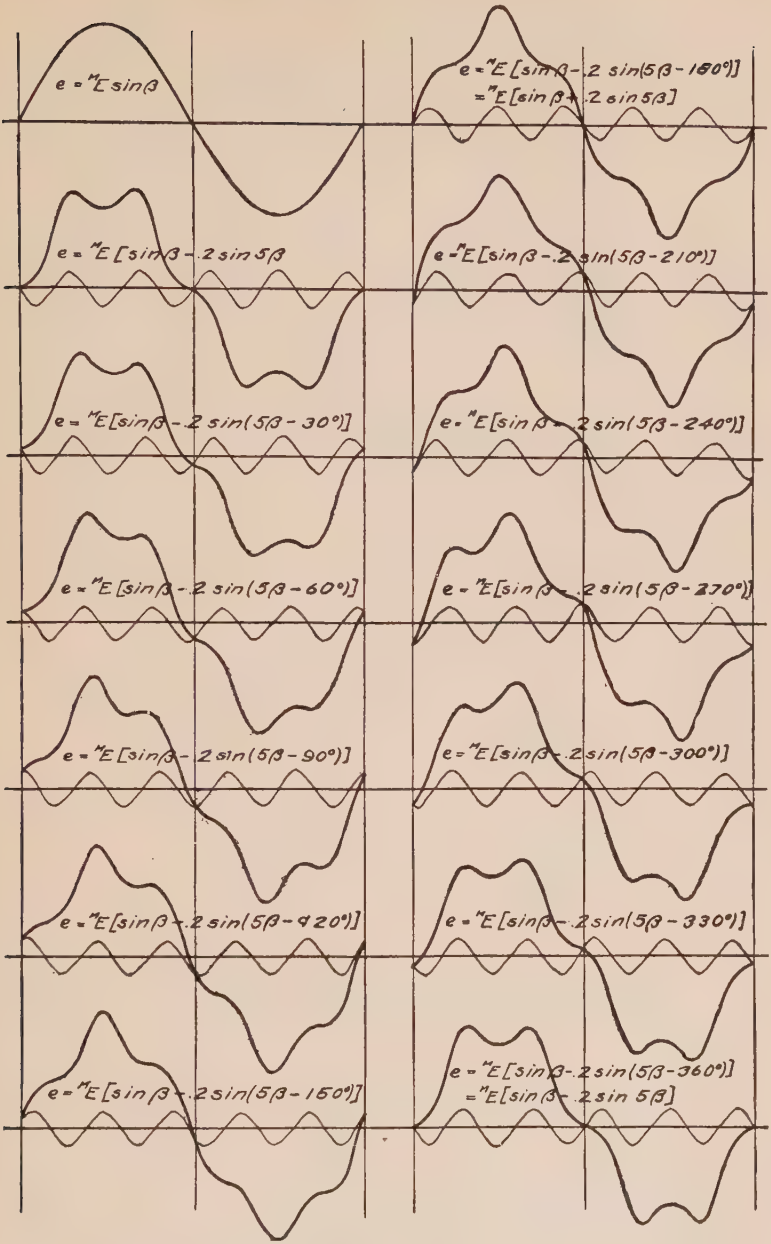

$e = {}^mE \sin\beta$
$e = {}^mE[\sin\beta - .2 \sin 5\beta$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 30°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 60°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 90°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 120°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 150°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 180°)]$
$= {}^mE[\sin\beta + .2 \sin 5\beta]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 210°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 240°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 270°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 300°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 330°)]$
$e = {}^mE[\sin\beta - .2 \sin(5\beta - 360°)]$
$= {}^mE[\sin\beta - .2 \sin 5\beta]$

Fig. 5.24.

a very great variety of wave forms. Often the shape of the wave makes it quite evident what harmonics are present and an approximate solution may be found by inspection. In general, a systematic analysis is required to determine the magnitude and phase relations of the harmonics. To the operating engineer it often becomes of great importance to know just what harmonics are present in the voltage or current waves. To gain this information is usually the first step in determining the cause of any periodic disturbance in the system. Knowing the relative magnitude of the harmonics present and noting which one is specially prominent, the problem of finding the cause of the difficulty is simplified. It is of importance to distinguish between higher harmonics of voltage and higher harmonics of current. In general, they are interdependent and the distorting effects of circuit conditions may affect one or both. Thus, if a sine voltage wave is impressed upon a transformer the hysteresis of the iron core distorts the current wave; and, conversely, if a current of sine wave form is passed through the same transformer the hysteresis causes a distortion of the voltage wave.

(*c*) **Generated Voltage Waves.**—In any circuit the first source of voltage wave distortion is in the alternator. With no current flowing, that is, on open circuit, the instantaneous voltage at the generator terminals is directly proportional to the instantaneous rate of cutting lines of force. A simple conductor revolving in a uniform field, Fig. 8.2, at uniform velocity, generates a sine voltage wave. Distortions may therefore be caused by:

1. Lack of uniformity or pulsation of the field.
2. Variation in the speed.
3. The distribution of the armature conductors connected in series.

In present-day alternators the speed is always so nearly uniform that this possible source of wave distortion need not be considered.

The slotted armature necessarily affects the distribution of the magnetic flux and produces pulsations in the field that, in turn, cause distortions in the voltage wave. Thus, in Fig. 6.24, is shown a voltage wave having a pronounced twenty-fifth harmonic produced by the pulsations of the magnetic flux. In alternators with few slots and teeth per pole the movement of the slots across the field pole produces pulsations of the magnetic reluctance of the field circuit and hence with constant excitation a pulsation

in the magnetic flux. Therefore, in a machine having s slots per pole the magnetic flux pulsates with $2s$ frequency.

Assuming that the pulsations follow a simple sine law and have $k{}^{M}\Phi$ amplitude, the instantaneous flux interlinked with the armature coil is:

$$\phi = {}^{M}\Phi \cos (\omega t)[1 + k \cos (2s\omega t - \gamma)] \qquad (7.24)$$

The voltage generated is:

amplitude of superim
ϕ is a variable

$$e = -n\frac{d\phi}{dt} \qquad (8.24)$$

From equations (7.24) and (8.24):

$$e = 2\pi f n {}^{M}\Phi\{\sin (\omega t)[1 + k \cos (2s\omega t - \gamma)] + 2sk \cos (\omega t) \sin (2s\omega t - \gamma)\} \qquad (9.24)$$

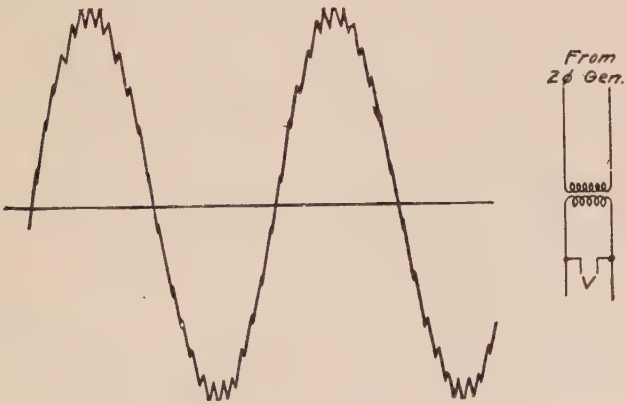


FIG. 6.24.

From trigonometry:

$$\left.\begin{aligned} 2 \sin x \cos y &= \sin (x + y) + \sin (x - y) \\ 2 \cos x \sin y &= \sin (x + y) - \sin (x - y) \end{aligned}\right\} \qquad (10.24)$$

Hence:

$$e = 2\pi f n {}^{M}\Phi\left\{\sin \omega t + k\frac{2s - 1}{2} \sin [(2s - 1)\omega t - \gamma] + k\left(\frac{2s + 1}{2}\right) \sin [(2s + 1)\omega t - \gamma]\right\} \qquad (11.24)$$

The pulsation of the magnetic flux due to the armature slots is therefore the source of two harmonics in the voltage wave of frequencies $2s + 1$ and $2s - 1$ times the fundamental.

For $s = 1$

$$e = 2\pi f n^{\prime\prime}\Phi\left[\sin(\omega t) + \frac{k}{2}\sin(\omega t - \gamma) + \frac{3k}{2}\sin(3\omega t - \gamma)\right] \qquad (12.24)$$

For $s = 2$

$$e = 2\pi f n^{\prime\prime}\Phi\left[\sin(\omega t) + \frac{3k}{2}\sin(3\omega t - \gamma) + \frac{5k}{2}\sin(5\omega t - \gamma)\right] \qquad (13.24)$$

Similarly, for three slots the pulsations of the field flux introduce a fifth and a seventh harmonic into the generated voltage. While the pulsations of field flux to some extent distort the voltage wave, the main factors determining the wave shape are the flux distribution and the arrangement of the armature conductors. A single conductor or a bundle of conductors in a single slot produces a voltage wave of the same shape as the flux distribution curve. However, the voltage for each phase of an alternator armature is ordinarily made up of the vector sum of the electromotive forces of several conductors in series, since the adjacent slots are displaced by an angle corresponding to the slot pitch. The voltage wave at the terminals of the generator therefore differs from the flux distribution curve and depends upon:[1]

1. The fractional pitch of the armature winding.
2. The number of slots per pole.
3. The number of coils per slot.
4. The angular span of the single-phase belt.

(*d*) **Distorted Current Waves. Constant R, L and C.**—In order to determine the current wave shape produced by any given impressed voltage wave the circuit conditions must be known. In many commercial circuits the so-called *circuit-constants*, the resistance, inductance and condensance, fluctuate or pulsate in a more or less irregular way during each cycle of the voltage or current waves. Thus in any circuit having iron, the inductance (and hence the reactance) varies, while the current changes from its zero to its maximum value. Moreover, the variation is not the same for decreasing as for increasing values of the current. Likewise, the resistance in the vapor of an arc lamp decreases with increase of the current. Similarly, under conditions where corona is produced at the crest of the voltage wave the condensance pulsates during the voltage cycle. The discussion of current wave shapes therefore naturally falls into

[1] Adams, C. A., "Wave Shape of Alternators," *Trans. Am. Inst. Elec. Eng.*, Vol. 28, p. 1053.

two groups: first, in circuits with constant r, L and C; second, in circuits with r, L or C pulsating or changing in magnitude during each cycle.

In Chap. VI it was shown that the instantaneous value of the voltage in a series circuit was given by the differential equation:

$$e = ri + L\frac{di}{dt} + \frac{1}{C}\int idt \tag{14.24}$$

If the voltage is a sine wave, the current is likewise simple harmonic and of the same frequency but differing in magnitude and phase position.

For

$$e = {}^{M}E \sin(\omega t),\ i = {}^{M}I \sin(\omega t - \theta) \tag{15.24}$$

in which

$${}^{M}E = z\,{}^{M}I \tag{16.24}$$

and

$$\theta = \tan^{-1}\frac{{}_{L}x - {}_{C}x}{r} \tag{17.24}$$

Since the voltage equation is single valued, the law of superposition may be applied; and, as each voltage wave produces a corresponding current of the same frequency, it follows that *in a complex voltage wave each harmonic produces its own current independent of the fundamental and all the other harmonics.*

Hence,

$$\begin{aligned} e &= e_1 + e_3 + e_5 + \cdots + e_{2n-1} \\ &= {}^{M}E_1 \sin(\omega t + \gamma_1) + {}^{M}E_3 \sin(3\omega t + \gamma_3) + {}^{M}E_5 \sin(5\omega t + \gamma_5) \\ &\qquad + \cdots + {}^{M}E_{2n-1} \sin[(2n-1)\omega t + \gamma_{2n-1}] \end{aligned} \tag{18.24}$$

and

$$\begin{aligned} i &= i_1 + i_3 + i_5 + \cdots + i_{2n-1} \\ &= \frac{{}^{M}E_1}{\sqrt{r^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} \sin\left[\omega t + \gamma_1 - \tan^{-1}\left(\frac{\omega L}{r} - \frac{1}{r\omega C}\right)\right] \\ &+ \frac{{}^{M}E_3}{\sqrt{r^2 + \left(3\omega L - \dfrac{1}{3\omega C}\right)^2}} \sin\left[3\omega t + \gamma_3 - \tan^{-1}\left(\frac{3\omega L}{r} - \frac{1}{3r\omega C}\right)\right] \\ &+ \frac{{}^{M}E_5}{\sqrt{r^2 + \left(5\omega L - \dfrac{1}{5\omega C}\right)^2}} \sin\left[5\omega t + \gamma_5 - \tan^{-1}\left(\frac{5\omega L}{r} - \frac{1}{5r\omega C}\right)\right] \end{aligned}$$

$$+ \cdots + \frac{{}^{M}E_{2n-1}}{\sqrt{r^2 + \left[(2n-1)\omega L - \frac{1}{(2n-1)\omega C}\right]^2}} \sin\left\{(2n-1)\omega t + \gamma_{2n-1} - \tan^{-1}\left[\frac{(2n-1)\omega L}{r} - \frac{1}{(2n-1)r\omega C}\right]\right\} \quad (19.24)$$

In order to abbreviate the notation in equation (19.24), let

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{r} - \frac{1}{nr\omega C}\right) \quad (20.24)$$

$${}^{M}I_n = \frac{{}^{M}E_n}{\sqrt{r^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}} \quad (21.24)$$

Then,

$$i = {}^{M}I_1 \sin(\omega t + \gamma_1 - \theta_1) + {}^{M}I_3 \sin(3\omega t + \gamma_3 - \theta_3) + {}^{M}I_5 \sin(5\omega t + \gamma_5 - \theta_5) + \cdots + {}^{M}I_{2n-1} \sin[(2n-1)\omega t + \gamma_{2n-1} - \theta_{2n-1}] \quad (22.24)$$

Comparing the expression for the voltage in equation (18.24) with the resulting current as expressed in either equation (19.24) or (22.24) it is seen that the current wave form differs from the voltage due to a change in both phase position and relative magnitude of the harmonic wave. Thus the phase position of the current for the third harmonic is θ_3 behind the corresponding third harmonic of the voltage wave. Similarly, the fifth harmonics differ by θ_5, while θ_3 is not equal to θ_5. The relative magnitude of the harmonics in the current is not the same as for the voltage wave, for in each case the maximum current for each harmonic equals the maximum voltage divided by the corresponding impedance as shown in equation (23.24).

$${}^{M}I_n = \frac{{}^{M}E_n}{z_n} = \frac{{}^{M}E_n}{\sqrt{r^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}} \quad (23.24)$$

It is readily seen that the relative values of r, L and C are of great importance in determining the current wave form resulting from a given impressed voltage wave.

Thus in a circuit having resistance only,

$${}^{M}I_n = \frac{{}^{M}E_n}{r} \text{ and } \theta_n = 0 \quad (24.24)$$

The current is in phase with the voltage wave and has the same wave form. This is illustrated in Fig. 7.24. The oscillogram

shows a voltage wave e having a pronounced third and fifth harmonic. This voltage was impressed upon a circuit having resistance only and the curve i in the oscillogram shows the current wave.

In a circuit having both resistance and inductance the current wave is different from the impressed voltage.

$$^{M}I_n = \frac{^{M}E_n}{\sqrt{r^2 + (n\omega L)^2}}, \text{ and } \theta_n = \tan^{-1}\left(\frac{n\omega L}{r}\right) \qquad (25.24)$$

The higher the harmonic the less current flows for the same impressed voltage, since the reactance increases with the fre-

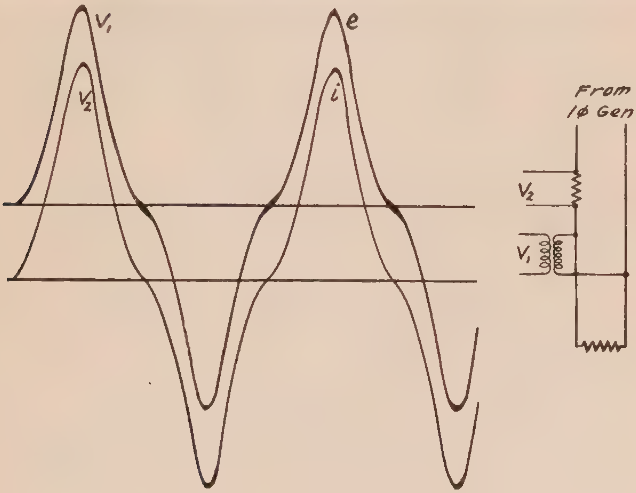


FIG. 7.24.

quency. Hence, the current harmonics are smaller than those in the voltage, and the current wave more nearly approaches the sine form. This is illustrated in Fig. 8.24. The same voltage wave e, as in Fig. 7.24, was impressed on the second circuit, having both resistance and inductance. The current wave is less peaked and its maximum value lags behind the voltage. The relative magnitudes of the third and fifth harmonics of the current are reduced proportionately to the impedance and hence are less than the corresponding harmonics in the voltage. The phase displacements also cause a reduction in the peak, as the maxima of the three component current waves do not coincide as in the voltage waves. In a circuit having resistance and con-

densance the effect is just the reverse, since an increase in the frequency reduces the reactance and impedance and therefore causes a proportionate increase in the current.

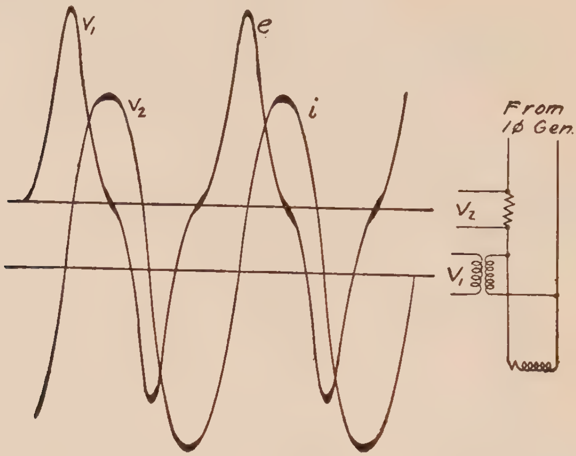


FIG. 8.24.

$$^{M}I_n = \frac{^{M}E_n}{\sqrt{r^2 + \left(\frac{1}{n\omega C}\right)^2}} \text{ and } \theta_n = \tan^{-1}\left(\frac{1}{rn\omega C}\right) \qquad (26.24)$$

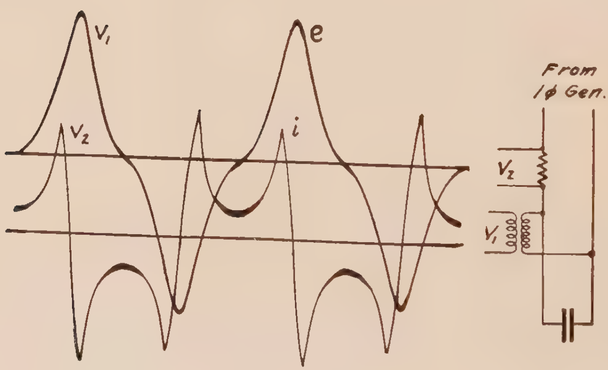


FIG. 9.24.

In Fig. 9.24 is shown the same voltage as in 7.24 and 8.24 impressed upon a circuit having resistance and condensance.

For the higher harmonics the effect of inductance or condensance is more marked. Thus, in Figs. 10.24 and 11.24 are shown

the voltage wave of a small generator having a strong eleventh harmonic. In the current, passing through a circuit having a constant resistance and inductance, Fig. 10.24, the eleventh harmonic

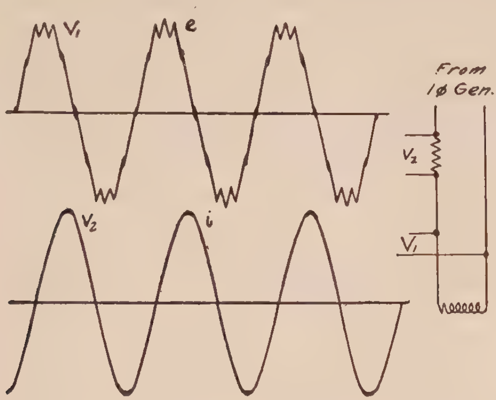


FIG. 10.24.

is practically eliminated; while the current flowing into a condenser, Fig. 11.24, from the same generator shows a very strong

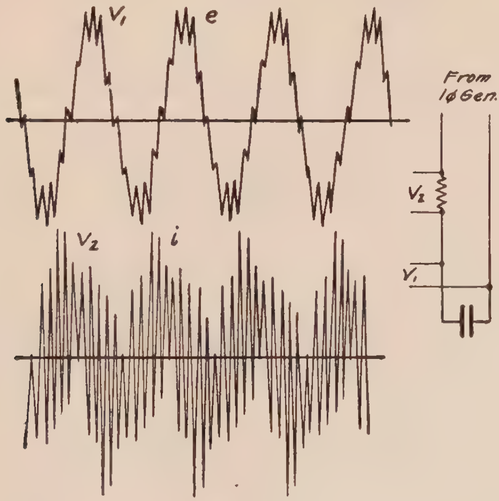


FIG. 11.24.

eleventh harmonic. The current distortion in Fig. 10.24 is only one-eleventh as large as for the voltage wave, while in Fig. 11.24 the current distortion is eleven times as large as for a non-reac-

tive circuit. By comparing the voltage waves in Figs. 10.24 and 11.24 it is seen that the lagging and leading currents pıoduced different armature reactions in the generator and thus the terminal voltages were not quite the same in the two cases.

In Fig. 12.24 is shown the voltage wave of a large machine having a pronounced twenty-fifth harmonic and also the condenser current wave. The circuit diagrams in each case show the connections of the oscillograph vibrators in the circuits.

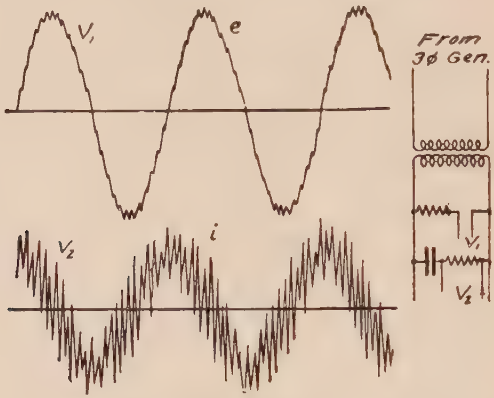


FIG. 12.24.

(*e*) **Distorted Current Waves in Circuits with Pulsating Inductance, Condensance and Resistance.** 1. *Pulsating Inductance.*—The main sources of pulsations in the inductive reactance in a circuit are: first, the variation of the reluctance around the armature conductors due to synchronous rotation; and, second, the variation in the permeability and the hysteresis with the flux density in iron-clad circuits.

The pulsations of the generator field caused by the relative position of the armature slots are discussed in the first part of this chapter in relation to the distribution of the field flux and hence as a primary cause of distortion of the voltage wave. The relative position of the field pole and armature slots also affects the armature reactance, producing pulsations that cause distortions in the current wave as compared to the impressed voltage. It is evident that a similar effect is produced by a synchronous motor, or any other synchronous apparatus with slotted armature, in the circuit. Since the cycle of variations in magnetic reluctance is completed for each pole, the pulsations of the induc-

tive reactance complete a cycle for each half wave of the fundamental frequency. Moreover, as the distortion cycle is applied alternately to a positive and a negative fundamental half wave, the harmonics introduced have a frequency of $2s - 1$ and $2s + 1$, in much the same way as for the voltage wave, but differing in magnitude and phase position. With a large air gap, large numbers of slots per pole, proper shaping and spacing of the armature slots and by using fractional pitch in the winding, the distortions produced by pulsations in the synchronous reactance may be reduced to practically negligible values.

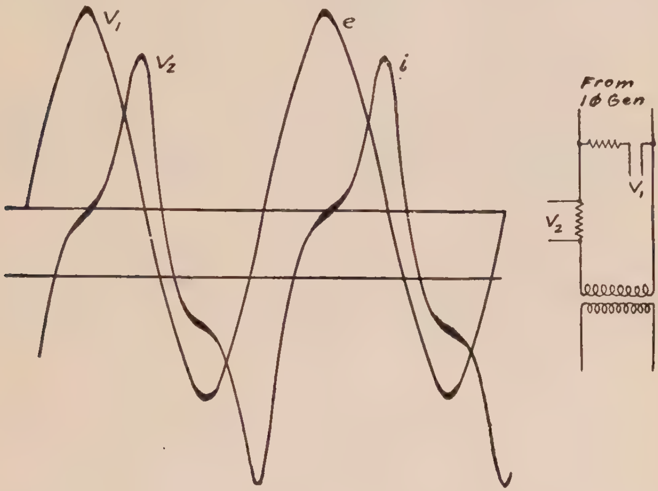


FIG. 13.24.

The second factor producing pulsations of the inductive reactance is present in all iron-clad circuits. In Chap. XI it is shown that with an impressed sine voltage the form of the current wave is determined by the shape of the hysteresis loop, Fig. 5.11. Conversely, the hysteresis loop of an iron-clad circuit may be found from the exciting current produced by a sine voltage wave. In Fig. 13.24 is shown an oscillogram of a sine voltage wave and the corresponding exciting current of a potential transformer.

$$E = 110; f = 60; I = 2.0; \text{10-kw. transformer.}$$

Since the upper and lower halves of the hysteresis curve are equal, the cycle of distortion is complete for each half of the fundamental wave. As the cycle of variation is applied alternately to the positive and negative halves of the fundamental

wave the distortion consists essentially of the third and fifth harmonics. For this reason the distortion of the current wave

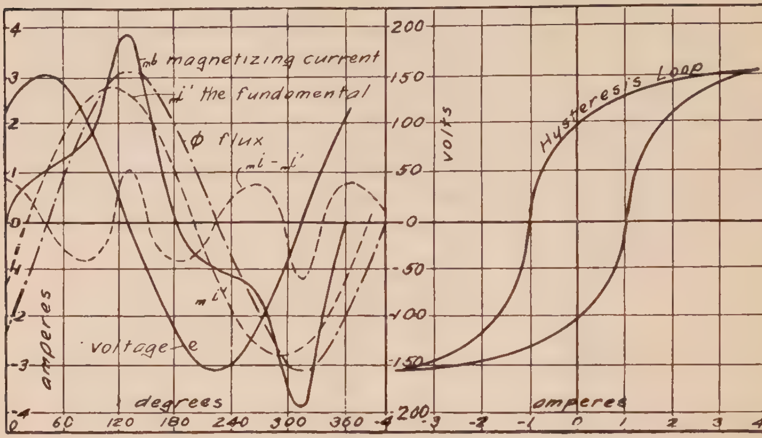


FIG. 14.24.

in Fig. 13.24 consists mainly of a third harmonic in combination with a smaller fifth harmonic. Analyzing the current wave in Fig. 13.24, the component fundamental, with the third and fifth

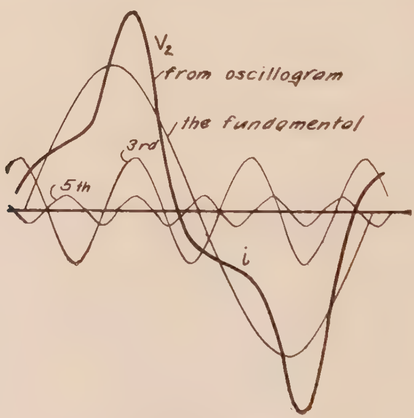


FIG. 15.24.

harmonics are represented graphically in Figs. 14.24 and 15.24 and analytically by equation (27.24).

$$i = 100 \sin (\omega t) + 37.0 \sin (3\omega t + 114°) + 10.7 \sin (5\omega t + 245°) \tag{27.24}$$

It is apparent that if a current of sine wave form is sent through an iron-clad circuit the distortions produced by the hysteresis of the iron will appear in the voltage wave. Thus, in Fig. 16.24 is shown the voltage wave produced by sine wave exciting current passing through an iron-clad circuit having a hysteresis loop of the shape shown in the figure. The third and fifth harmonics therefore appear in the voltage wave when a sine current is flowing in the circuit in precisely the same manner and for the same reason as the distortion of the current wave with an impressed sine voltage wave. In single-phase, constant-potential systems the distortion produced by the hysteresis of the iron

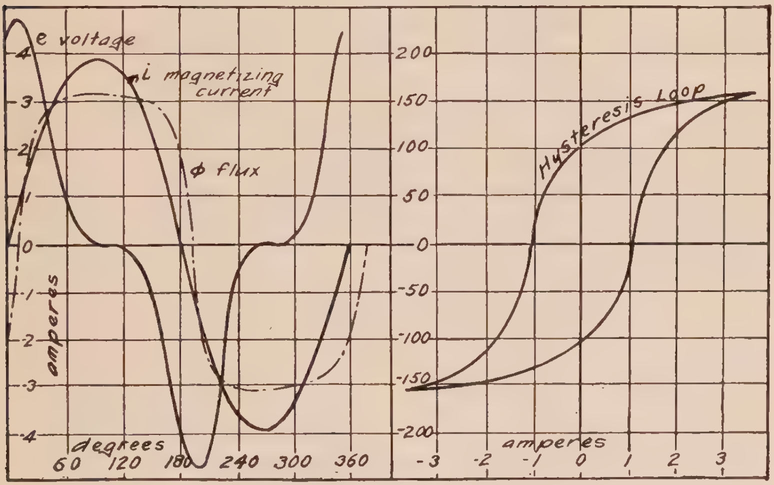


Fig. 16.24.

appears chiefly in the current wave. Secondary reactions of the distorted current wave upon the generator field may cause corresponding distortions in the voltage wave, but the current wave is primarily affected by the hysteresis distortion.

In polyphase circuits the manner of interlinking the component circuits, in a large measure, determines which part of the distortion shall appear in the current or voltage. The delta and star connections in three-phase circuits are of special importance in commercial systems. Since the fundamental waves in a three-phase system are 120° apart the third harmonics in the three phases are in phase with each other. This is shown in Fig. 17.24. Hence in a star connection the current cannot have a third

harmonic, and likewise in a delta connection the third harmonic must be absent from the voltage wave. The same phase rela-

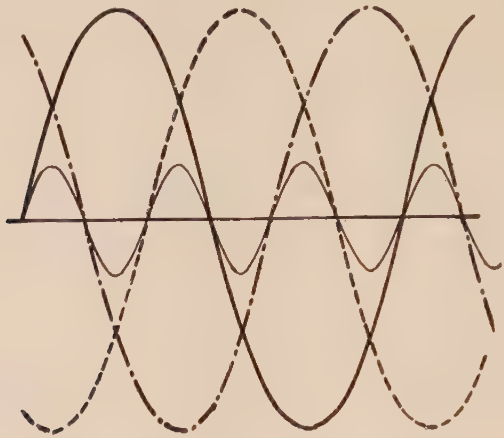


Fig. 17.24.

tions exist for any odd multiple of the third harmonic. Therefore, in a star connection the third, ninth, fifteenth, etc., harmonics

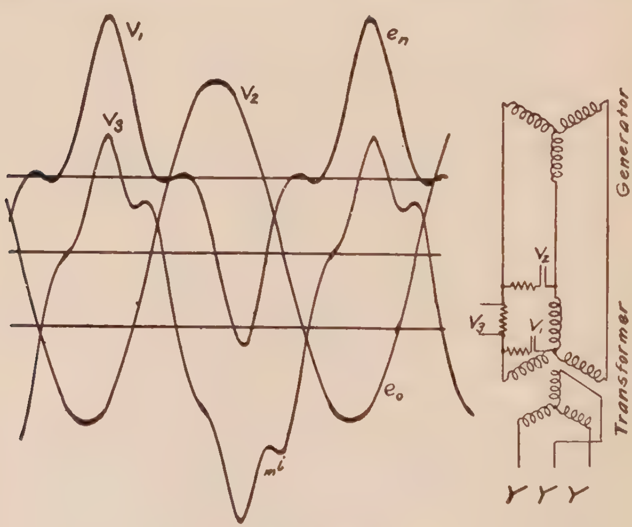


Fig. 18.24.

cannot exist in the current wave; and similarly the third, ninth, fifteenth, etc., harmonics cannot appear in the delta-connected

voltage wave. The fifth, seventh, eleventh, etc., harmonics, however, may exist in both current and voltage waves for both star and delta connections.

The effect of a star connection in distributing the harmonics produced by the hysteresis of the transformer iron is shown in Fig. 18.24. The circuit diagram shows the star connection of both the generator and the three transformers, and the locations in the

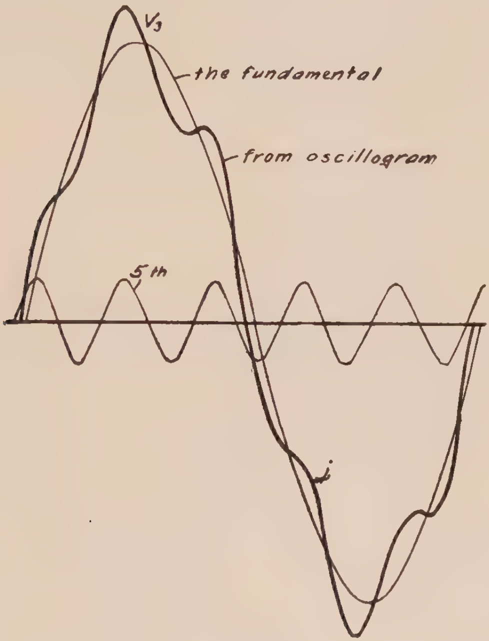


FIG. 19.24.

circuit of the three vibrators of the oscillograph. The secondaries of the transformers are open so the transformer exciting current only is flowing in the circuits. Hence vibrator 1 gives the voltage from one main to neutral, vibrator 2 the voltage between mains, and vibrator 3 the exciting current in one transformer. The voltage between mains as impressed on the transformers closely approximates a sine wave. The exciting current consists of a fundamental and a fifth harmonic, as shown in Fig. 19.24 and by equation (28.24):

$$_{m}i = 100 \sin (\omega t - 5°) + 15 \sin (5\omega t + 28°) \qquad (28.24)$$

The third harmonic produced by the hysteresis distortion appears in the voltage wave to neutral. The components of the voltage to neutral wave are shown in Fig. 20.24 and expressed by equation (29.24).

$$ {}_ne = 100 \sin (\omega t - 26°) + 44 \sin (3\omega t + 98°) \qquad (29.24) $$

In the star connection the third harmonic cannot exist in the current and hence only the fifth harmonic appears in the exciting current, while the third harmonic appears in the voltage to neutral wave. Since the voltage between the mains is the vector difference of the voltages of each pair of circuits, the third harmonics in the circuits neutralize. This may be illustrated

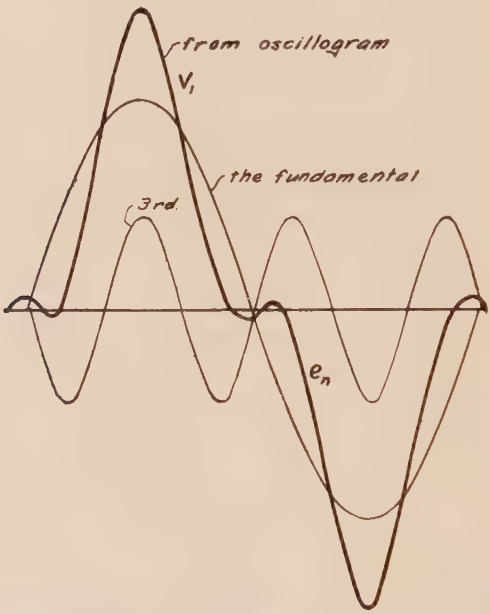


FIG. 20.24.

graphically by the diagram in Fig. 21.24. Let the triangle ABC represent the balanced voltage between the mains. If the voltage to neutral is also a sine wave, the lines OA, OB, OC with O as the center represent the corresponding circuit voltages. With a third harmonic in the three circuits the neutral point may be represented by the point N revolving around O with a triple frequency. The corresponding magnetic-flux wave and hysteresis cycle are shown in Fig. 22.24.

If the neutral of the generator be connected to the neutral point in the star-connected transformers, then the point N in Fig. 21.24 is fixed and coincides with the center O. Under these conditions there can be no third harmonic in the voltage to

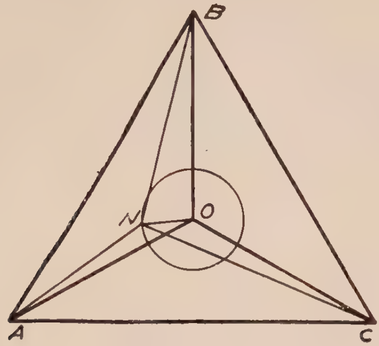


FIG. 21.24.

neutral, but a current of triple frequency flows in the neutral. In Fig. 23.24 are shown the circuit diagram and connections for the oscillograph, with the corresponding oscillograms. Vibrator

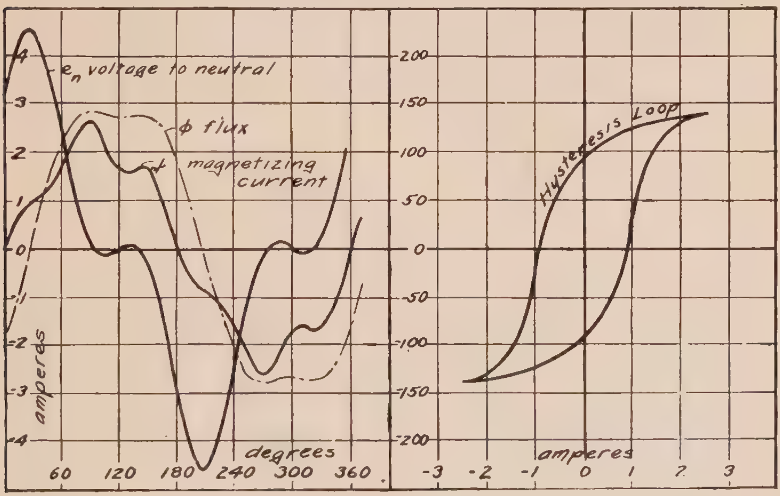


FIG. 22.24.

1 gives the voltage to neutral, vibrator 2 the voltage between mains, vibrator 3 the exciting current and vibrator 4 the current in the neutral. With the neutral connected, the hysteresis distortion produces a triple harmonic current in the neutral,

which is the sum of the three triple harmonics in the exciting currents of the three transformers.

As a third illustration of the effect of transformer connections on the wave shapes in three-phase circuits take the star-star-delta connection as shown in the circuit diagram of Fig. 24.24. The star-connected generator is again connected to the low-tension side of the three star-connected transformers, and the neutral is left open. The high-voltage sides of the transformers are connected in delta and without load. As indicated in the diagram, Fig. 24.24, vibrator 1 gives the voltage to neutral, vibrator

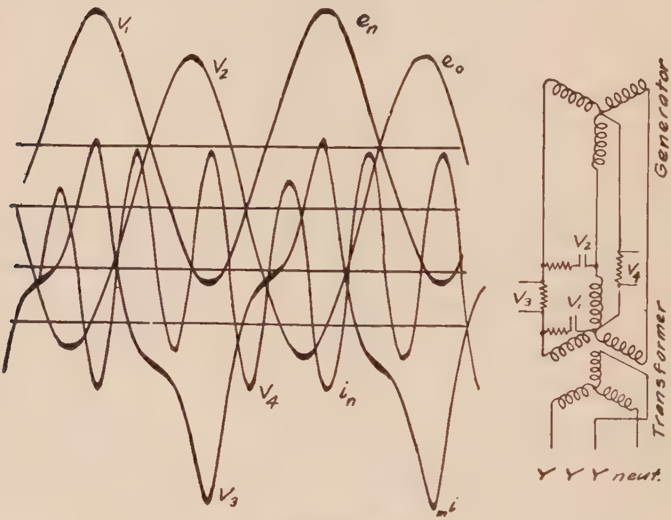


FIG. 23.24.

2 the voltage between mains, vibrator 3 the line current and vibrator 4 the delta current. The corresponding oscillograms show that the two voltages have only slight distortions, while the fifth harmonic appears in the line current. The third harmonic required by the hysteresis distortion is supplied by the current of triple frequency flowing in the secondary delta connection of the transformers. From the above it is apparent that, while the hysteresis of the iron introduces a third and a fifth harmonic, the phase connections largely determine whether the distortion shall appear in the current or voltage waves.[1]

[1] *Proc. Am. Inst. Elec. Eng.*, Vol. 33, pp. 753, 771, 785, 791, 1153.

2. *Pulsating Condensance.*—Within ordinary voltages the properties of the generally used dielectrics remain fairly constant and do not show any marked variation of the condensance of the circuit. Leaky condensers, polarization cells and possibly dielectric hysteresis cause a distortion similar to that of magnetic hysteresis. The magnitude of such distortion is, however, comparatively small and of little commercial importance. With higher voltages, particularly under conditions producing the corona or brush discharge, air as a dielectric undergoes periodic changes that cause important pulsations in the condensive reactance as well as

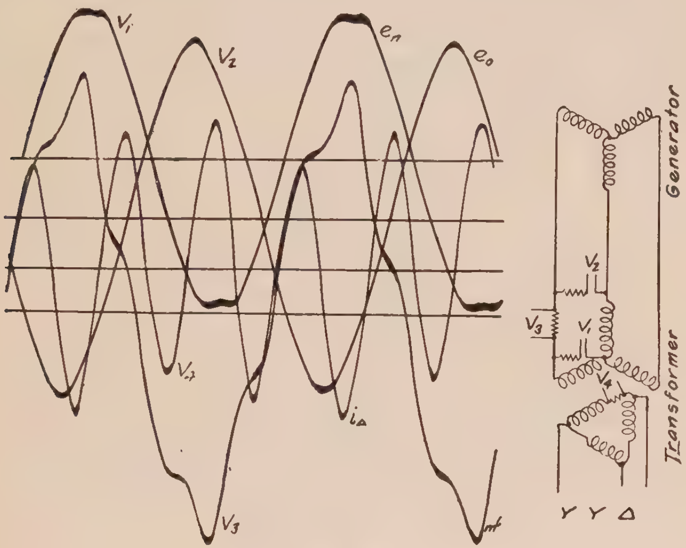


FIG. 24.24.

marked variations in the insulating properties of the material surrounding the conductor. The dielectric strength of air and the law of the corona are discussed in Chaps. XX and XXI. It was explained how the corona appeared only during that part of the voltage cycle when the instantaneous voltage exceeded the critical value. It was also noted that the positive and negative halves of the voltage wave do not produce similar corona effects. Since the extent of the corona is affected by such factors as the shape, size and distance apart of the conductors, the temperature, barometric pressure and moisture of the atmosphere, etc., it is evident that the effects upon a commercial system operating under varying conditions may be difficult to determine.

However, the presence of the corona increases the condensance and leakage loss of the system and hence causes a pulsation in the condensive reactance. As the variation is not the same for successive half waves, this pulsation may introduce even harmonics into the circuit.

In the same manner as magnetic hysteresis produces distortion primarily in the exciting current of iron-clad circuits, so the pulsations of the condensive reactance primarily introduce pulsations in the charging current of the circuit. If the condensance or charging current is of sufficient relative magnitude, secondary effects may cause distortions in the voltage wave. Moreover, the connections of the circuits in polyphase systems may shift the distortion in much the same manner as was explained for pulsations of inductive reactance.

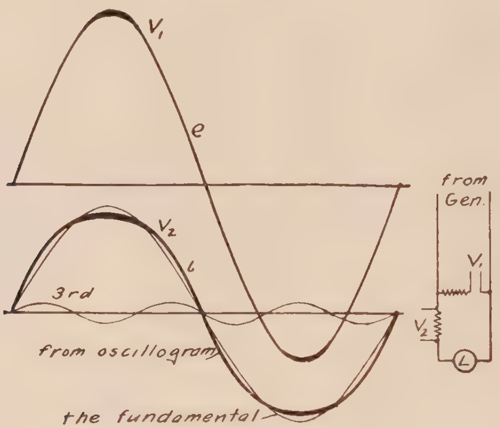


Fig. 25.24.

3. *Pulsating Resistance.*—As a rule, the pulsations in reactance are accompanied by similar pulsations in the resistance. This is indicated by the *hysteretic angle of advance* in iron-clad circuits having a pulsation of inductive reactance; and similarly in the dielectric by pulsations in the leakage current coincident with the appearance of the corona.

Rapid changes in the temperature of a conductor, as, for example, in an incandescent lamp filament, cause corresponding changes in the resistance. Thus in Fig. 25.24 are shown oscillograms of the sine voltage wave impressed upon the terminals of a tungsten incandescent lamp, and the current waves flowing in the lamp. Vibrator 1 gives the voltage wave and vibrator

2 the current in the lamp. The distortion consists mainly of a third harmonic. While periodic changes in temperature introduce pulsations in the resistance of solid conductors, the distortion of the wave shapes is small under commercial conditions and, as a consequence, of little importance.

The most important example of a synchronously pulsating resistance is the alternating-current arc. In a gaseous conductor the resistance decreases with an increase of current, being high for small currents and low for large currents. The gaseous conductor between the carbons or metallic electrodes of an arc lamp has a rapidly varying resistance, pulsating at double the frequency of the current. Assuming that the current flowing in the arc is a sine wave, and that the fluctuations of the resistance vary inversely with the current, the circuit relations may be expressed as in equations (24.24) to (32.24).

$$i = {}^{M}I \sin(\omega t) = \sqrt{2}I \sin(\omega t) \tag{30.24}$$

$$r = r_0[1 + k \cos(2\omega t)] \tag{31.24}$$

$$\begin{aligned} e = ri &= \sqrt{2}Ir_0 \sin(\omega t)\,[1 + k\cos(2\omega t)] \\ &= \sqrt{2}r_0 I\left[\left(1 - \frac{k}{2}\right)\sin(\omega t) + \frac{k}{2}\sin 3(\omega t)\right] \end{aligned} \tag{32.24}$$

The distortion of the voltage wave therefore consists, under the given assumption, of a third harmonic in phase with the fundamental when passing zero value, and hence 180° out of phase at the maximum value of the fundamental. This is illustrated in Fig. 26.24 for the following values of current and resistance:

$$i = 10 \sin(\omega t) \tag{33.24}$$

$$r = 4[1 + 0.6 \cos(2\omega t)] \tag{34.24}$$

$$e = 28 \sin(\omega t) + 0.43 \sin(3\omega t) \tag{35.24}$$

With a sine wave of current the voltage wave becomes double-peaked, having an abrupt rise near the zero value of the current. Conversely, if a sine wave of voltage is impressed across the terminals of the arc the resulting current wave in the arc is peaked at the maximum points and flat near the zero values of the voltage. While the assumption of a variation in the resistance inversely proportional to the current illustrates the main feature of the phenomenon, the relations in the actual arc are much more complex. Many factors affecting the resistance variation are difficult to determine. The material used in the

electrodes radically modifies the resistance variation. A cored or impregnated set of carbons gives an entirely different law of resistance variation than would apply to hard carbons or for carbides; similarly, different metals or electrodes radically change the resistance of the arc. The temperature of the surrounding space, the vaporization point of the materials in the electrodes, the size and shape of the electrodes, enclosed or open condition of the arc, length of the arc, current density, frequency and wave shape of the current or of the impressed voltage and other

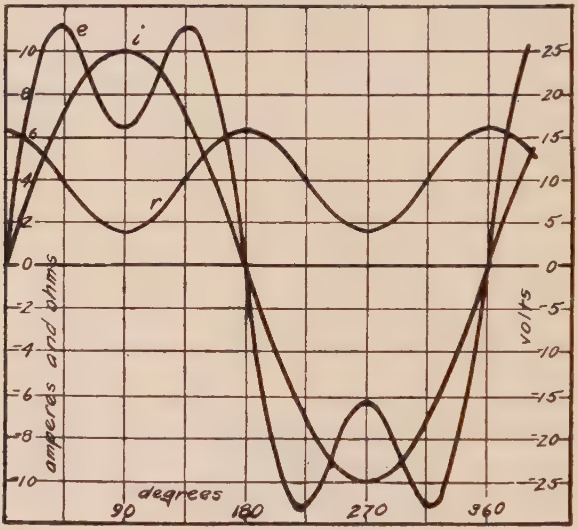


FIG. 26.24.

factors affect the resistance and may produce an infinite variety of resistance fluctuations. In all this diversity of electric-arc phenomena, the double-peaked nature of voltage wave is the most important. This characteristic is so marked that if in an investigation of an electric circuit by the oscillograph such wave shapes are found, the existence of an arc or arcing ground may be suspected. It is of special importance in high-tension systems since any arc or arcing ground is a most fruitful source for high-frequency oscillations producing dangerous voltages.

(*f*) **Analysis of Wave Forms.** *First Method.*—A number of equally spaced ordinates are measured on the half wave to be analyzed. The number of ordinates used must be one greater than the highest harmonic. Thus, if the eleventh harmonic is

the highest, 12 equally spaced ordinates are selected, while if the seventeenth harmonic is the highest, 18 equally spaced ordinates must be measured. These measured values are tabulated and a series of operations performed as prescribed in the corresponding chart[1] for the number of ordinates measured. In this way the numerical values of the sine and cosine coefficients are derived for the fundamental and each of the harmonics. Having the values of the sine and cosine coefficients, the numerical values may be substituted in the equation. As this determines both the magnitude and phase position of the several component waves, the curves may be plotted. The method is, however, laborious and requires painstaking care.

Second Method.—Several mechanical devices, which have been invented for analyzing complex waves, operate in a more or less

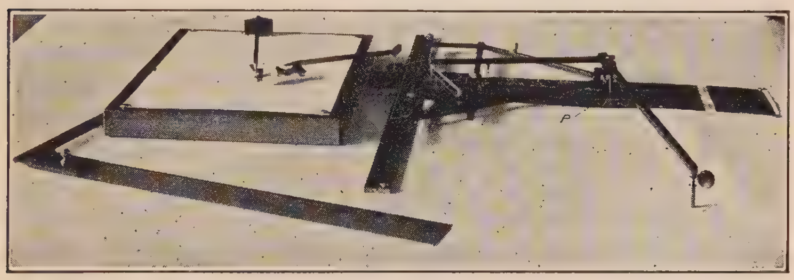


FIG. 27.24.

satisfactory manner. The harmonic analyzer devised by Dr. O. Mader is an excellent example of a simple mechanical contrivance by which the complex waves may be rapidly analyzed. The instrument is shown in Fig. 27.24 in connection with an ordinary planimeter for obtaining quantitative values. It may be noted that the values of the sine and cosine coefficients are taken separately for each of the component waves. After the limits of the carriage movement have been adjusted to the size of wave to be analyzed, the process is as follows: Select the proper cog wheel for the fundamental. Place the podium of the planimeter in the small hole marked s on the cog wheel. Move the point P to the left end of the wave, adjust the cog wheel to the zero position and read the planimeter. Next let the point P trace the complete wave and return to the starting point along the X-axis in Fig. 27.24. Again read the planimeter. The differ-

[1] BEDELL and PIERCE, "Direct and Alternating-current Manual," *U. S. Bur. Stand. Bull.* 203, p. 331.

ence of the two planimeter readings gives directly the sine component of the coefficient for the fundamental wave.

To obtain the cosine component, merely move the podium of the planimeter to the small hole marked c on the cog wheel; read the planimeter; move the point P through the complete cycle as before; again read the planimeter. The difference in the two planimeter readings gives directly the cosine component of the fundamental.

For any of the harmonics select the proper cog wheel and take similar readings for the sine and cosine components. The adjustments can be made quickly and the readings taken rapidly. With care the errors will be less than 5 per cent, which is about the accuracy that can be obtained by the first method.

(*g*) **Effective Values of Currents and Voltages of Distorted Wave Shapes.**—Since the effective value of voltage or current represents the square root of the mean square of the instantaneous values, and since the successive cycles are alike, the effective values may be expressed by equations (36.24) and (37.24).

$$E = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} e^2 d(\omega t)} \tag{36.24}$$

and

$$I = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} i^2 d(\omega t)} \tag{37.24}$$

From equations (3.24) and (22.24)

$$e = {}^M E_1 \sin(\omega t + \gamma_1) + {}^M E_3 \sin(3\omega t + \gamma_3) + {}^M E_5 \sin(5\omega t + \gamma_5) + \text{etc.} \tag{38.24}$$

$$i = {}^M I_1 \sin(\omega t + \gamma_1 - \theta_1) + {}^M I_3 \sin(3\omega t + \gamma_3 - \theta_3) + {}^M I_5 \sin(5\omega t + \gamma_5 - \theta_5) + \text{etc.} \tag{39.24}$$

From calculus, for any positive integral values of m and n:

$$\left.\begin{aligned}
&\int_{-\pi}^{+\pi} \cos mx \sin nx dx = 0 \\
&\int_{-\pi}^{+\pi} \cos mx \cos nx dx = \begin{cases} 0 \text{ when } m \lessgtr n \\ 2\pi \text{ when } m = n = 0 \\ \pi \text{ when } m = n > 0 \end{cases} \\
&\int_{-\pi}^{+\pi} \sin mx \sin nx dx = \begin{cases} 0 \text{ when } m \lessgtr n \\ 0 \text{ when } m = n = 0 \\ \pi \text{ when } m = n > 0 \end{cases}
\end{aligned}\right\} \tag{40.24}$$

and it is evident that, in the integration of e^2dt and i^2dt, all terms except those that contain a sine squared are equal to zero. Hence,

$$E = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} e^2 d(\omega t)} = \sqrt{\frac{1}{2}({}^ME_1{}^2 + {}^ME_3{}^2 + {}^ME_5{}^2 \cdots \text{etc.})}$$

$$= \sqrt{E_1{}^2 + E_3{}^2 + E_5{}^2 + \cdots \text{etc.}} \qquad (41.24)$$

and

$$I = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} i^2 d(\omega t)} = \sqrt{\frac{1}{2}({}^MI_1{}^2 + {}^MI_3{}^2 + {}^MI_5{}^2 + \cdots \text{etc.})}$$

$$= \sqrt{I_1{}^2 + I_3{}^2 + I_5{}^2 + \cdots \text{etc.}} \qquad (42.24)$$

Graphic Method.—The effective value of a complex voltage or current wave may be found graphically without the necessity of analyzing the curve into the component harmonics. The half-wave cycle is plotted in polar coördinates and the area of the curve obtained by means of a planimeter. Draw a circle having the same area. Then the diameter of this circle is equal to the maximum value of the equivalent sine wave. The effective value is therefore $\sqrt{2}R$, where R is the radius of the circle. Fairly accurate results may be obtained even without the aid of a planimeter, as a circle of approximately the same area can readily be drawn over the polar wave diagram.

(*h*) **Form Factor, Curve Factor, Peak Factor, Harmonic Factor and Distortion Factor for Complex Waves.** *Form Factor.*—The form factor is the ratio between the effective and average values. The average value may be found by plotting the wave in rectangular coördinates, obtaining the area by means of a planimeter and dividing the area by the base, thus obtaining an average ordinate. For sine waves the form factor = 1.11.

Curve Factor.—The curve factor is the ratio between the effective values of the complex wave and of the fundamental.

$$\text{Curve factor} = \frac{E}{E_1} = \frac{\sqrt{E_1{}^2 + E_3{}^2 + E_5{}^2 + \cdots}}{E_1} \qquad (43.24)$$

Peak Factor.—The peak or crest factor is the ratio between the maximum and the effective values.

$$\text{Peak factor} = \frac{{}^ME}{\sqrt{E_1{}^2 + E_3{}^2 + E_5{}^2 + \cdots}} \qquad (44.24)$$

This ratio is of importance in the study of corona, insulation tests, etc.

Harmonic Factor.—The harmonic factor is the ratio of the effective value of all the harmonics to the effective value of the fundamental.

$$\text{Harmonic factor} = \frac{\sqrt{E_3^2 + E_5^2 + E_7^2 + \cdots}}{E_1} \tag{45.24}$$

Distortion Factor.—The distortion factor is the ratio of the effective value of the first derivative of the wave, with respect to time, to the effective value of the first derivative of the equivalent sine wave. If a condenser be connected across the terminals of a circuit, on which a complex voltage wave is impressed, the current flowing will be greater than for a simple sine voltage wave of effective value equal to the equivalent effective value of the complex wave. The true reactance of the condenser at the frequency of the fundamental is

$$_cx_1 = \frac{E_1}{I_1} \tag{46.24}$$

The apparent reactance for the complex wave is

$$_cx_0 = \frac{\sqrt{E_1^2 + E_3^2 + E_5^2 + \cdots}}{\sqrt{I_1^2 + I_3^2 + I_5^2 + \cdots}} \tag{47.24}$$

$$\text{Distortion factor} = \frac{_cx_1}{_cx_0} \tag{48.24}$$

(*i*) **Power in Circuits with Distorted Current and Voltage Waves. Equivalent Sine Wave. Power Factor.**—The power delivered in a circuit is

$$P = \frac{1}{T}\int_0^T eidt \tag{49.24}$$

Substituting the values of e and i from equations (38.24) and (39.24) and integrating the product, all terms vanish except those containing a sine squared, in the same manner as for the effective values of voltage and current.

$$\begin{aligned} P &= \tfrac{1}{2}({}^ME_1{}^MI_1 \cos\theta_1 + {}^ME_3{}^MI_3 \cos\theta_3 + {}^ME_5{}^MI_5 \cos\theta_5 + \cdots) \\ &= E_1I_1 \cos\theta_1 + E_3I_3 \cos\theta_3 + E_5I_5 \cos\theta_5 + \cdots \qquad (50.24) \\ &= P_1 + P_3 + P_5 + \text{etc.} \cdots \end{aligned}$$

It is important to note that as regards the power all the harmonics are independent of each other. The current of any harmonic is wattless with respect to the pressures of all other harmonics. Each harmonic has its own power factor and each follows independently the same laws as already explained for simple sine waves. The total power is the summation of the powers of the several harmonics. Hence each harmonic may be

treated by itself and the effects calculated separately by the laws and graphic constructions already deduced. In commercial work the wave shape is seldom known with any close degree of exactness. The analysis of waves is a tedious process and the power and power factor are usually determined by the wattmeter, voltmeter and ammeter readings. It is generally assumed that the actual conditions may be represented by an *equivalent sine wave* of current and voltage and that the *power factor* is the ratio of the watts to the volt-amperes.

$$P = EI\cos\theta = rI^2 \tag{51.24}$$

In equation (51.24) r is the effective resistance, I the *equivalent sine current wave* and $\cos\theta$ the measured *power factor.*

From equations (19.24), (20.24) and (31.24) and considering the effective resistance constant for all frequencies:

$$\cos\theta = \frac{rI}{E} = r\frac{\sqrt{I_1^2 + I_3^2 + I_5^2 + \ldots}}{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots}} \tag{52.24}$$

$$= r\frac{\sqrt{\dfrac{E_1^2}{r^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} + \dfrac{E_3^2}{r^2 + \left(3\omega L - \dfrac{1}{3\omega C}\right)^2} + \dfrac{E_5^2}{r^2 + \left(5\omega L - \dfrac{1}{5\omega C}\right)^2} + \ldots}}{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots}} \tag{53.24}$$

$$= \sqrt{\frac{E_1^2\cos^2\theta_1 + E_3^2\cos^2\theta_3 + E_5^2\cos^2\theta_5 + \ldots}{E_1^2 + E_3^2 + E_5^2 + \ldots}} \tag{54.24}$$

It should be noted that, while the resistance is constant for all frequencies (in all cases to which the above equations apply), the reactance is affected by the frequency, and hence the relative value of the reactances for the several frequencies depends upon whether the circuit is inductive or condensive. If x_n be the reactance for the nth harmonic, its value is given in equation (55.24) in terms of the fundamental frequency and the inductance and condensance of the circuit.

$$x_n = n\omega L - \frac{1}{n\omega C} = n_L x_1 - \frac{{}_c x_1}{n} \tag{55.24}$$

Hence,

$$x_3 = 3_L x_1 - \frac{{}_c x_1}{3};\ x_5 = 5_L x_1 - \frac{{}_c x_1}{5};\ \text{etc.} \tag{56.24}$$

$$\tan\theta_n = \frac{x_n}{r} = \frac{n\omega L}{r} - \frac{1}{rn\omega C} \tag{57.24}$$

$$\cos\theta_n = \frac{r}{\sqrt{r^2 + x_n^2}} = \frac{r}{Z_n} = \frac{rI_n}{E_n} \tag{58.24}$$

(*j*) **Resonance in Circuits with Distorted Current and Voltage Waves.**—Since any complex wave may be separated into a series of sine waves of multiple frequency, and each harmonic acts independently, the effect of a distorted voltage wave is equivalent to the resultant of the component sine waves. While the inductance and condensance are constant, the inductive and condensive reactances are different for the several harmonics. For resonance the inductive and condensive reactances or susceptances must be equal. Hence resonance can occur only for one frequency. This is illustrated in Fig. 28.24. A voltage wave repre-

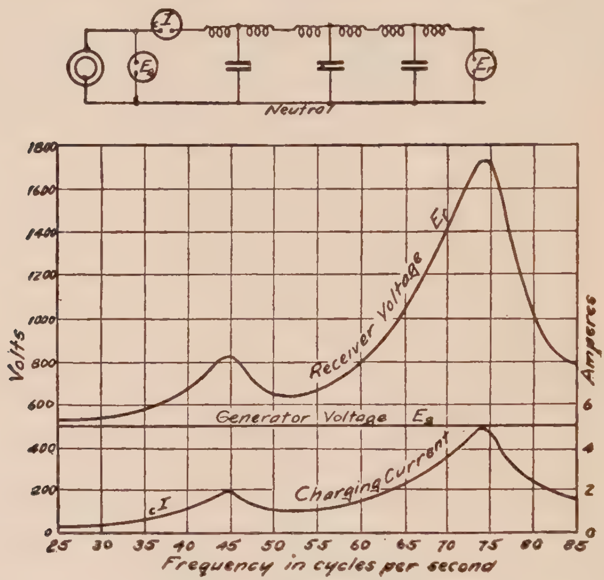


FIG. 28.24.

sented by equation (59.24) was impressed on one end of an artificial transmission line with the ammeter and voltmeters connected as shown in the circuit diagram.

$$e = 100 \sin(\omega t) - 28.2 \sin(3\omega t) + 5.4 \sin(5\omega t) \quad (59.24)$$

By varying the frequency of the impressed voltage and reading the charging current at the generator end and the voltage at the receiver end the data for the two curves were obtained. The natural period of the line was 223 cycles. Note the increase in $_cI$ and E_r when $f = 44.5$ and for $f = 74$. Also that $44.5 \times 5 = 222.5$; and $74 \times 3 = 222$. That is, for $f = 44.5$, the circuit was

in resonance with the fifth harmonic, and for $f = 74$, resonance was produced by the third harmonic. The increase is approximately proportional to the relative magnitude of the two harmonics in the impressed voltage wave.

It should also be noted that for some frequencies the current may be leading for one harmonic and lagging for another. Thus for frequencies giving resonance for the third harmonic,

$$x_3 = 3\omega L - \frac{1}{3\omega C} = 3\,{}_Lx_1 - \frac{{}_cx_1}{3} = 0 \qquad (60.24)$$

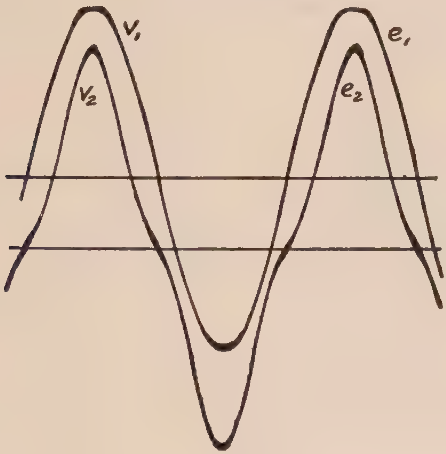


FIG. 29.24.

At this frequency the fundamental has a leading current, since

$$x_1 = \omega L - \frac{1}{\omega C} \qquad (61.24)$$

must be negative and therefore condensive; while the fifth, seventh, etc., harmonics must have lagging currents, since

$$x_5 = 5\,{}_Lx_1 - \frac{{}_cx_1}{5};\ x_7 = 7\,{}_Lx_1 - \frac{{}_cx_1}{7};\ \text{etc.} \qquad (62.24)$$

are positive and therefore inductive.

(k) **The Standard Wave Form.**—In small plants operating at low potential the current and voltage wave forms are of minor importance. As this was the general condition in the early development of the electrical industry little attention was given to wave forms. The vast extension of electrical distribution systems, the development of enormous central stations and the necessity of operating long-distance transmission lines at high

potentials demand very careful attention to all factors that may produce wave distortion.

Two requirements may be considered as fundamental for the wave form in constant-potential systems:

1. In parallel operation the waves generated should be equal at all instants so as to avoid cross-currents.

2. The differentials and the integrals of the curve should have the same shape as the generated wave.

For alternating currents, generated by rotating machinery, both requirements are met by the simple sine wave. Consider

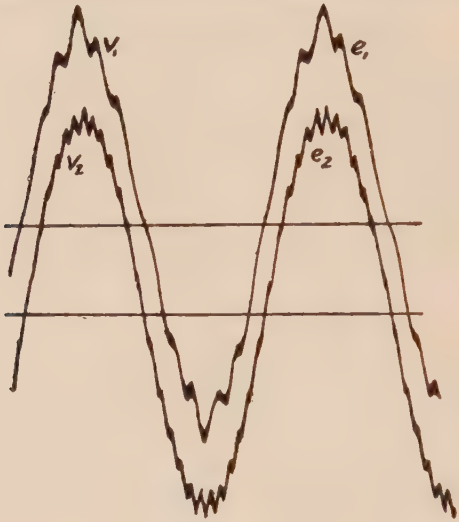


Fig. 30.24.

the two voltage waves in Fig. 29.24, an oscillogram taken from two small generators (35 and 60 kv.a.) operating in parallel in an isolated plant. Machine 1, e_1, gives almost a simple sine wave, while in machine 2, e_2, both the third and fifth harmonics are present. It is evident that for all instants during which the two waves are not equal the difference in the voltages causes cross-currents to flow through the two armatures. The additional RI^2 losses reduce the efficiency and also raise the temperature above the value which would obtain for the same load if the machines operated independently. Since the permissible output, or rating, of well-designed electrical machines is based on the temperature rise, the cross-currents reduce both the rating and the efficiency of the alternator. In Fig. 30.24 is shown an

oscillogram of two larger units (500 and 1,500 kv.a.) operating in parallel on the same busbars. Machine 1, e_1, has a marked eleventh, and machine 2, e_2, has a strong twenty-fifth harmonic, and instantaneous values of the two waves differ at most points.

In the induction motor the harmonics are likewise undesirable. In a rotating field the fundamental alone produces the useful torque, while the harmonics increase the losses and may cause undesirable stresses on the dielectric.

As already explained, the harmonics may produce entirely different effects from the fundamental while passing through an extensive distribution system. For instance, resonance condi-

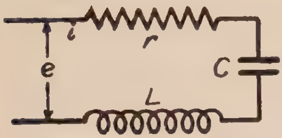


FIG. 31.24.

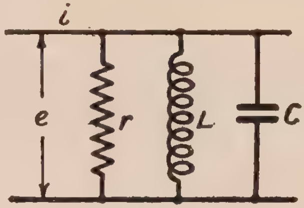


FIG. 32.24.

tions for one harmonic in some section of the system may greatly magnify the distortion and cause excessive pressures or abnormally large currents.

The basis for the second requirement is found in the fundamental law of magnetic and dielectric induction, as may be readily seen from equations (63.24) and (64.24). In a simple series circuit, Fig. 31.24, the instantaneous voltage is:

$$e = ri + L\frac{di}{dt} + \frac{\int idt}{C} \tag{63.24}$$

For a simple parallel circuit, as in Fig. 32.24, the instantaneous current is:

$$i = \frac{e}{r} + \frac{\int edt}{L} + C\frac{de}{dt} \tag{64.24}$$

Letting $i = {}^{M}I \sin(\omega t)$ for equation (63.24), and similarly $e = {}^{M}E \sin(\omega t)$ for equation (64.24), the total voltage and current waves are of the same simple sine form, of the same frequency and are merely displaced in time-phase position. For any other wave form there is distortion so that the total voltage wave in

equation (63.24) is not of the same shape as the current; nor is the total current wave in equation (64.24) like the impressed voltage.

The chief unavoidable factor producing distortion in the wave forms is the variable permeability of iron. The effect is, of course, greatest in iron-clad circuits. Many other causes, which offer a constant menace to satisfactory operation, are often present in commercial systems due to faulty design or construction. The generated voltage wave should approximate very closely the standard sinusoidal form, and the system should be so constructed as to eliminate as completely as possible all sources of wave distortion.

PROBLEMS

1.24. A generator having an effective inductance of 0.01 henry and a resistance of 0.1 ohm is connected to a series circuit consisting of a 10-mf condenser, a 0.1-henry inductance and a 10-ohm resistance. The generated wave of the generator is represented by

$$e_0 = 100 \sin wt + 20 \sin (3wt + 30°) + 10 \sin (5wt + 15°)$$

The frequency is 60 cycles per sec. for the fundamental. Plot waves of current and voltage across the inductance in rectangular coordinates.

2.24. What are the effective voltages across each unit in the series circuit in problem 1.24? What is the effective current? What is the average power? What is the relation between the cosine of the angle of phase difference between fundamental current and terminal voltage and the average power divided by the effective volt-amperes?

3.24. In Fig. 32.24

Let

$e_0 = 100 \sin wt + 30 \sin (5wt + 15°)$.

$r = 10$ ohms.

$L = 0.1$ henry.

$C = 100$ mf.

$f = 60$ cycles.

(a) Find E_0, ${}_cI$, ${}_LI$, ${}_rI$ and I_0.

(b) Write the equation for i_0.

(c) Plot curves in rectangular coördinates for e and i.

CHAPTER XXV

PROTECTIVE APPLIANCES

In all electric plants safety devices are installed for protection against abnormal conditions. Even under normal operating conditions the stresses may vary greatly under changes of load, and especially while starting or stopping machines may reach several times the average values. The extent of this variation may be calculated and the maximum stresses must be provided for in the design of the apparatus. Auxiliary apparatus, like starting autotransformers, are used to lower the impressed voltage; and circuit-breakers and fuses are inserted to protect against excessive currents. Speed control is centered in the governor of the prime mover. Any changes in the load must be balanced by a similar change in the amount of energy supplied by the prime mover. The action of the governor cannot, however, be instantaneous and there will always be a certain lag in the adjustment of the energy delivered by the generator to the amount received by the load. Hence after a decrease in load the generators and motors receive more energy than is delivered; and conversely, for an increase in the load the amount delivered is greater than that received. The difference must be stored in or taken from the system and causes changes chiefly in the kinetic energy of the rotating machinery. Any change in the load must, therefore, be counteracted by rapid and effective action of the governor of the prime mover in order to obtain, as nearly as possible, the constant speed required for constant frequency.

Abnormal conditions are, of course, transient phenomena, and an extended discussion does not come within the scope of this book. However, a brief statement of two important groups of dangerous disturbances will be given in order that students may more readily understand the limitations imposed on operating conditions for alternating-current systems.

(*a*) Lightning.

(*b*) Short-circuits and arcing grounds.

(*a*) **Lightning.**—Lightning is a complex phenomenon and for many of its manifestations no satisfactory explanations have been found. The results of extensive investigations, still in progress, have determined with sufficient accuracy the more important factors, so that apparatus can be constructed that effectively protect electric systems against damage from lightning, except in extreme cases. The process by which electric energy becomes stored dielectrically in a cloud is not fully determined. Rapidly rising air currents moving against the falling rain drops is held to be a factor for increasing the voltage, although, most likely, of minor importance. Under atmospheric conditions the potential differences between the cloud and earth, or between two clouds, or between parts of the same cloud, are very large, often reaching millions of volts before disruptive action, or a lightning stroke, suddenly relieves the dielectric stress. The cloud volume is a non-conductor, there being practically no free electrons or ions in the interspaces of the tiny water globules of which the cloud is composed. In order to release the dielectric energy stored in the cloud a portion of the cloud or some path in the cloud must become conducting; that is, ionization must be initiated at some point from which the conducting path may extend into the cloud volume, thus forming the channel through which the dielectrically stored energy can discharge. The duration of the lightning stroke is extremely short, a few microseconds, but during this brief interval of time the current, and especially the power, may be very large, in many cases probably millions of kilowatts. However, the amount of energy released in the dielectric circuit, and suddenly transformed into light, heat, sound, mechanical work and chemical reactions, varies widely, and may be only a few kilowatt-hours. It is the *extremely rapid rate of transformation*, the *steepness of the current and voltage wave fronts*, that makes lightning so destructive, although in most cases the total amount of energy involved may be relatively small. The collapse of the conducting channel, when the discharge is completed, is likewise abrupt; and all parts of the cloud rapidly (in a few seconds) regain their pre-stroke, non-conducting property and potential characteristics. The readjustment of the energy distribution, although a secondary effect, may induce violent dielectric disturbances that may cause damage to electrical equipment at a considerable distance from the actual stroke. This is particularly true in electric power systems, as the trans-

mission-line conductors form special rapid-transit routes for equalizing the energy distribution.

The polarity of a thunder cloud with respect to earth may be either positive or negative; or two parts of the same cloud may differ in polarity with respect to earth and with respect to each other. In most cases of lightning strokes between earth and cloud, the positive terminal of the conducting channel is at the earth, with the negative terminal or terminals of the branching lightning stroke in the cloud.

If it be assumed that lightning discharges originate in regions of high positive potential, the lightning channel would be formed in much the same manner as the streamers, or slides, for positive Lichtenberg figures, or the positive brushes of glide sparks. That is, the voltage gradient wave in a lightning stroke originates at a point of positive potential and *travels away* from the starting point. At successive points the voltage gradient draws or drives electrons towards the positive starting point. The electrons, rapidly increasing in number, due to ionization by collision, produce grooves, or slides, that form the highly conducting paths of the lightning discharge. Aside from the speed (in the order of 10^8 cm. per sec.) at which the conducting channel is formed and the ionization process, the lightning discharge may well be likened to a river, which formed by erosion develops from its mouth upwards into several branches and numerous rills, but in which the water that develops the channel, flows in the opposite direction. In general, the lightning cycle may be considered as consisting of the following stages:

1. A gradual increase in the potential difference between the cloud and the earth or between two clouds or parts of clouds with a corresponding increase in the electrically stored energy and in the dielectric stress. During this stage the cloud volume is a non-conductor of electricity.

2. When at any point of positive potential the *voltage gradient* becomes sufficiently high, so as to cause ionization, a lightning stroke may be started, which rapidly develops a highly conducting path in the electrically charged cloud. Through this conducting path or stroke the energy stored in a limited space near the conducting channel is discharged and transformed at an extremely rapid rate into light, heat, sound, mechanical energy and chemical reactions. The lightning strokes or conducting channels extend from the starting point into the cloud until at

the end of the channel the voltage gradient falls below the ionizing potential. At the instant this point is reached, and the ionization at the negative terminal of the path ceases, the conducting channel suddenly collapses as the electron stream falls into the positive terminal.

3. A rapid readjustment of the energy distribution in all parts of the complex dielectric system; the cloud again becomes a non-conductor and regains its pre-stroke, potential characteristics.

The cycle then repeats: A gradual increase in the electric charge and stress until, at some point the ionizing potential gradient is reached; rapid ionization and sudden extension of conducting paths on the surface of the cloud and into the cloud volume accompanied by a discharge of the electrically stored energy, followed by an abrupt collapse of the conducting paths; a rapid readjustment of the stored energy distribution and potential stress in the surrounding region. Only the energy stored in a limited space near the conducting channels is released by any one lightning stroke. So many variables, as the polarity of the cloud, potential gradients in the cloud volume and to different points on the earth; the size and shape of the cloud; the conductivity of the air surrounding the cloud; changes in temperature, humidity, precipitation; direction and intensity of wind, and other related factors, may vary widely and so modify local conditions that quantitative data for specific cases are, in the main, unobtainable.

(*b*) **Short-circuits. Arcing Grounds.**—Frequently the internal disturbances produce a breakdown of the insulation followed by a short-circuit, or an arcing ground. Other causes, such as the sudden operation of switches or circuit-breakers, or the occurrence of resonance may also produce high voltages with disastrous results. Usually the disturbances from internal causes are of comparatively low voltage and low frequency but often of very large current. Under certain conditions, as switching heavy loads, high voltages may result. Isolated systems are subject to greater stresses than grounded systems. On constant-potential systems the generators and prime movers automatically tend to hold the voltage at the normal value, and hence under short-circuit or arcing-ground conditions enormous currents may flow. With the entire power of the station forcing energy into the break, the effects may be very destructive. Intermittent arcing grounds are especially dangerous because they impose a succession of

severe strains on the apparatus. The arc may start oscillations of almost any frequency, and it is therefore extremely dangerous, since it may cause resonance or destructive surges in almost any system.

PROTECTIVE APPARATUS

In electric power systems, whether the source of the disturbance be from external or internal causes, or a combination of both, the dangerous abnormal conditions consist of high voltages, excessively large currents or high frequencies. Since these factors all refer to a transient flow of energy, the abnormal condition may be transferred from one factor to another in passing through parts of the electric apparatus. Thus a high-frequency current passing through an inductive reactance, as a transformer winding, generates in the first few turns an abnormally high voltage that may puncture the insulation. Although most of the dangerous disturbances are caused by external forces, as lightning and wind storms, and first affect the transmission line, the protective appliances are installed primarily to shield the station apparatus. Three fundamental ideas that underlie protective installations should be kept in mind:

1. To open the circuit automatically and thus cut off the portion of the system having trouble.

2. To provide a low-resistance path to ground by which the energy in the transient may be removed from the system.

3. To limit the energy flow between sections of the system, until the short-circuited portion may be isolated.

(*a*) **Circuit-breakers.**—Considered as protective devices, circuit-breakers operate primarily on the first of the above stated basic principles. The duty of the circuit-breaker is to open the circuit having trouble and thereby protect the main, or unaffected part of the system. With reference to the materials directly affecting the arc extinction in the opening of a circuit, there are three main types of circuit-breakers.

1. Carbon contacts breaking in air.

2. Copper contacts breaking in oil.

3. Deion breakers in which the arc, in segments, moves rapidly over metal plates until extinguished.

In the first type the main contacts, which open first, are of copper but the actual breaking of the circuit occurs a fraction of a

second later when the smaller carbon contacts separate. This type is limited to low-voltage circuits, 2,300 volts or less.

In the second type the contact jaws open under oil, which becomes an important factor in quenching or extinguishing the arc. The oil circuit-breaker is generally used in high voltage and large power circuits.

In the deion[1] circuit-breaker, the arc is drawn in air but forced magnetically into a deionizing chamber, consisting of a number of closely spaced parallel metal plates, where it is broken up into a number of short arcs that travel at high velocity over annular paths until extinguished.

(*b*) **Special Ground Wire.**—Long-distance transmission lines are often provided with an extra wire or stranded cable supported on the tops of the poles above the regular transmission wires. This conductor is grounded at frequent intervals, preferably at every pole. The *ground* is thereby brought above the transmission line and hence the dielectric stress in the air, due to lightning and other atmospheric disturbances, near the transmission wires and insulators is greatly reduced. Since the ground wire is parallel to the transmission wires, it also dampens any impulses or surges that travel along the line, thus aiding the station arresters. Although the ground wire is on top of the transmission line, its chief service is to prevent external disturbances from reaching the station apparatus, the protection of the transmission line itself being of secondary importance.

(*c*) **Arresters.**—The function of the arrester is to protect the apparatus by relieving the system of voltages that may break down the insulation. The lightning arresters safeguard the insulation by automatically providing a by-path to ground for the transient energy impulse when the voltage exceeds a predetermined value. Energy impulses, produced by lightning, switching, arcing grounds, or other violent electric disturbances, produce voltage waves of very steep wave fronts. At the open end of a line or where a line terminates in a high surge impedance, as in a transformer, the voltage waves are reflected; but in the process the surge voltage at the terminal rises to nearly double its previous magnitude. A lightning arrester connected to the

[1] Slepian, J., "Theory of the Deion Circuit Breaker," *Trans. Am. Inst. Elec. Eng.*, Vol. 48, p. 523.

ine at a point near the transformer would prevent the rise in voltage by automatically providing a by-path through which the transient energy is diverted to earth.

The arrester therefore *operates as a valve* which automatically opens when the voltage in the line exceeds a predetermined value, and promptly closes when the voltage drops to normal operating value. This *valve action* was first observed as a property of thin aluminum-oxide films and forms the basis of the electrolytic lightning arrester. Like valve action has later been found in several other materials, on the basis of which the oxide film, pellet, autovalve and thyrite lightning arresters have been developed. The several types of arresters that have proved to be of practical value in some phase of circuit protection may be listed as follows:

1. Horn gap.
2. Sphere gap.
3. Non-arcing metal .
4. Compression chamber.
5. Electrolytic.
6. Pellet.
7. Oxide film.
8. Autovalve.
9. Thyrite.

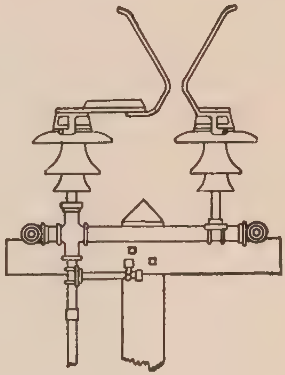


Fig. 1.25.

1. *Horn Gap.*—Two curved wires, bending away from each other, as shown in Fig. 1.25, form the horn-gap arrester. One side is connected to the line wire, and the other to ground, usually in series with a resistance or some form of valve arrester. If the voltage gradient exceeds the spark-over value, a discharge occurs across the narrowest part of the gap and an arc is formed. The path of the current through the lower portion of the horn-shaped wires and the bridging arc is in the shape of an inverted U, and the magnetic field on the inside of this curve forces the arc upward, thereby increasing its length, until the arc breaks. The heat liberated by the arc produces air currents that, to some extent, aid the magnetic forces in breaking the arc.

2. *Sphere Gap.*—The sphere-gap arrester is similar in construction to the horn-gap arrester, except that two hemispheres are used in place of the curved wires or horns. One of the hemispheres is connected to the line wire and the other to ground

through a resistance in series with some form of valve-action arrester.

3. *Non-arcing Metal Arresters.*—When a discharge occurs between copper electrodes, a vapor stream of low resistance bridges the gap. The high resistance of the air gap is thus changed into a fairly good conductor. Similar results follow if the electrodes are made of other metals. But if certain alloys of zinc are used as terminals, the vapor formed by spark-over has a rectifying property which tends to prevent the current from flowing in the reverse direction during the next half cycle. This property of the so-called *non-arcing metal* tends to limit an alternating-current arc to one-half cycle. The non-arcing metal arrester consists essentially of a series of knurled cylinders of a non-arcing alloy separated by small air gaps. In general, the series of air gaps is combined with resistance in order to extend the voltage range of the arrester.

4. *Compression Chamber Arrester.*—The essential element of the compression chamber arrester is a small air gap between two electrodes of non-arcing metal (zinc alloy) separated by a porcelain spacer. During discharge the gases, held in the small chamber formed by the electrodes and spacer, become somewhat compressed and assist in extinguishing the arc. This feature gives the arrester its name. This type of arrester is designed for protection of secondary lighting and power circuits.

5. *Electrolytic Arresters.*—The elemental unit in the electrolytic arrester is a cone-shaped aluminum tray immersed in an electrolyte and so placed that the current flowing in the electrolyte passes through the tray. The current passing through the aluminum forms a thin surface layer of aluminum hydroxide having a very high resistance. For higher impressed voltages the hydroxide film becomes thicker and the resistance increases up to a *critical voltage.* For pressures above the critical voltage the hydroxide film breaks down and the high resistance is then removed from the circuit. While the range between partial and complete breakdown of the hydroxide layer is small, the critical voltage is a fairly definite value, as shown by the sharp bend in the curve in Fig. 2.25. If the current passes through several trays in series, the total resistance and likewise the total breakdown voltage are increased in proportion to the number of trays. Arresters designed for service on high-voltage systems, therefore, have a sufficient number of trays arranged in series so

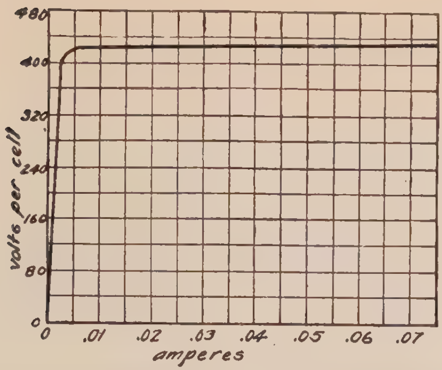


FIG. 2.25.—Volt-ampere characteristic curve of electrolytic arrester for direct currents.

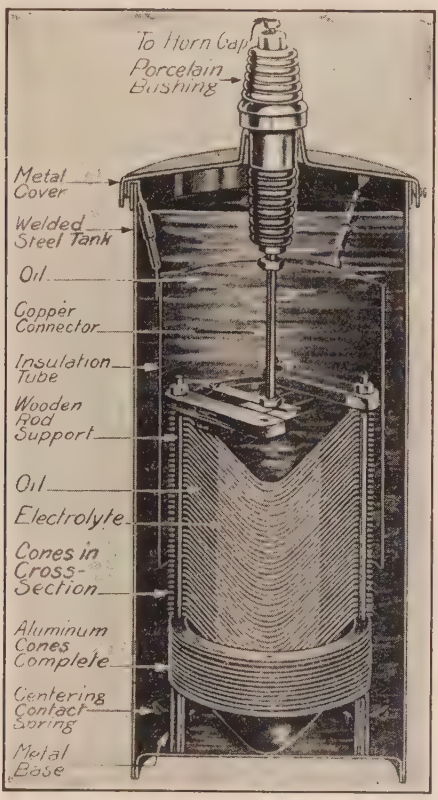


FIG. 3.25.—Cross-section of an aluminum arrester. (*General Electric Company.*)

that the voltage per tray is less than the critical voltage. For alternating currents it is customary to let the normal operating

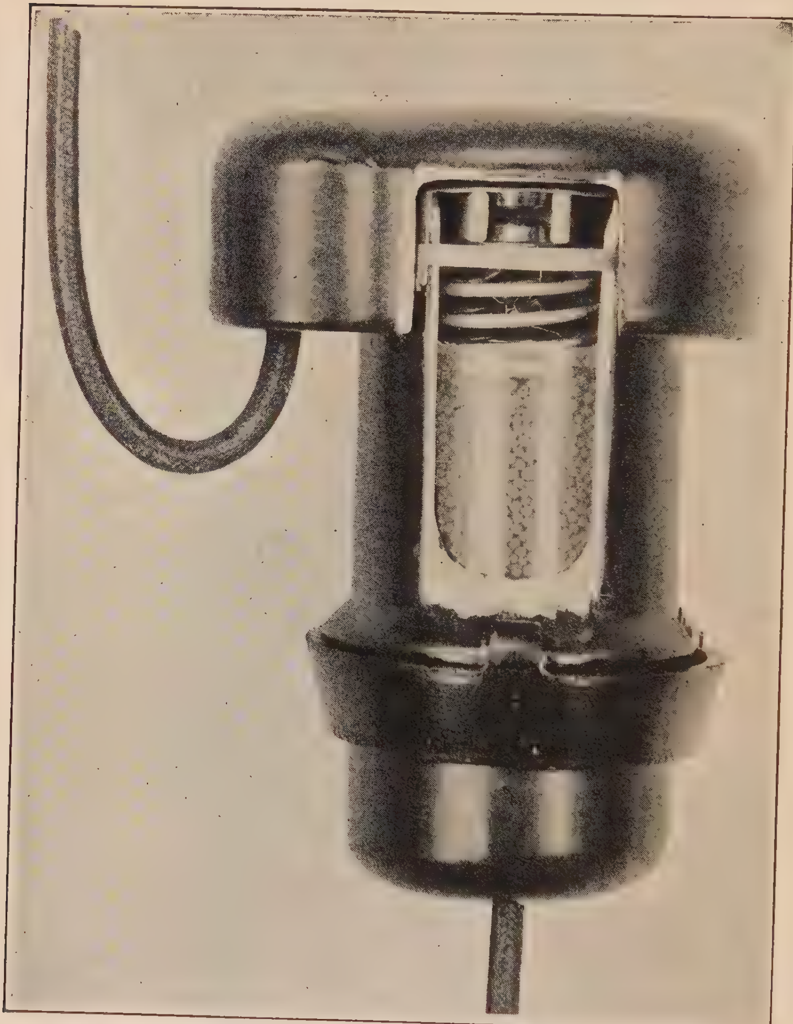


Fig. 4.25.—Sectional view of pellet lightning arrester rated at 1,000 to 3,000 volts. Type OF. (*General Electric Company.*)

voltage be 300 volts or less per tray. Hence, for a system having 44,000 volts to neutral the electrolytic arrester should have at least 150 trays.

The shape and arrangement of the aluminum trays, the space filled by the electrolyte, the oil insulation and terminal leads are shown in Fig. 3.25. To eliminate both the energy loss and undesirable rise in temperature a horn gap is installed between the electrolytic arrester and the line.

The formation of a uniform and continuous hydroxide film requires great care and, moreover, the hydroxide film, once formed, is slowly dissolved by the electrolyte. To keep the film in good condition it is therefore necessary to send at intervals a current through the arrester for a few seconds. This is called *charging the arrester*, and is accomplished by short-circuiting the horn gap through a resistance. For commercial systems it is customary to *charge the arresters* once every 24 hr.

Although the combination of horn gaps and electrolytic arresters protects the plant for practically all types of high-voltage and high-frequency disturbances coming from the transmission line, this type of arrester is being replaced by the more effective and more easily operated thyrite and autovalve arresters, Figs. 5.25, and 8.25.

6. *Pellet Arrester.*—In the pellet arrester, Fig. 4.25, the necessary valve action is provided by the thin lead-oxide film, coating the pellets.

The enclosed sphere gap, shown in the upper end of the arrester, Fig. 4.25, is connected in series with the pellets to ground. The sphere gap prevents leakage currents from flowing during normal operating conditions.

7. *Oxide-film Arrester.*—The arrester consists of a set of oxide-film cells connected in series in a manner similar to the aluminum disks in the electrolytic arrester. The cells are disk-shaped, about 7½ in. in diameter and ⅝ in. thick. Each cell is made of two circular sherardized steel plates crimped firmly to the edges of an annular piece of porcelain. A powder, lead peroxide, which has very low resistance, compactly fills the space between the metal plates. The inside of the metal plates is covered with a varnish film which is an insulator. The number of cells used in an arrester is such that the voltage per cell is approximately 300 volts.

8. *Autovalve Arrester.*—A sectional view of the autovalve arrester is shown in Fig. 5.25. The arrester, proper, consists of a series of low-resistance disks separated by mica spacers to form

air gaps of approximately 0.0003 in. length, each having a breakdown voltage at atmospheric pressure of about 350 volts.

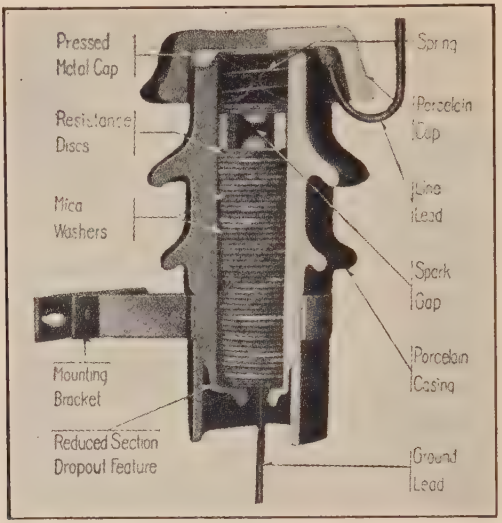


FIG. 5.25.—Sectional view of an autovalve distribution-type arrester. (*Westinghouse Electric and Manufacturing Company.*)

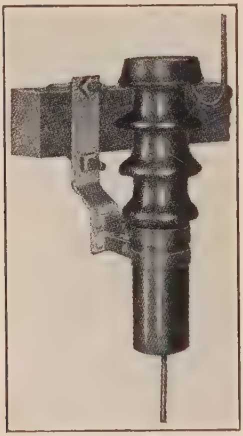


FIG. 6.25.—A 15,000-volt, LV distribution-type arrester. (*Westinghouse Electric and Manufacturing Company.*)

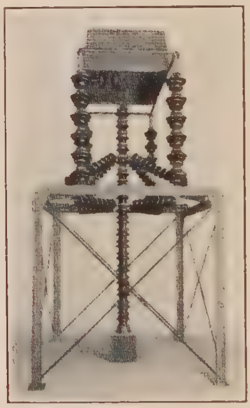


FIG. 7.25.—Phase leg of 220-kv., open-gap, station-type SV autovalve arrester. (*Westinghouse Electric and Manufacturing Company.*)

The spacing of the disks, that is the length of the air gap, determines whether the current-flow shall be in the form of an arc or of a glow. In the autovalve arrester the break-down voltage

of 350 volts is maintained by a glow discharge. The disks are made of a composition, mechanically similar to porcelain, having electrical conductivity which in combination with the air gap provides the valve action in the arrester.

The voltage rating of the autovalve arrester is proportional to the number of air gaps in series, each air gap representing 350 volts.

The LV distribution-type arrester, Fig. 6.25, consists of a column of disks, the number depending on the line voltage, with a sphere spark-gap in series; all enclosed in a porcelain casing. The LV-type arrester is used on distribution systems mainly for the protection of transformers.

The SV station-type arrester, Fig. 7.25, consists of several elements or units in series with a sphere spark-gap. Each element, or section, consists of four stacks of autovalve disks in parallel, enclosed or housed in a porcelain casing. The number of sections required depends on the line voltage. The 220-kv. arrester, shown in Fig. 7.25, has 18 sections. The SV type is applied at power plants or substations or large isolated transformers requiring the highest degree of protection.

9. *Thyrite Arrester.*—Thyrite, which forms the basic constituent of the thyrite arrester, is a dense, homogeneous, inorganic compound of ceramic nature, which can be molded into any desired shape. For lightning arresters, the thyrite is formed into disks, 6 in. in diameter and ¾ in. thick, with metal sprayed on both sides to provide uniform electrical contact.

The thyrite arrester is built in units, each of which consists of 11 disks in series with a sphere-gap assembly; all housed in a glazed porcelain container, as illustrated in Fig. 8.25. The gap assembly is placed at the bottom of the arrester unit, with cushion contacts at both ends. The thyrite disks have no appreciable time lag. The gap assembly likewise has a negligible time lag and consists of 12 small gaps in combination with small condensers and high resistances.

The volt-ampere characteristic of the arrester unit is governed by the properties of the thyrite material of which the arrester disks are composed. This relation is expressed by the equation, $R(\text{ohms}) = 580I^{-0.72}$ (amp.), for each thyrite disk. When plotted on log-log paper the volt-ampere characteristic becomes a straight line, as shown, for one 11.5-kv., thyrite unit, in Fig. 9.25. For two arrester units in series the volt-ampere characteristic

would be represented by a parallel straight line with double voltage values. The volt-ampere characteristic for arresters of

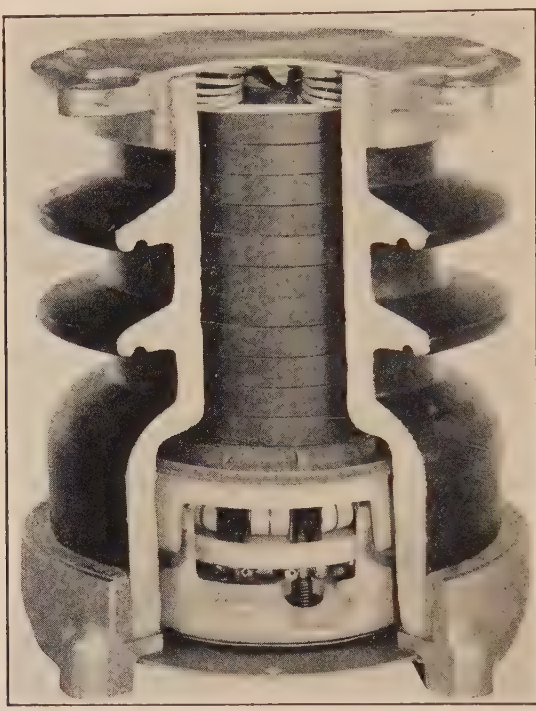


Fig. 8.25.—Sectional view of thyrite arrester unit; 11.5 kv. (*General Electric Company.*)

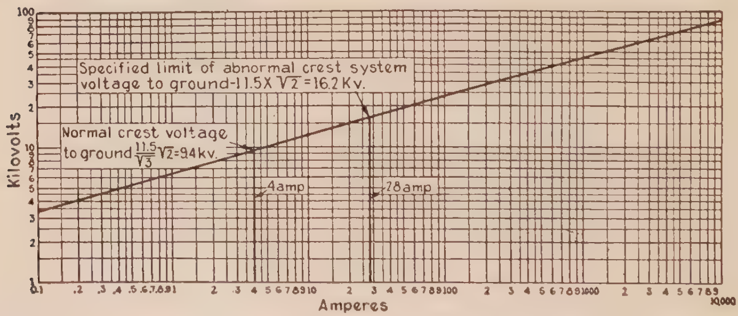


Fig. 9.25.—Volt-ampere characteristic of 11.5-kv., thyrite arrester unit. (*General Electric Company.*)

any number of units can therefore readily be determined from the single-unit curve in Fig. 9.25.

For three-phase systems a separate, single-pole, thyrite arrester is used for each phase, as illustrated in Fig. 10.25.

(*d*) **Relays.**—Aside from protection to life, continuity of service is of paramount importance in the operation of all electric systems. The larger the system the more important is the necessity for

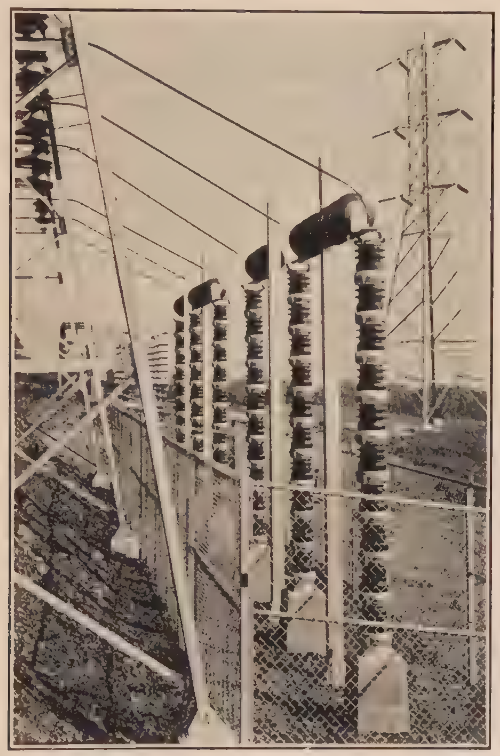


Fig. 10.25.—Thyrite lightning arresters on three-phase, 115-kv., grounded neutral system (*General Electric Company.*)

continuous service. Likewise, the larger the system the more frequent will be the occurrence of short-circuits that may cause shutdown. The enormous increase in the size of central stations during the past few years has, in a measure, been due to the successful operation of selective apparatus that automatically cuts out any part of the system in which a short-circuit develops, thus permitting the greater portion of the plant to operate without interruption. This automatic, selective action is obtained

by means of some form of relay, which may be defined as an auxiliary device, that operates and transmits its action to another independent piece of apparatus on the occurrence of some predetermined condition in the electric circuit. Mechanically a relay consists: first, of a coil, or coils, connected to the circuit it controls; second, a movable part whose motion is determined by the current in the coils; and, third, a contact device operated by the movable part, controlling the circuit that operates a switch or circuit-breaker. The relay must operate only for those disturbances for which it has been designed and only when the disturbances are in the part of the system under its control. Relays of great variety are in commercial use, and may be classified in several ways. The nature of the disturbance is a fundamental factor and may be used as a basis for classification; also, whether the switch or circuit-breaker controlled by the relay opens or closes the circuit.

In power systems three basic factors, the current, the voltage and the power must be kept within predetermined limits. Relays to operate circuit-breakers or other protective and regulating devices in power systems may be classified on the basis of the factor controlled.

1. Current Relays:
   a. Overcurrent.
   b. Undercurrent.
   c. Reverse current.
2. Voltage Relays:
   a. Overvoltage.
   b. Undervoltage.
   c. Phase reversal.
3. Power Relays:
   a. Overpower.
   b. Underpower.
   c. Reverse power.

Many other factors, as temperature, speed, starting and stopping operations, are likewise controlled by relays designed to meet specific requirements in each case. In the operation of relays the time element involved is in many types of service of prime importance. Relays may be required to operate instantaneously, or in other cases a definite time interval must elapse from the instant the relay starts until the circuit-breaker opens. On the time-element basis, relays may be classified into three

groups whether operating on the current, the voltage or the power in the circuit.

1. Instantaneous relays.
2. Definite-time relays.
3. Inverse time-limit relays.

The nature of the disturbance and the general plan of the system determine which type should be installed at various places in the system.

1. *Instantaneous Relays.*—In principle this is the simplest form and usually consists simply of a solenoid with a plunger acting on a trip mechanism. For currents in excess of a predetermined maximum or below a fixed minimum the relay operates as nearly instantaneously as the inertia of the moving parts permits.

2. *Definite-time Relays.*—In most cases the disturbances do not cause a complete breakdown, and the excessive flow of current is of short duration, causing no damage to the apparatus. The continuity of service may be improved by using a relay so constructed that a definite time must elapse from the occurrence of the disturbance to the operation of the contact devices. While a number of devices have been used to give the desired time lag, only three types are in general use:

a. Clock mechanism.
b. Leather bellows.
c. Magnetic induction.

If two types of apparatus for the same service give satisfactory results, the higher priced machine will be forced from the market. On account of the relatively high cost of the clock type of relay its use is limited to special cases, since satisfactory services may be secured in most cases from the bellows and induction types. The definite-time limit is secured, in the second type, by means of leather bellows filled with air, under normal operating conditions. When the increased current operates the relay, a spring is released, which then presses against the bellows, forcing the air out through a needle valve. The size of the opening in the needle valve and the pressure exerted by the spring may be adjusted so that it takes a predetermined length of time to empty the bellows. The third type is built on the same principle as an induction wattmeter. The rotating field produces the torque, and the *drag* is obtained by means of an aluminum disk moving between the poles of permanent magnets. A small transformer

(compensator) is inserted between the series transformer and the relay. The core of this transformer is of such cross-section that the saturation point is reached at full-load current. Hence, for overloads the current in the relay does not increase so fast as in the mains and the torque may be adjusted so that a definite time elapses before the relay operates.

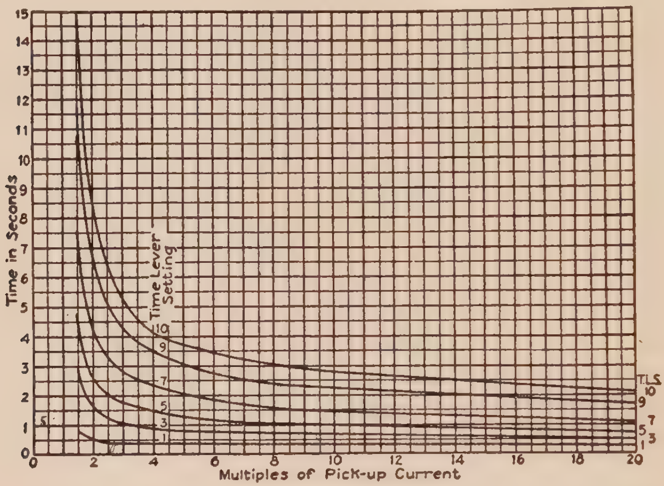


FIG. 11.25.—Time-induction overcurrent relay; typical time-current curves at various lever settings. (*General Electric Company.*)

3. *Inverse Time-limit Relays.*—In the inverse time-limit relay of the bellows type, the core acts directly on the bellows; thus a pressure, proportional to the overload, forces the air through the needle valve. The time for compressing the bellows is, therefore,

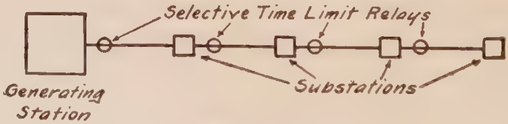


FIG. 12.25.

inversely proportional to the overload. Characteristic curves for induction overload relay are shown in Fig. 11.25

The location of the relays in the system depends on many factors. In general, inverse time-limit relays are placed nearest the generators, while definite time-limit relays control the substation feeders.

The time setting depends on the characteristics of the generators and the position in the system. Among the factors that must be known, the more important are:

a. The instantaneous short-circuit current.

b. The sustained short-circuit current.

c. The safe circuit-opening capacity of the circuit-breakers.

d. The time characteristics of the relays and the circuit-breakers.

e. The position in the system.

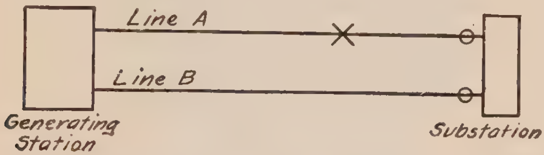


FIG. 13.25.

If several relays are in series on a system, as in Fig. 12.25, the one farthest from the generator must operate in the shortest time, with each relay in the series toward the generator requiring longer time than the one preceding it.

The characteristics of both the definite-time and the inverse time-limit relays are sometimes combined in one apparatus for use in special cases.

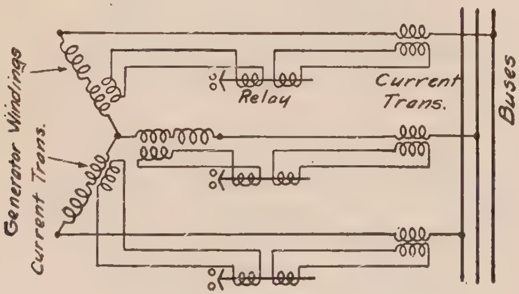


FIG. 14.25.

4. *Reverse Power Relays.*—If power is transmitted from a generating station to a substation over two lines connected to the same busbars, Fig. 13.25, a short-circuit, as at X on the line A, will cause power to flow not only from the generator on line A, but also on line B through the substation and back on line A to X. The excessive current should operate the relays and open the circuit of the line A at the generator. In order to continue the

service, it is also necessary that the line A be disconnected at the substation. This is accomplished by means of reverse-current relays. One form in common use is based on the dynamometer principle. The stationary coils are connected to series

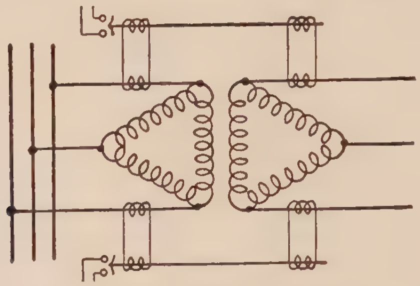


FIG. 15.25.

current transformers and the movable element to potential transformers connected to the busses. While the energy flows in the desired direction the contact lever operated by the shaft carrying the movable coil is held against a dead stop by a spring. When the direction of energy flow reverses, the movable coil tends to turn in the opposite direction and throws the contact lever against a contact completing the tripping circuit for the oil switch. It is evident that this apparatus is not affected by overloads and that it operates only on the reversal of the direction of the energy flow.

For the protection of generators or transformers against short-circuits inside the machine or for a short between the machine and the busbars a reverse-current relay, based on the differential principle, is used. The connections are shown in Figs. 14.25 and 15.25. Series transformers are connected in opposition as shown in the diagrams. If a short-circuit occurs between the two series transformers connected to the relay, the flow of current in one is reversed and thus the sum of the two currents produces a heavy m.m.f. that operates the relay. Various connections are used for special cases to give the desired action, but fundamentally the reverse-power relays are based on either the dynamometric or the differential principles.

(*e*) **Power-limiting Reactances.**—In large central stations apparatus is installed that will automatically limit the amount of power that can be developed under short-circuit conditions in order to save the machines from self-destruction.

As explained in Chap. XIV and illustrated by Fig. 27.14, the instantaneous short-circuit current is several times the value of the sustained short-circuit. Since the magnetic forces vary as the square of the current, the mechanical forces due to the instantaneous short-circuit current may tear the alternator or the transformer windings apart and wreck the station.

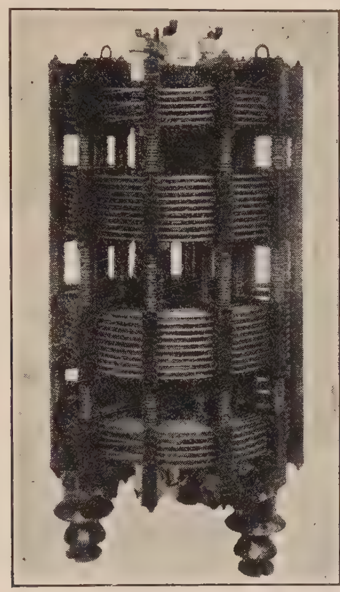


Fig. 16.25.—119-kv.a., three-phase, 160-cycle, current-limiting reactor. 220 amp., 3 per cent and 11,400-volt lines. (*Westinghouse Electric and Manufacturing Company.*)

Another factor that must be considered is the maximum current for which the circuit-breaking switches can safely operate. Under excessive currents the switch would be destroyed. A third factor is the necessity for limiting the amount of energy that can be transmitted from the station to the short-circuit. In stations of 100,000-kw. capacity or more the amount of power that could be concentrated in a short-circuit is enormous, probably several million kilowatts.

To protect the alternators, transformers and switches against excessive short-circuit currents and to limit the power that can be delivered by the station, power-limiting reactances are placed in the generator leads.

Typical sets for three-phase generators are shown in Figs. 16.25 and 17.25.

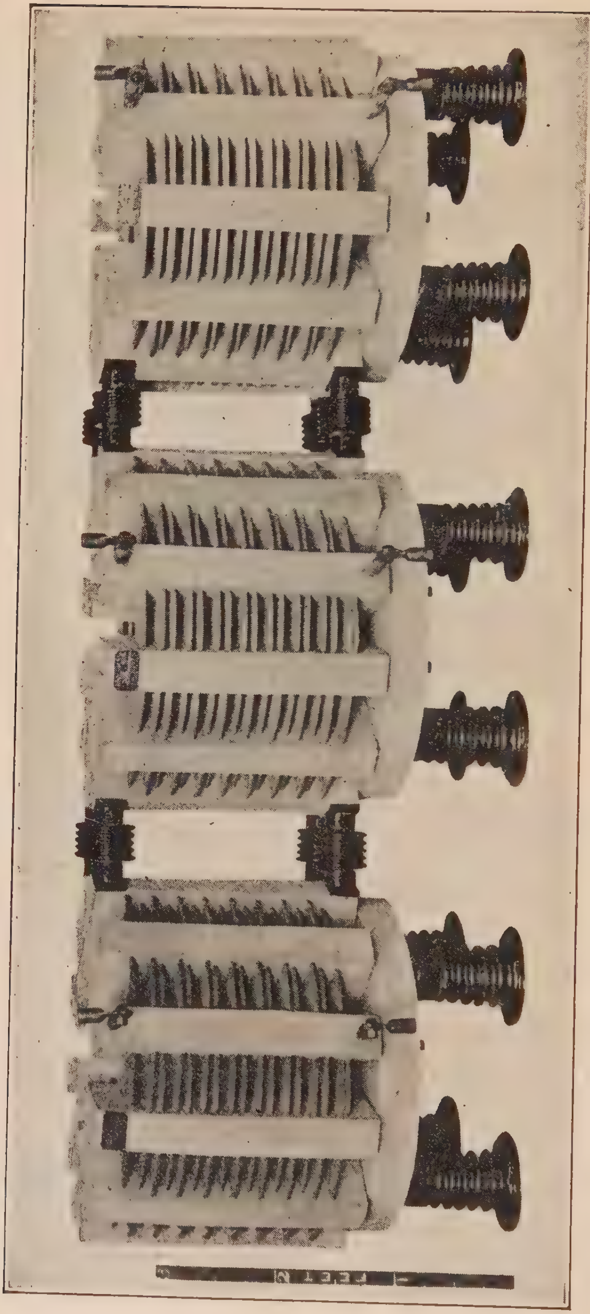


FIG. 17.25.—Type CLS-25-125-228 volt indoor reactor. (*General Electric Company.*)

The core is of concrete, and the winding consists of stranded copper cable held in place by wooden supports bolted to the core.

In addition to the protection given to the station machinery, power-limiting reactances are also used to enable the large central station to give continuity of service. The busbars in the station are sectionalized and reactances are placed between the several sections, as illustrated for a typical case in Fig. 18.25. These reactances limit the power that flows between the sections so that, if a short-circuit occurs on one section, only the circuit-breakers for that section will operate. The generators in the

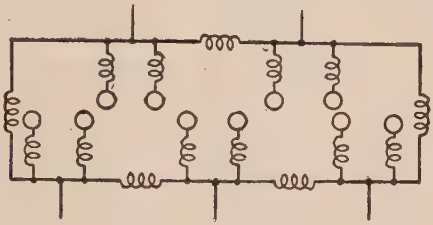


FIG. 18.25.

other sections carry an overload until the operator isolates the section on which the short-circuit occurred. With 25 per cent reactance between the sections, neither the drop in voltage nor the phase displacement is large enough to cause synchronous motors or converters to drop their load. Similar reactances are sometimes placed in the feeders and in the tie lines between the stations.

(*f*) **Light-sensitive Cells.**—In a rapidly increasing number of cases protective devices are controlled or actuated by some form of light-sensitive cells. The electrical properties of several materials, as selenium, are markedly affected by exposure to light; and certain substances, as the alkali metals, have the property of directly converting light into electric energy. The devices containing the active light-sensitive material, arranged to be connected into an electric circuit, are usually called *cells;* or more specifically *light-sensitive cells, photo-electric cells, photo-electronic cells,* and *photo-voltaic cells.* Under the general term of light-sensitive cells there are three groups or types based on three different physical processes.

1. Photo-electric or photo-electronic cells.
2. Photo-electric or photo-voltaic cells.
3. Light-sensitive or photo-conductive cells.

In the *photo-electric* or *photo-electronic cells* the active materials are the alkali metals, particularly caesium, potassium, sodium and rubidium, or some of their compounds. A cross-section of a typical photo-electric cell is shown in Fig. 19.25. On the inside surface of a glass bulb a layer of silver is deposited, covering the greater part of the surface but leaving a clear section or window through which the light beam may reach the sensitive material. The inside silver surface is then coated with a very thin layer of caesium or potassium anhydride, or some other light-sensitive substance. The sensitive film is extremely thin, in some cases approaching to a single layer of atoms over the silver surface. An electrode connected to the middle part of the silver coating forms the cathode of the cell. A centrally located filament, Fig. 19.25, insulated from the silver coating, forms the anode. Under light exposure the sensitive film covering the silver coating liberates or emits electrons thereby generating a potential difference between the cathode and the anode. The current produced by the ordinary photo-electric cell is small, only a few micro-amperes, but the operating characteristics, as regards sensitivity, speed of response, reproducibility and direct proportionality of the current to the light intensity, are highly desirable for many forms of control and protective appliances.

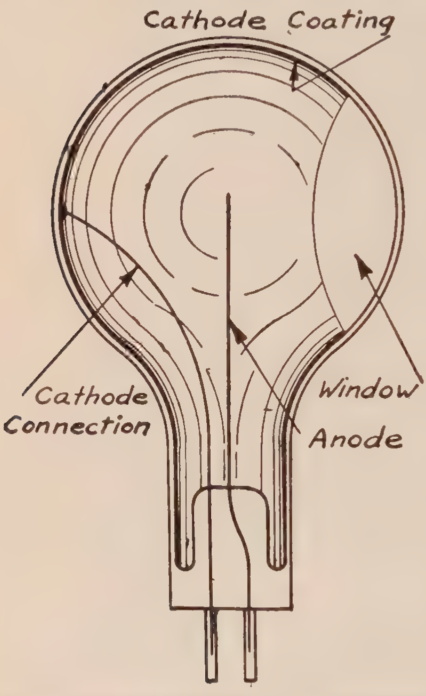


FIG. 19.25.—Cross-section of a photo-electric cell.

The *photo-voltaic cells* have the appearance of an electric battery, in that the two electrodes are immersed in an electrolyte. In one form, cuprous oxide and lead form the electrodes, and a solution of lead nitrate the electrolyte. On exposing the cuprous oxide to light, voltage is generated in the cell. Larger currents

are produced than by the photo-electric cell; the output being approximately 4 milliamperes per sq. in. of electrode surface.

In the *third type* of light-sensitive cells, exposure to light greatly varies the resistance of the active element. Selenium is the material generally used in *photo-conductive cells.* The change in the resistance of the cell in the dark or exposed to light may be more than 10:1. Sufficient current to operate relays may be passed through selenium cells.

The applications of light-sensitive cells are very numerous and may be classified into several groups, as control of processes, safety devices, sorting of materials on a color basis, evaluation and quantitative measurements of light intensities and indicators of sequence of events or change in conditions.

PROBLEMS

1.25. True reproduction of current on the secondary of a current transformer is desirable for both value and phase relation. In any commercial design the ratio and the phase angle, or phase displacement, are affected by a change in secondary loading with fixed primary current, and a change in primary current with fixed secondary loading.

(*a*) What effect have the following losses on these factors:

(1) Primary copper loss?
(2) Secondary copper loss?
(3) Iron loss?

(*b*) With a fixed secondary loading and a fixed primary current what is the effect on ratio and phase angle of:

(1) Change in power factor of secondary load?
(2) Change in power factor of system load?

2.25. Transformer banks are frequently protected against internal trouble by means of differential protection, which consists of balancing the currents on the two sides of the bank through current transformers and an overload relay, so that, in event of unbalance of current produced by an internal short, the relay will be operated, but not in the case of trouble outside the bank. A transformer bank is connected star-delta with the current transformers on the star side connected in delta, and on the delta side connected in star. Trace the currents through the windings and determine which leads should be connected together in order to get the proper action.

3.25. (*a*) Why are current-limiting reactors made without an iron core? (*b*) Why is the cylindrical form the best for use in reactors?

CHAPTER XXVI

POLYPHASE SYSTEMS

Any polyphase system may be analyzed by separating the network into its elemental branches or phases. The voltage generated in any one phase causes currents to flow in the network that may be calculated independently of the voltages in the other phases. The current flowing at any point and at any instant in an interconnected system is the vector sum or the resultant of the currents produced by all the voltages in the several phases. Ohm's and Kirchhoff's laws apply to the complicated network in the same manner as for simple series or parallel circuits. In order to emphasize certain characteristics of commercial systems under operating conditions, it is customary to make the following classifications:

(*a*) Symmetrical and unsymmetrical systems.
(*b*) Independent and interlinked systems.
(*c*) Balanced and unbalanced systems.
(*d*) Symmetrical components of unbalanced systems.
(*e*) Systems having grounded or floating neutral.

(*a*) **Symmetrical and Unsymmetrical Systems.**—If a polyphase system is composed of n phases having voltages of equal magnitude and differing in time phase successively by $\frac{1}{n}$th of a cycle, the system is symmetrical; but if either the magnitude or the time-phase displacement of the voltages or both differ, the system is unsymmetrical.

For symmetrical systems:

$$e_1 = {}^M E \sin(\omega t)$$

$$e_2 = {}^M E \sin\left(\omega t - \frac{2\pi}{n}\right)$$

$$e_3 = {}^M E \sin\left(\omega t - 2\frac{2\pi}{n}\right)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$e_n = {}^M E \sin\left[\omega t - (n-1)\frac{2\pi}{n}\right] \qquad (1.26)$$

For $n = 1$, the single-phase, two-wire system:

$$e_1 = {}^{M}E \sin (\omega t) \tag{2.26}$$

For $n = 2$, the single-phase, three-wire (Edison) system:

$$\begin{aligned} e_1 &= {}^{M}E \sin (\omega t) \\ e_2 &= {}^{M}E \sin (\omega t - 180°) \end{aligned} \tag{3.26}$$

For $n = 3$, the three-phase, three-wire system:

$$\begin{aligned} e_1 &= {}^{M}E \sin (\omega t) \\ e_2 &= {}^{M}E \sin (\omega t - 120°) \\ e_3 &= {}^{M}E \sin (\omega t - 240°) \end{aligned} \tag{4.26}$$

For $n = 4$, the four-phase, or quarter-phase, four-wire system:

$$\begin{aligned} e_1 &= {}^{M}E \sin (\omega t) \\ e_2 &= {}^{M}E \sin (\omega t - 90°) \\ e_3 &= {}^{M}E \sin (\omega t - 180°) \\ e_4 &= {}^{M}E \sin (\omega t - 270°) \end{aligned} \tag{5.26}$$

Similarly, for $n = 6$ and $n = 12$, we have the six-phase and the twelve-phase systems.

Unsymmetrical Systems.—The two-phase system and the three-phase open-delta or V connection are the more common unsymmetrical systems.

(*b*) **Independent and Interlinked Systems.**—Each phase of an n-phase system may be arranged so as to form a closed circuit, the whole system, therefore, consisting of n independent single-phase systems. The two phases of a four-wire, two-phase system are often operated as two independent single-phase circuits. In most cases the several phases of commercial polyphase systems are interconnected, forming networks or interlinked systems through which the currents flow. In practice the phases are either star- or ring-connected, or the network consists of a combination of star and ring connections.

In a star-connected system the starting points of all the phases are connected together, forming the so-called *neutral point.* This common point is connected either to earth or to another neutral point in the same system. It is customary to consider

the neutral point as having zero potential. The voltage e_n from the neutral to the line wire is the phase or circuit voltage. For any phase q, of an n-phase system,

$$e_n = {}^M E_n \sin\left[\omega t - (q-1)\frac{2\pi}{n}\right] \tag{6.26}$$

Between the terminals of any two adjacent phases is the line voltage e_m, or voltage between the mains. For any two adjacent phases, as q and $q+1$,

$$\begin{aligned} e_m &= {}^M E_n \sin\left[\omega t - (q-1)\frac{2\pi}{n}\right] - {}^M E_n \sin\left(\omega t - q\frac{2\pi}{n}\right) \\ &= 2\,{}^M E_n \sin\frac{\pi}{n}\left\{\cos\left[\omega t - (2q-1)\frac{\pi}{n}\right]\right\} \end{aligned} \tag{7.26}$$

$$E_m = 2E_n \sin\frac{\pi}{n} \tag{8.26}$$

In any star-connected system the voltage between the mains is the vector difference of the voltages between the two adjacent circuits, and the current is the same as for the corresponding phase circuit.

In a ring-connected system the starting point of one phase is connected to the ending point of the next preceding phase. The sum of the voltages generated at any instant in the several phases connected in series should be zero, otherwise the resultant voltage causes currents to circulate in the ring connection. In symmetrical polyphase systems having simple harmonic voltage waves the sum of the voltages for all the phases is at all instants equal to zero and therefore either the ring or the star connections may be used. The voltage between successive mains is the same as the voltage in the corresponding circuit. The current in the mains is the vector difference of the currents in the adjoining phases. For the main between the two adjacent phases q and $q+1$ the current is

$$\begin{aligned} i_m &= {}^M I \sin\left[\omega t - (q-1)\frac{2\pi}{n}\right] - {}^M I \sin\left(\omega t - q\frac{2\pi}{n}\right) \\ &= 2\,{}^M I \sin\frac{\pi}{n}\cos\left[\omega t - (2q-1)\frac{\pi}{n}\right] \end{aligned} \tag{9.26}$$

$$I_m = 2I \sin\frac{\pi}{n} \tag{10.26}$$

The interlinked systems of commercial importance are described in Chaps. XII and XVI.

(c) **Balanced and Unbalanced Systems.**—Any polyphase system that develops a constant flow of energy is balanced, and any system in which the power varies or pulsates during each voltage or current cycle is unbalanced.

In single-phase circuits,

$$e = {}^{M}E \sin(\omega t) \tag{11.26}$$
$$i = {}^{M}I \sin(\omega t - \theta) \tag{12.26}$$
$$p = EI[\cos\theta - \cos(2\omega t - \theta)] \tag{13.26}$$

The power pulsates with double the frequency of the voltage or current, and the system is unbalanced.

In two-phase circuits having the voltages, currents and power factors of the two circuits alike, the power

$$p = p_1 + p_2 = 2EI\cos\theta, \text{ a constant} \tag{14.26}$$

Under these conditions the two-phase system is balanced although unsymmetrical. However, if the voltage, current or power factor of phase A is not equal to the corresponding factor in phase B, the flow of energy pulsates and the system is unbalanced.

In the three-phase system, for either delta or star connections, with the currents, voltages and power factors in the three phases of like value the power is constant and the systems are balanced.

$$p = p_1 + p_2 + p_3 = \sqrt{3}EI\cos\theta \tag{15.26}$$

However, if the voltages, the currents or the power factors of the three phases differ, the power pulsates and the system is unbalanced; similarly for six-phase and twelve-phase or any symmetrical n-phase system. If the voltages, currents and power factors are alike for the several phases, the power is constant throughout the complete voltage cycle and the systems are balanced. For satisfactory operation the balanced condition is the most desirable. In commercial systems this is an ideal which is seldom attained.

The chief disadvantages resulting from unbalanced load conditions are: (1) poor voltage regulation, (2) unequal voltage stresses in the several phases and (3) unequal heating in the circuits. The more common cause for unbalanced-load conditions is the operation of essentially single-phase load, as incandescent lamps or single-phase motors, on polyphase systems.

With sufficient data given, the values of the currents and voltages in the several phases and their time-phase relations may be found by the method of *symmetrical components*, as explained in the next section, or by direct application of Ohm's and Kirchhoff's laws. This may be illustrated by the solution of a few problems.

Problem 1.26. Given a three-phase system, star-connected, floating neutral and having a non-reactive unbalanced load. Circuit diagram as in Fig. 1.26. $E_{A-D} = E_{B-A} = E_{D-B} = 220$ volts; $r_A = 3$; $r_B = 4$; $r_D = 5$ ohms.

Find (*a*) E_A, E_B, E_D.

(*b*) I_A, I_B, I_D.

(*c*) $\theta_{A,A}$; $\theta_{B,B}$; $\theta_{D,D}$; time-phase angles between E_A and I_A, etc.

(*d*) $\theta_{A-D,A}$; $\theta_{B-A,B}$; $\theta_{D-B,D}$, time-phase angles between E_{A-D} and I_A, etc.

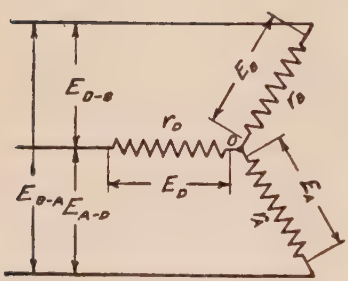


FIG. 1.26.

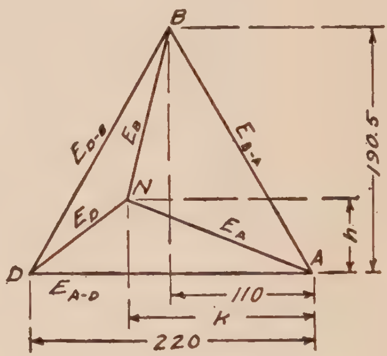


FIG. 2.26.

(*e*) The total kilowatts.

(*f*) Draw to scale the vector diagram for the currents and voltages.

Let the voltages between the mains be represented by the sides of an equilateral triangle, Fig. 2.26. Since the common point O is not grounded and the resistances in the three circuits are unequal, the voltages from the mains to the common neutral point must be unequal. The vector difference of the voltages in any two circuits must be equal to the voltage between the corresponding pair of mains. Hence, by assuming some point N in the diagram as the neutral position and drawing the vectors NA, NB and ND, we have a graphic representation of the

voltages in the circuits and in the mains. The actual position of the point N may be found by the application of Kirchhoff's laws. Through N draw rectangular coördinates with the X-axis parallel to the line DA. Denote the coördinates of the point A by k and h. The following equations may then be derived directly from the diagram:

$$E_A = k - jh \tag{16.26}$$

$$E_B = (k - 110) + j(190.5 - h) \tag{17.26}$$

$$E_D = -(220 - k) - jh \tag{18.26}$$

$$I_A = \frac{k - jh}{3} \tag{19.26}$$

$$I_B = \frac{(k - 110) + j(190.5 - h)}{4} \tag{20.26}$$

$$I_D = \frac{(k - 220) - jh}{5} \tag{21.26}$$

From Kirchhoff's laws the real and quadrature components may be separately equated to zero.

$$\frac{k}{3} + \frac{k - 110}{4} + \frac{k - 220}{5} = 0 \tag{22.26}$$

$$\frac{-h}{3} + \frac{190.5 - h}{4} + \frac{-h}{5} = 0 \tag{23.26}$$

Hence $k = 91.3$ and $h = 60.8$.

(*a*) $E_A = 110.0$ volts.
$E_B = 131.0$ volts.
$E_D = 142.2$ volts.
(*b*) $I_A = 36.7$ amp.
$I_B = 32.8$ amp.
$I_D = 28.5$ amp.
(*c*) $\theta_A = \theta_B = \theta_D = 0$, or the power factor is unity.

(*d*) The time-phase angles between the currents and the corresponding voltages between the mains may be found directly from the above equations.

The time-phase angle of the current in line A and the voltage between the mains A and D,

$$\theta_{A-D,A} = \tan^{-1}\frac{h}{k} = 33° \, 40' \tag{24.26}$$

The time-phase angle of the current in line B and the voltage between the mains B and A,

$$\theta_{B-A,B} = 30° + \tan^{-1}\frac{-110 + k}{190.5 - h} = 21° 45' \qquad (25.26)$$

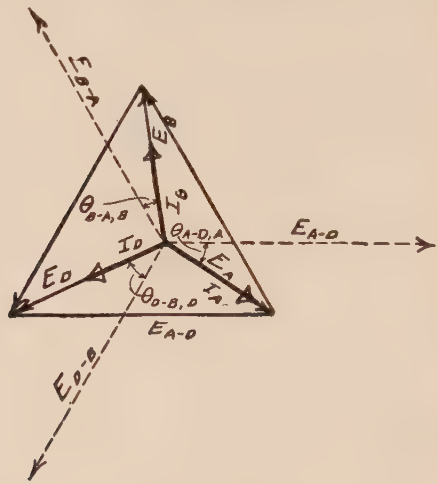


FIG. 3.26.

The time-phase angle of the current in line D and the voltage between the mains D and B,

$$\theta_{D-B,D} = 60° - \tan^{-1}\frac{-h}{-220 + k} = 34° 40' \qquad (26.26)$$

(e) The total power $= E_AI_A + E_BI_B + E_DI_D = 12.35$ kw.

(f) The vector diagram drawn to scale is shown in Fig. 3.26.

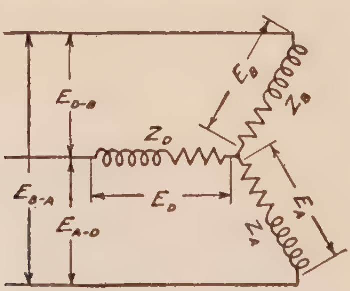


FIG. 4.26.

Problem 2.26.—Given a three-phase system having an inductive load, star-connected and with floating neutral, Fig. 4.26.

$E_{A-D} = E_{B-A} = E_{D-B} = 440$ volts; $z_A = 2 + j4$; $z_B = 3 + j2$; $z_D = 1 + j3$; $f = 60$ cycles.

Find (*a*) E_A, E_B, E_D.
(*b*) I_A, I_B, I_D.
(*c*) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.
(*d*) $\theta_{A-D,A}$, $\theta_{B-A,B}$, $\theta_{D-B,D}$.
(*e*) The total kilowatts.
(*f*) The total reactive power.
(*g*) Draw to scale the voltage and current vector diagram.

Following the same method as explained in problem 1.26, draw the equilateral triangle ABD, similar to Fig. 2.26; assume a neutral point N, as before, for the origin and let k and h be the coördinates of the point A. The complex equations are:

$$E_A = k - jh \qquad (27.26)$$
$$E_B = (k - 220) + j(381 - h) \qquad (28.26)$$
$$E_D = (k - 440) - jh \qquad (29.26)$$
$$I_A = \frac{k - jh}{2 + j4} \qquad (30.26)$$
$$I_B = \frac{(k - 220) + j(381 - h)}{3 + j2} \qquad (31.26)$$
$$I_D = \frac{(k - 440) - jh}{1 + j3} \qquad (32.26)$$

Equating to zero the real and quadrature components of the currents passing through the common point, we have:

$$k - 1.52h - 83.7 = 0 \qquad (33.26)$$
$$jh + j1.52k - j590 = 0 \qquad (34.26)$$

Hence $k = 296$ and $h = 140$.

(*a*) $E_A = 328$ volts.
$E_B = 253$ volts.
$E_D = 201$ volts (35.26)

(*b*) $I_A = 73.3$ amp.
$I_B = 70.3$ amp.
$I_D = 63.3$ amp. (36.26)

(*c*) $$\theta_{A,A} = \tan^{-1}\frac{4}{2} = 63° \, 25' \qquad (37.26)$$

$$\theta_{B,B} = \tan^{-1}\frac{2}{3} = 33° \, 40' \qquad (38.26)$$

$$\theta_{D,D} = \tan^{-1}\frac{3}{1} = 71° \ 35' \quad (39.26)$$

$$(d) \quad \theta_{A-D,A} = 63° \ 25' + \tan^{-1}\frac{h}{k} = 88° \ 45' \quad (40.26)$$

$$\theta_{B-A,B} = 33° \ 40' + \tan^{-1}\frac{k - 220}{381 - h} + 30° = 81° \ 10' \quad (41.26)$$

$$\theta_{D-B,D} = 71° \ 35' + 60° - \tan^{-1}\frac{h}{440 - k} = 87° \ 25' \quad (42.26)$$

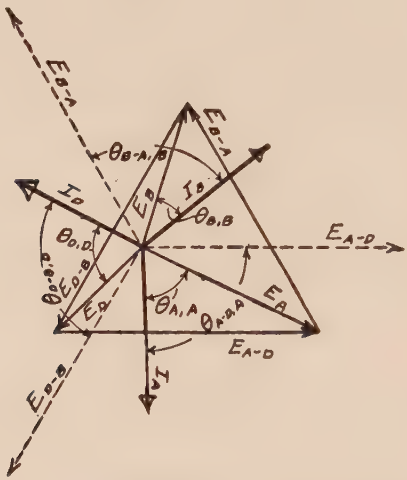


FIG. 5.26.

(e) The total power $= E_A I_A \cos \theta_{A,A} + E_B I_B \cos \theta_{B,B}$
$+ E_D I_D \cos \theta_{D,D} = 29.9$kw. (43.26)

(f) The total reactive power $= E_A I_A \sin \theta_{A,A}$
$+ E_B I_B \sin \theta_{B,B} + E_D I_D \sin \theta_{D,D} = 44.0$ kv.a (44.26)

(g) The vector diagram is drawn to scale in Fig. 5.26.

Problem 3.26.—Given the same data as in problem 2.26, except that the neutral is grounded with the voltage in each circuit to ground $= \frac{440}{\sqrt{3}}$ volts.

In the vector diagram the point N is therefore fixed at the center of the equilateral triangle.

Find (a) I_A, I_B, I_D.

(b) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.

(c) $\theta_{A-D,A}$, $\theta_{B-A,B}$, $\theta_{D-B,D}$.

(d) The total power, in kw.

(e) The total reactive power.

(*f*) Draw to scale the vector diagram for the currents and voltages.

The voltage in each circuit is known, hence:

$$(a)\qquad I_A = \frac{254}{2 + j4} = 56.8 \text{ amp.}$$

$$I_B = \frac{254}{3 + j2} = 70.5 \text{ amp.}$$

$$I_D = \frac{254}{1 + j3} = 80.4 \text{ amp.} \qquad (45.26)$$

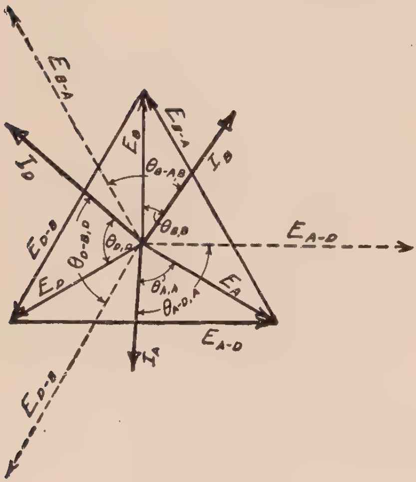


Fig. 6.26.

(*b*) $\theta_{A,A} = 63° 25'$.
$\theta_{B,B} = 33° 40'$.
$\theta_{D,D} = 71° 35'$ (46.26)

(*c*) $\theta_{A-D,A} = 93° 25'$.
$\theta_{B-A,B} = 63° 40'$.
$\theta_{D-B,D} = 101° 35'$ (47.26)

(*d*) Total power = 27.7 kw.

(*e*) The reactive power = 42.2 kv.a.

(*f*) The vector diagram in Fig. 6.26.

Problem 4.26.—Given a three-phase circuit, star-connected, floating neutral, similar to Fig. 4.26.

$E_{A-D} = 120$ volts; $E_{B-A} = 115$ volts; $E_{D-B} = 118$ volts; $z_A = 2 + j4$; $z_B = 3$; $z_D = 2 + j3$; $f = 60$ cycles.

Find (a) E_A, E_B, E_D.
(b) I_A, I_B, I_D.
(c) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.
(d) $\theta_{A-B,A}$, $\theta_{B-A,B}$, $\theta_{D-B,D}$.
(e) The total kilowatts.
(f) The total reactive power.
(g) Draw to scale the vector diagram for the voltages and currents.

From Fig. 7.26 the complex equations are:

$$E_A = k - jh \tag{48.26}$$
$$E_B = -(57.5 - k) + j(99.8 - h) \tag{49.26}$$
$$E_D = -(120 - k) - jh \tag{50.26}$$

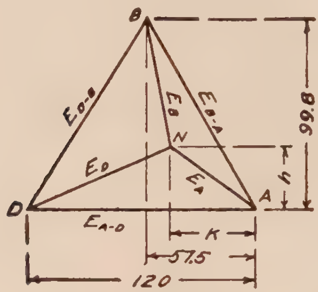


FIG. 7.26.

$$I_A = \frac{k - jh}{2 + j4} \tag{51.26}$$
$$I_B = \frac{-(57.5 - k) + j(99.8 - h)}{3} \tag{52.26}$$
$$I_D = \frac{-(120 - k) - jh}{2 + j3} \tag{53.26}$$

Equating real and quadrature components to zero, we have:

$$k - 0.733h - 64.5 = 0 \tag{54.26}$$
$$jk + j1.36h - j141.4 = 0 \tag{55.26}$$

Hence $k = 91.3$; $h = 37.0$.

(a) $E_A = 98.6$ volts.
$E_B = 71.5$ volts.
$E_D = 46.8$ volts. (56.26)

(b) $I_A = 22.05$ amp.
$I_B = 23.8$ amp.
$I_D = 13.0$ amp. (57.26)

(c) $\theta_{A,A} = 63° 25'$; $\theta_{B,B} = 0°$; $\theta_{D,D} = 56° 20'$. (58.26)

(d) $\theta_{A-D,A} = 82° 47'$; $\theta_{B-A,B} = 58° 12'$; $\theta_{D-B,D} = 63° 8'$.

(e) The total power = 3.00 kw.

(f) The total reactive power = 2.45 kv.a.

(g) The corresponding vector diagram is shown in Fig. 8.26.

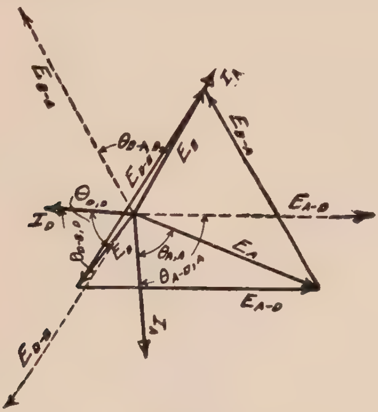


FIG. 8.26.

Problem 5.26.—Given a three-phase, delta-connected system as in Fig. 9.26.

$$E_A = E_B = E_D = 120 \text{ volts}; f = 25 \text{ cycles}.$$
$$z_A = 3 + j4; z_B = 4 + j2; z_D = 3 \text{ ohms}.$$

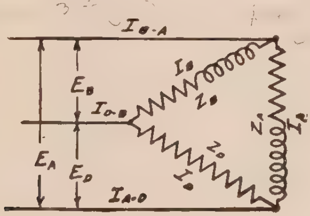


FIG. 9.26.

Find (a) I_A, I_B, I_D.

(b) I_{A-D}, I_{B-A}, I_{D-B}.

(c) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.

(d) $\theta_{A,A-D}$, $\theta_{B,B-A}$, $\theta_{D,D-B}$.

(e) The total kilowatts.

(*f*) The total reactive power.

(*g*) Draw the vector diagram.

Since the three voltages are equal, the three circuit currents may be solved directly.

(*a*) $I_A = 24.0$ amp.; $I_B = 26.9$ amp.; $I_D = 40.0$ amp.

(*b*) The currents in the mains may be found graphically from the vector diagram or by applying the law of cosines.

$$I_{A-D} = 37.6 \text{ amp.}; \ I_{B-A} = 49.0 \text{ amp.}; \ I_{D-B} = 64.0 \text{ amp.} \qquad (59.26)$$

(*c*) $\theta_{A,A} = 53° \, 10'$; $\theta_{B,B} = 26° \, 34'$; $\theta_{D,D} = 0°$ (60.26)

(*d*) $\theta_{A,A-D} = -24° \, 30'$; $\theta_{B,B-A} = 11° \, 10'$; $\theta_{D,D-B} = -13° \, 10'$.

(*e*) The total power = 9.0 kw.

(*f*) The total reactive power = 3.6 kv.a.

(*g*) The vector diagram is drawn to scale in Fig. 10.26.

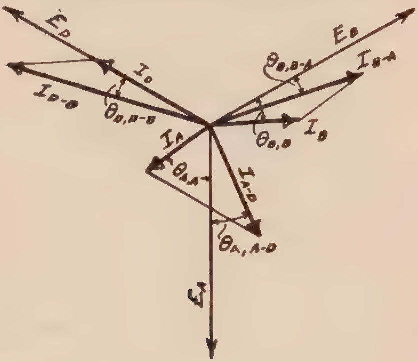


FIG. 10.26.

Problem 6.26.—Same circuit and data as in problem 2.26, except that the frequency of the impressed voltage = 25 cycles.

Problem 7.26.—Same circuit and data as in problem 4.26, except that the frequency of the impressed voltage = 40 cycles.

Problem 8.26.—Same circuit and data as in problem 5.26, except that the frequency of the impressed voltage = 50 cycles.

Problem 9.26.—A three-phase delta-connected distribution system has the following load:

(*a*) Three-phase induction motor, 50 kw., 85 per cent power factor.

(*b*) Incandescent lamp load of 30 kw. on phase *A*, 22 kw. on phase *B* and 20 kw. on phase *D*.

(*c*) Single-phase motor on phase *D*, 4 kw., 80 per cent power factor. The line voltage is 2,200 volts. Find the line currents.

(*d*) **Symmetrical Components of Unbalanced Systems.**[1]—Solutions for certain problems in unbalanced, three-phase systems can be obtained to best advantage by resolving the given unbalanced,

[1] SHUCK, GORDON R., *Bull.* "Metering Symmetrical Components."

three-phase vector system into two, or in some cases three, sets of equivalent component vectors. The method is based on the principle that the vectors of an unbalanced, three-phase system may be resolved into three component sets of symmetrically balanced vec-

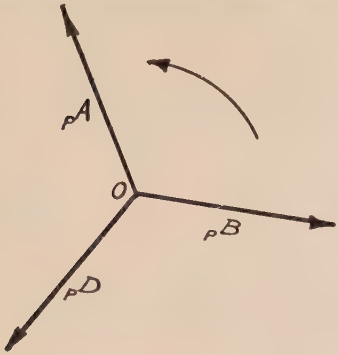


FIG. 11.26.—Balanced, three-phase system. Positive sequence: $_P\dot{A}$–$_P\dot{B}$–$_P\dot{D}$.

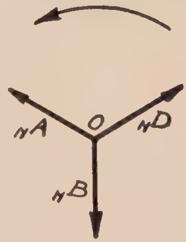


FIG. 12.26.—Balanced, three-phase system. Negative sequence: $_N\dot{A}$–$_N\dot{D}$–$_N\dot{B}$.

tors: the first, a balanced, three-phase system having the same (positive) sequence of vectors as the original unbalanced system; the second, likewise a balanced, three-phase system but having the reverse (negative) vector sequence; the third, a uniphase system

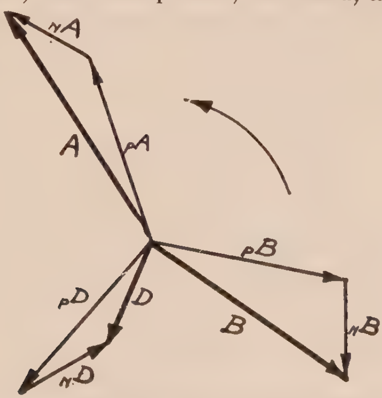


FIG. 13.26.—Unbalanced, three-phase, resultant system. Positive sequence: $\dot{A}$–$\dot{B}$–$\dot{D}$.

of three equal in-phase vectors. In some problems, as illustrated by Fig. 13.26 only the first two sets of symmetrical components enter; that is, the vectors in the uniphase system are of zero value. To grasp more readily the basic principle, let two

symmetrical-component, three-phase vectors, as in Figs. 11.26 and 12.26, be combined to form the unbalanced, three-phase system in Fig. 13.26. In the two sets of balanced, three-phase vectors shown in Figs. 11.26 and 12.26 the notation is in the same direction (counterclockwise), but the vector sequence in Fig. 12.26 is in the reverse order to that of Fig. 11.26. In Fig. 13.26 are shown the resultant vectors, in which $\dot{A}$, $\dot{B}$ and $\dot{D}$ represent the sum of the corresponding component vectors in Figs. 11.26 and 12.26. It is evident that:

$$\dot{A} = {}_P\dot{A} + {}_N\dot{A} \tag{61.26}$$

$$\dot{B} = {}_P\dot{B} + {}_N\dot{B} \tag{62.26}$$

$$\dot{D} = {}_P\dot{D} + {}_N\dot{D} \tag{63.26}$$

The phase sequence of the resultant unbalanced, three-phase system is $\dot{A} - \dot{B} - \dot{D}$; that is, positive, or in the same sequence as for the vectors in Fig. 11.26.

In practical problems the known data relate to the unbalanced system, as $\dot{A}$, $\dot{B}$, $\dot{D}$ in Fig. 13.26, and it is required to find the corresponding symmetrical components, as ${}_P\dot{A}$, ${}_P\dot{B}$, ${}_P\dot{D}$ and ${}_N\dot{A}$, ${}_N\dot{B}$, ${}_N\dot{D}$ in Figs. 11.26 and 12.26. That is, in equations (61.26), (62.26) and (63.26) let the values for $\dot{A}$, $\dot{B}$ and $\dot{D}$ be given, and let it be required to express the values of ${}_P\dot{A}$, ${}_N\dot{A}$, ${}_P\dot{B}$, ${}_N\dot{B}$, ${}_P\dot{D}$ and ${}_N\dot{D}$ in terms of $\dot{A}$, $\dot{B}$ and $\dot{C}$.

In order to find the expression for $\dot{A}$, transform equations (62.26) and (63.26) into terms of ${}_P\dot{A}$ and ${}_N\dot{A}$ by means of the operators $j^{4/3}$ and $j^{8/3}$, which indicate vector rotation of 120° and 240°, respectively.

$$\dot{A} = {}_P\dot{A} + {}_NA \tag{64.26}$$

$$\dot{B} = j^{8/3}\,{}_P\dot{A} + j^{4/3}\,{}_N\dot{A} \tag{65.26}$$

$$\dot{D} = j^{4/3}\,{}_P\dot{A} + j^{8/3}\,{}_N\dot{A} \tag{66.26}$$

Multiply both sides of equation (65.26) by $j^{4/3}$ and likewise equation (66.26) by $j^{8/3}$ and add the resulting equations and equation (64.26).

$$\dot{A} + j^{4/3}\dot{B} + j^{8/3}\dot{D} = {}_P\dot{A} + {}_N\dot{A} + j^{12/3}\,{}_P\dot{A} + j^{8/3}\,{}_N\dot{A} + j^{12/3}\,{}_P\dot{A} + j^{16/3}\,{}_N\dot{A} \tag{67.26}$$

Since the operator j represents a rotator of 90° in the counterclockwise direction it is evident that $j^{12/3} = 1$; $j^{16/3} = (-0.5 + j0.866)$ and $j^{8/3} = (-0.5 - j0.866)$. Simplifying equation (67.26) and solving for ${}_P\dot{A}$ gives equation (68.26).

$${}_P\dot{A} = \frac{\dot{A} + j^{4/3}\dot{B} + j^{8/3}\dot{D}}{3} \tag{68.26}$$

By the same method, to find the value for ${}_N\dot{A}$, multiply both sides of equation (65.26) by $j^{8/3}$ and also equation (66.26) by $j^{4/3}$, and proceed by adding the resulting equations and equation (64.26).

$$\dot{A} + j^{8/3}\dot{B} + j^{4/3}\dot{D} = {}_P\dot{A} + {}_N\dot{A} + j^{16/3}\,{}_P\dot{A} + j^{12/3}\,{}_NA + j^{8/3}{}_P\dot{A} + j^{12/3}\,{}_N\dot{A} \quad (69.26)$$

Simplifying equation (69.26) by eliminating ${}_P\dot{A}$, and solving for ${}_N\dot{A}$ gives equation (70.26).

$${}_N\dot{A} = \frac{\dot{A} + j^{8/3}\dot{B} + j^{4/3}D}{3} \quad (70.26)$$

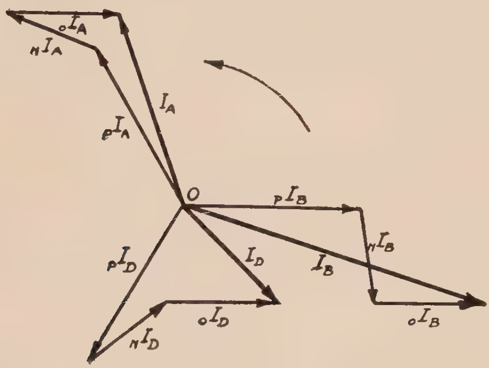


FIG. 14.26.—Vectors $\dot{I}_A$, $\dot{I}_B$ and $\dot{I}_D$ represent the given unbalanced system. Sequence: $\dot{I}_A$–$\dot{I}_B$–$\dot{I}_D$.

In like manner the expressions for the symmetrical components of $\dot{B}$ and $\dot{D}$ may be determined.

$${}_P\dot{B} = \frac{\dot{B} + j^{8/3}\dot{A} + j^{4/3}\dot{D}}{3} \quad (71.26)$$

$${}_N\dot{B} = \frac{\dot{B} + j^{4/3}\dot{A} + j^{8/3}\dot{D}}{3} \quad (72.26)$$

$${}_P\dot{D} = \frac{D + j^{8/3}\dot{B} + j^{4/3}\dot{A}}{3} \quad (73.26)$$

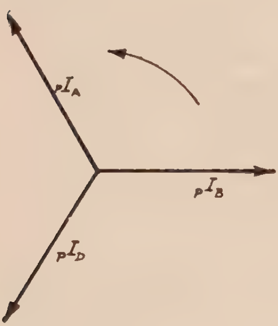
$${}_N\dot{D} = \frac{D + j^{4/3}\dot{B} + j^{8/3}\dot{A}}{3} \quad (74.26)$$

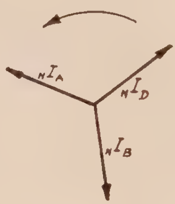
A somewhat more direct method for finding ${}_P\dot{B}$, ${}_N\dot{B}$, ${}_P\dot{D}$ and ${}_N\dot{D}$, after having determined the expressions for ${}_P\dot{A}$ and ${}_N\dot{A}$, as in

equations (68.26) and (70.26), is obtained by merely multiplying ${}_P\dot{A}$ and ${}_N A$ by the operators $j^{4/3}$ and $j^{8/3}$ as follows:

$$ {}_P\dot{B} = j^{8/3}\,{}_P\dot{A} \tag{75.26}$$
$$ {}_N\dot{B} = j^{4/3}\,{}_N\dot{A} \tag{76.26}$$
$$ {}_P\dot{D} = j^{4/3}\,{}_P\dot{A} \tag{77.26}$$
$$ {}_N\dot{D} = j^{8/3}\,{}_N\dot{A} \tag{78.26}$$

In the preceding problem it was assumed that the sum of the three vectors $\dot{A}$, $\dot{B}$ and $\dot{D}$ is zero, as would be the case for the voltages in a delta-connected system or the line currents in a three-phase, three-wire system. However, in some three-phase systems the sum of the original vectors is not equal to zero, as for example, the line currents in a four-wire, three-phase system, illustrated by the vectors $\dot{I}_A$, $\dot{I}_B$ and $\dot{I}_D$ in Fig. 14.26. Let the





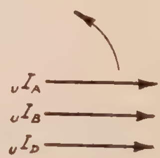


FIG. 15.26. FIG. 16.26. FIG. 17.26.

FIG. 15.26.—Positive sequence, symmetrical components for Fig. 14.26. Sequence: ${}_P\dot{I}_A - {}_P\dot{I}_B - {}_P\dot{I}_D$.

FIG. 16.26.—Negative sequence, symmetrical components for Fig. 14.26. Sequence: ${}_N\dot{I}_A - {}_N\dot{I}_D - {}_N\dot{I}_B$.

FIG. 17.26.—Uniphase, zero sequence components for Fig. 14.26.

three component systems be represented by the vectors in Figs. 15.26, 16.26 and 17.26; and the sums of the components for each phase equal to the original vectors, as indicated in Fig. 14.26. The three equal uniphase vectors ${}_U\dot{I}_A$, ${}_U\dot{I}_B$, and ${}_U\dot{I}_D$ form the third set of components, into which the unbalanced, three-phase system may be resolved.

If $\dot{I}_A$, $\dot{I}_B$ and $\dot{I}_D$ represent the three line currents, the resultant current in the neutral or fourth wire,

$$ \dot{I}_R = \dot{I}_A + \dot{I}_B + \dot{I}_D \tag{79.26}$$

From Fig. 14.26:

$$\dot{I}_A = {}_P\dot{I}_A + {}_N\dot{I}_A + {}_U\dot{I}_A \tag{80.26}$$
$$\dot{I}_B = {}_P\dot{I}_B + {}_N\dot{I}_B + {}_U\dot{I}_B \tag{81.26}$$
$$\dot{I}_D = {}_P\dot{I}_D + {}_N\dot{I}_D + {}_U\dot{I}_D \tag{82.26}$$
$${}_U\dot{I}_A = {}_U\dot{I}_B = {}_U\dot{I}_D = \frac{\dot{I}_R}{3} \tag{83.26}$$

Expressions for the positive and negative symmetrical components in terms of the original vectors $\dot{I}_A$, $\dot{I}_B$ and $\dot{I}_D$ may be

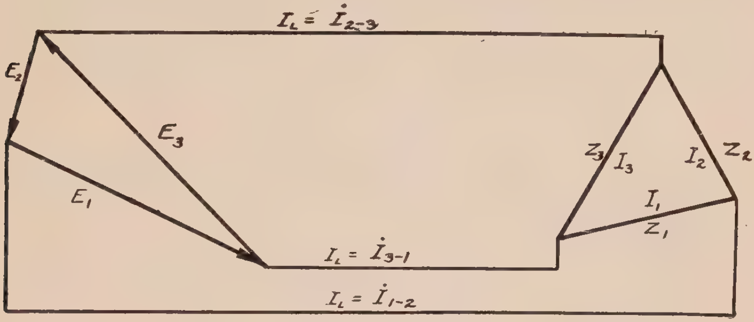


FIG. 18.26.—Original, unbalanced, three-phase system.

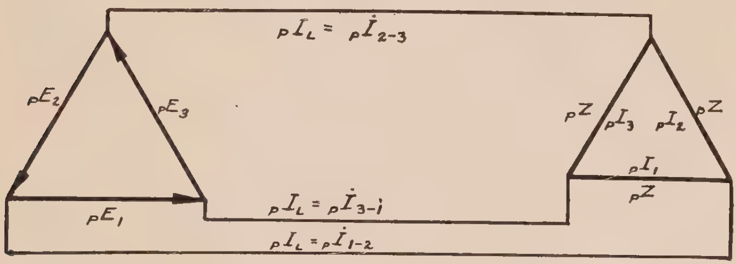


FIG. 19.26.—Positive-sequence, symmetrical components for circuit in Fig. 18.26.

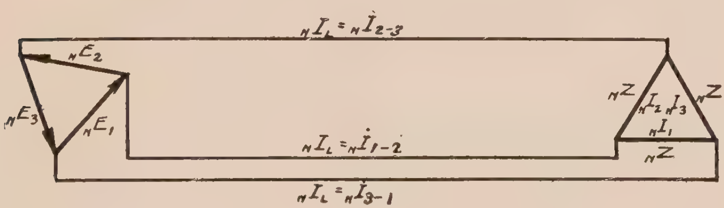


FIG. 20.26.—Negative-sequence, symmetrical components for circuit in Fig. 18.26.

obtained by the same method as used in deriving equations (68.26) to (79.26) in the previous case.

In the application of symmetrical components to the solution of practical problems the vectors may represent either voltages or currents. For example, the delta-connected system shown in Fig. 18.26 may be resolved into a balanced, positive-sequence system and a corresponding negative-sequence set of vectors, as illustrated by Figs. 19.26 and 20.26, respectively. Each component system is complete in itself, having balanced, delta-connected voltages; balanced, three-phase line currents; delta-connected impedances, and power factor.

Since the two symmetrical-component systems are balanced, the power consumed is represented by equations (84.26) and (85.26)

$$_PP = \sqrt{3}\,_PE\,_PI \cos {}_P\theta \tag{84.26}$$

$$_NP = \sqrt{3}\,_NE\,_NI \cos {}_N\theta \tag{85.26}$$

For systems, in which the uniphase components exist, the corresponding power,

$$_UP = 3\,_UE\,_UI \cos {}_U\theta \tag{86.26}$$

The total power in the original systems is therefore,

$$P = {}_PP + {}_NP + {}_UP \tag{87.26}$$

In nearly all commercial three-phase systems, delta or star three-wire, or star four-wire, either $_UE$ or $_UI$ is zero. As a consequence, $_UP$ equals zero and the total power is the sum of the positive-sequence power, $_PP$, and the negative-sequence power, $_NP$, of the symmetrical components.

In Table XVIII are listed the symmetrical components that may exist in different parts of commercial three-phase systems.

Let:

P denote positive-sequence, symmetrical components.
N denote negative-sequence, symmetrical components.
U denote zero-sequence, or uniphase components.

As an illustration of the symmetrical-components method of analysis, both for gaining physical concepts of the operating conditions and for obtaining quantitative solutions of specific problems, consider an induction motor receiving power from an unbalanced, three-phase system. The effect produced by unbalanced voltages is as if two sets of balanced voltages were impressed on the motor, one of the same phase sequence as the line voltage, tending to turn the rotor in the normal direction, the

TABLE XVIII

Line voltage	Circuit voltage	Line current	Circuit current
(a) Three-wire; delta-connected			
P N	P N	P N	P N U
(b) Three-wire; star-connected			
P N	P N U	P N	P N
(c) Four-wire; star-connected			
P N	P N U	P N U	P N U

other of negative vector sequence, tending to make the rotor revolve in the reverse direction. As a consequence, two torques act on the rotor, one positive and the other negative; each produced by balanced sets of voltages, currents and fluxes. The resultant or useful torque is therefore the difference of the torques produced by the positive and the negative systems of symmetrical components. It is evident that unbalanced line conditions necessarily reduce the efficiency and increase the heating of induction motors at all loads.

The following quantitative solution of a specific problem is given in full to illustrate the symmetrical-components method of analysis.

Problem 10.26.—Given the line voltages of an unbalanced, three-phase system supplying power to an unbalanced delta-connected load. The same notation is used as in Fig. 18.26. In order to avoid fractional exponents for the operator j, the equivalent values, $(-0.5 + j0.866)$ for $j^{4/3}$ and $(-0.5 - j0.866)$ for $j^{8/3}$, are used.

Given: $\dot{E}_1 = 198 + j0$ volts. $\qquad Z_1 = 6 + j0$ ohms.
$\dot{E}_2 = -67 - j110.5$ volts. $\qquad Z_2 = 1 + j7$ ohms.
$\dot{E}_3 = -131 + j110.5$ volts. $\qquad Z_3 = 4 + j3$ ohms.

(a) Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, ${}_P\dot{I}_{1-2}$, ${}_P\dot{I}_{2-3}$, ${}_P\dot{I}_{3-1}$, ${}_N\dot{I}_{3-1}$, ${}_N\dot{I}_{1-2}$, ${}_N\dot{I}_{2-3}$; ${}_P\dot{E}_1$, ${}_P\dot{E}_2$, ${}_P\dot{E}_3$, ${}_N\dot{E}_1$, ${}_N\dot{E}_2$, ${}_N\dot{E}_3$; ${}_PZ$, ${}_NZ$; $\cos {}_P\theta$, $\cos {}_N\theta$; ${}_PP$, ${}_NP$, P.

(b) Find the absolute values of the several current and voltage vectors listed under (a).

$$\dot{I}_1 = \frac{198 + j0}{6 + j0} = 33 + j0;\ I_1 = 33.0 \text{ amp.}$$

$$\dot{I}_2 = \frac{-67 - j110.5}{4 + j3} = -16.8 + j7.17;\ I_2 = 18.25 \text{ amp.}$$

$$\dot{I}_3 = \frac{-131 + j110.5}{4 + j3} = -7.7 + j33.4;\ I_3 = 34.3 \text{ amp.}$$

$$\dot{I}_{1-2} = 33 + j0 + 16.8 - j7.17 = 49.8 - j7.17 \text{ amp.}$$

$$\dot{I}_{2-3} = -16.8 + j7.17 + 7.7 - j33.4 = -9.1 - j26.23 \text{ amp.}$$

$$\dot{I}_{3-1} = -7.7 + j33.4 - 33 - j0 = -40.7 + j33.4 \text{ amp.}$$

$${}_P\dot{I}_{1-2} = \frac{49.8 - j7.17 + (-9.1 - j26.23)(-0.5 + j0.866) + (-40.7 + j33.4)(-0.5 - j0.866)}{3}$$

$$= 42.10 + j5.52;\ {}_PI_{1-2} = 42.45 \text{ amp.}$$

$${}_P\dot{I}_{2-3} = (42.10 + j5.52)(-0.5 - j0.866) = -16.27 - j39.16;$$

$${}_PI_{2-3} = 42.45 \text{ amp.}$$

$${}_P\dot{I}_{3-1} = (42.10 + j5.52)(-0.5 + j0.866) = -25.83 + j33.66;$$

$${}_PI_{3-1} = 42.45 \text{ amp.}$$

$${}_N\dot{I}_{1-2} = \frac{49.8 - j7.17 + (-9.1 - j26.23)(-0.5 - j0.866) + (-40.7 + j33.4)(-0.5 + j0.866)}{3}$$

$$= 8.03 - j12.69;\ {}_NI_{1-2} = 15.05 \text{ amp.}$$

$${}_N\dot{I}_{2-3} = (8.03 - j12.69)(-0.5 + j0.866) = 6.97 + j13.29;$$

$${}_NI_{2-3} = 15.05 \text{ amp.}$$

$${}_N\dot{I}_{3-1} = (8.03 - j12.69)(-0.5 - j0.866) = -15.005 - j0.605;$$

$${}_NI_{3-1} = 15.05 \text{ amp.}$$

$${}_P\dot{E}_1 = \frac{198 + j0 + (-67 - j110.5)(-0.5 + j0.866) + (-131 + j110.5)(-0.5 - j0.866)}{3}$$

$$= 162.8 + j18.50;\ {}_PE_1 = 163.8 \text{ volts.}$$

$${}_P\dot{E}_2 = (162.8 + j18.50)(-0.5 - j0.866) = -65.38 - j150.25;$$

$${}_PE_2 = 163.8 \text{ volts.}$$

$${}_P\dot{E}_3 = (162.8 + j18.50)(-0.5 + j0.866) = -97.42 + j131.75;$$

$${}_PE_3 = 163.8 \text{ volts.}$$

$$_N\dot{E}_1 = \frac{198 + j0 + (-67 - j110.5)(-0.5 - j0.866) + (-131 + j110.5)(-0.5 + j0.866)}{3}$$

$= 35.2 - j18.50;\ _NE_1 = 39.8$ volts.

$_N\dot{E}_2 = (35.2 - j18.50)(-0.5 + j0.866) = -1.58 + j39.72;$

$_NE_2 = 39.8$ volts.

$_N\dot{E}_3 = (35.2 - j18.50)(-0.5 - j0.866) = -33.62 - j21.22;$

$_NE_3 = 39.8$ volts.

The positive-sequence load impedance, which would cause the positive-sequence currents to flow under pressure of the positive-sequence voltage:

$$_PZ = \frac{\sqrt{3}\,_PE_1}{_PI_{1-2}(0.866 - j0.5)} = 5.85 + j3.245 \text{ ohms.}$$

$$\text{Cos}\ _P\theta = \frac{5.85}{\sqrt{\overline{5.85}^2 + \overline{3.245}^2}} = 87.6 \text{ per cent.}$$

The negative-sequence load impedance:

$$_NZ = \frac{\sqrt{3}\,_NE_1}{_NI_{1-3}(0.866 + j0.5)} = 4.6 - j0.008 \text{ ohms.}$$

$$\text{Cos}\ _N\theta = \frac{4.6}{\sqrt{\overline{4.6}^2 + \overline{0.008}^2}} = 99.99 \text{ per cent.}$$

$_PP$ = Positive-sequence power $= \sqrt{3} \times 163.8 \times 42.45 \times 0.876$
$= 10{,}540$ watts.

$_NP$ = Negative-sequence power $= \sqrt{3} \times 39.8 \times 15.05 \times 0.9999$
$= 1{,}038$ watts.

P = Total power in system $= 10{,}540 + 1{,}038 = 11{,}578$ watts.

The total power in the system may be checked by finding the sum of the RI^2 losses.

$$R_1I_1^2 + R_2I_2^2 + R_3I_3^2 = 6 \times \overline{33}^2 + 4 \times \overline{34.3}^2 + 1 \times \overline{18.25}^2 = 11{,}564 \text{ watts.}$$

Problem 11.26.—Given the following data for a four-wire, three-phase, Y-connected system:

E_1, E_2 and E_3 are voltages to neutral.

$$\dot{E}_1 = 30 + j60. \qquad Z_1 = 3 + j2.$$
$$\dot{E}_2 = 30 - j50. \qquad Z_2 = 5 + j0.$$
$$\dot{E}_3 = -50 + j20. \qquad Z_3 = 4 + j1.$$

Find: $_P\dot{E}_{1-2}$, $_P\dot{E}_{2-3}$, $_P\dot{E}_{3-1}$, $_N\dot{E}_{1-3}$, $_N\dot{E}_{3-2}$, $_N\dot{E}_{2-1}$, $_U\dot{E}_1$, $_U\dot{E}_2$, $_U\dot{E}_3$, $_N\dot{E}_1$, $_N\dot{E}_2$, $_N\dot{E}_3$, $_PE_1$, $_NE_1$; $_P\dot{I}_1$, $_P\dot{I}_2$, $_P\dot{I}_3$, $_N\dot{I}_1$, $_N\dot{I}_2$, $_N\dot{I}_3$, $_U\dot{I}$; $_PP$, $_NP$, $_UP$ and P.

(*e*) **Grounded and Floating Neutral.**—While many factors enter into the problem of whether a high-voltage system should be grounded or kept isolated, the main argument lies in the relative value of continuity of service as against an increased risk of damage to the apparatus. In any transmission system the apparatus operating on the lowest factor of safety needs most attention. Until the advent of the suspension insulator, line insulation was the weakest part of the system and on high-voltage transmission lines special care had to be taken in order that no excessive voltages should be impressed on the insulators. A partial protection is gained by grounding the neutral, as then only the fixed Y voltage (58 per cent of the line voltage) can come on the line insulators, as a ground on any of the lines would produce a short-circuit. If the neutral is floating, *i.e.*, the line isolated, a ground on one of the lines does not produce a short-circuit, but shifts the neutral point to the grounded wire and therefore increases the stress across the insulators on the other two lines to full-line voltage. If the factor of safety for the insulators is small, the increased stress may cause a breakdown with interruption of service. By means of the suspension type of insulator, sufficient insulation can be provided to give an ample factor of safety for the full-line voltage.

Continuity of service is of very great importance in practically all electric service systems. Because an isolated system can operate with one line grounded, while a system having the neutral grounded would be short-circuited and forced to shut down if one of the lines became grounded, it is generally assumed that more nearly continuous service is possible with the system isolated. However, if the increased stresses on the line insulators cause further breakdown, the reverse might be true. Many other factors, like the location and number of power stations in a system, the number of parallel trunk lines, the possible mechanical and electrical stresses incident to short-circuit conditions, the importance of providing continuous service, must also be taken into consideration. With a single power plant supplying energy over a single line, the isolated system is clearly desirable. With several power plants and several transmission lines forming a network, continuity may be secured even if one or more of the lines or plants are out of service. In the isolated system the single ground must be located and promptly removed, as a second ground would cause a short-circuit. The location of grounds,

particularly if of a partial or intermittent nature, is sometimes a difficult problem and, as a consequence, repairs must often be delayed. Under such conditions the isolated system is virtually grounded and also the insulation to ground is subjected to full line voltage on two of the lines.

The chief advantage gained by having the neutral grounded is that the voltage to ground cannot exceed the normal Y voltage of the system. In isolated systems having unbalanced loads the neutral shifts in position with changes in the load. In high-voltage lines the condensance of the system helps to keep the neutral fixed except when a ground develops on one of the lines. Short-circuit stresses become increasingly dangerous as the amount of power involved increases. In order to reduce the sudden flow of energy when a short-circuit occurs, the system is sometimes grounded through a resistance. While the short-circuit shock may be reduced by this device, two objections should be noted. First, it is difficult to install and maintain a reliable grounding resistance. Second, the voltages between the lines and ground would rise above the normal Y voltage until the circuit-breakers operate. As explained in Chap. XXV, power-limiting reactances are installed in large systems in order to keep the effects of short-circuits within permissible limits.

The question of whether the neutral should be grounded or floating is independent of the use of star or delta transformer connections. However, isolated systems are generally delta-connected and the grounded neutral systems star-connected. A ground on a delta-connected system may be obtained by using a special grounding device or by letting one set of transformers be star-connected. The neutral should be grounded at one place only. If connected at two or more places, currents will flow through the ground even under normal operating conditions. These ground currents may cause serious interference with the telephone service.

(*f*) **Transmission-line Efficiency.**—Omitting the transformers and taking only the transmission line into consideration, the efficiency of transmission depends practically on the RI^2 losses and the power delivered. For properly designed lines operating under normal conditions, both the leakage and the corona losses are negligible. In order to make a comparison of the several systems it is necessary first to assume a basis. The power delivered is the product of the current, voltage and power factor,

while the losses vary as the product of the resistance and the square of the current. As the power factor affects all systems alike, it may, for convenience, be taken as unity. Assuming the power delivered to be of the same magnitude in all cases, the line efficiencies may be compared either by the losses for the same amount of copper or as to the copper required for the same losses under specified voltage conditions. As the voltage and the insulation are directly related, the assumptions on which the comparison should be made must be different in low-tension distribution systems from that which would be desirable for long-distance transmission lines. Likewise, the stresses on the insulation in a high-voltage system differ, depending upon whether the neutral is floating or grounded. For application to commercial systems the comparison should be made under three voltage conditions:

1. The systems to have the same minimum effective voltage per circuit.
2. The systems to have the same maximum effective voltage between any conductor and ground.
3. The systems to have the same maximum effective voltage between any two conductors in each system.

In distribution networks ample insulation can be provided and hence the voltage per terminal circuit should be the basis for comparison. In high-tension distribution lines with grounded neutral the maximum voltage impressed on the insulators, or the voltage from conductor to ground, should be the same in the several systems. In cables the voltage between conductors is the desirable basis for comparison.

1. Low-potential Distribution Networks, Lighting Circuits, Etc.—(*a*) *Single-phase, two-wire system.*

Let

r = the resistance per wire and i = the current.

Power loss = $2ri^2$.

Copper required (two wires) = 100 per cent.

(*b*) *Single-phase, three-wire system. Neutral of same cross-section as either outside wire.*

Current in outside wire $= \frac{i}{2}$.

Copper required = 37.5 per cent.

(*c*) *Single-phase, three-wire system. Neutral of half the cross-section of one outside wire.*

Current in outside wire $= \dfrac{i}{2}$.

Copper required = 31.25 per cent.

(*d*) *Single-phase, three-wire system. Neutral of zero cross-section.*

Current in outside wire $= \dfrac{i}{2}$.

Copper required = 25 per cent.

(*e*) *Single-phase, five-wire system. All wires of the same cross-section.*

Current in outside wire $= \dfrac{i}{4}$.

Copper required = 15.63 per cent.

(*f*) *Single-phase, five-wire system. Each of the inside wires to have half the cross-section of one of the outside wires.*

Current in outside wire $= \dfrac{i}{4}$.

Copper required = 10.93 per cent.

(*g*) *Quarter-phase, four-wire system. All wires of the same cross-section.* Equivalent to two single-phase, two-wire systems. Same as (*a*).

Copper required = 100 per cent.

(*h*) *Quarter-phase, three-wire system. The third wire is the common return for the two phases. Its cross-section is $\sqrt{2}$ times that of either of the outside wires so as to have the same current density in the three wires.*

The resistance of one of the outside wires $= r_1$.

Current in each of the outside wires $= \dfrac{i}{2}$.

Current in the middle wire $= \dfrac{i}{\sqrt{2}}$.

Power loss in the middle wire $= \dfrac{r_1}{\sqrt{2}}\left(\dfrac{i}{\sqrt{2}}\right)^2 = \dfrac{r_1 i^2}{2\sqrt{2}}$.

Total loss in the three wires $= 2r_1\dfrac{i^2}{4} + \dfrac{r_1 i^2}{2\sqrt{2}} = r_1 i^2\left(\dfrac{2+\sqrt{2}}{4}\right)$.

Equating the total loss to that of the single-phase, two-wire system in (*a*)

$$r_1 i^2\left(\frac{2+\sqrt{2}}{4}\right) = 2ri^2$$

and solving for r_1, we have:

$$r_1 = \frac{8}{2 + \sqrt{2}} r.$$

Hence each outside wire must have a cross-section of $\frac{2 + \sqrt{2}}{8}$ and the middle wire $\frac{2 + 2\sqrt{2}}{8}$ times that of the equivalent single-phase wires.

$$\text{Copper required} = \frac{\frac{2(2 + \sqrt{2})}{8} + \frac{2 + 2\sqrt{2}}{8}}{2} = 72.9 \text{ per cent.}$$

(*i*) *Quarter-phase, six-wire system. All wires of equal cross-section.*

Equivalent to two circuits of (*b*).

Copper required = 37.5 per cent.

(*j*) *Quarter-phase, five-wire system. The fifth wire to be used as neutral for both phases. All wires of the same cross-section.*

Equivalent to two circuits of (*c*)

Copper required = 31.25 per cent.

(*k*) *Three-phase, three-wire system. All wires of the same cross-section.*

Let r_1 = resistance for each wire.

Current in each main $= \frac{i}{\sqrt{3}}$.

Loss in each main $= \frac{r_1 i^2}{3}$; total loss $= r_1 i^2 = 2ri^2$.

The cross-section is inversely as the resistance, and $r_1 = 2r$.

Copper required $= \frac{3(\frac{1}{2})}{2} = 75$ per cent.

(*l*) *Three-phase, four-wire system. The load star-connected. All wires of same cross-section. Voltage from main to neutral equal to the voltage between lines in* (*a*). *Voltage between mains is* $\sqrt{3}$ *times voltage to neutral. For balanced load no current flows in the neutral.*

Current in mains $= \frac{i}{3}$.

Loss in each wire $= r_1\frac{i^2}{9}$; total loss $\frac{r_1 i^2}{3} = 2ri^2$, and hence $r_1 = 6r$.

$$\text{Copper required} = \frac{4(\frac{1}{6})}{2} = 33.3 \text{ per cent.}$$

(*m*) *Three-phase, four-wire system. Same as in* (*l*) *except that the neutral has only half the cross-section of one of the mains.*

Copper required = 29.16 per cent.

2. High-voltage Circuits. Transmission Lines with Grounded Neutral.—The maximum voltage from the line to ground is the basis of comparison.

On this basis the Edison three-wire and five-wire systems give no advantage over the two-wire system. Whether the ground connection is used merely for maintaining a constant-potential difference between the ground and the line conductors, or as a return circuit-carrying current, all grounded systems are equivalent to a single-phase, two-wire system with grounded neutral, or to a single-phase circuit having one wire and ground return.

3. High-voltage Circuits. Isolated Transmission Lines, Cables, Etc.—The maximum voltage between any two conductors is taken as the basis of comparison. On this basis the Edison three-wire and five-wire systems offer no advantage over the single-phase, two-wire system.

(*a*) *Quarter-phase, four-wire system. All wires of the same cross-section.* The system is equivalent to two single-phase circuits in time quadrature.

Copper required = 100 per cent.

(*b*) *Quarter-phase, three-wire system. The cross-section of the middle wire is $\sqrt{2}$ times that of either outside wire.*

$$\text{Current in outside wire} = \frac{i}{\sqrt{2}}.$$

$$\text{Current in middle wire} = i.$$

$$\text{Power loss in outside wire} = \frac{r_1 i^2}{2}; \text{ in middle wire} = \frac{r_1 i^2}{\sqrt{2}}.$$

$$\text{Total loss in the three wires} = 2\left(\frac{r_1 i^2}{2}\right) + \frac{r_1 i^2}{\sqrt{2}} = r_1 i^2\left(\frac{2 + \sqrt{2}}{2}\right).$$

$$\text{Copper required} = \frac{\frac{2(2+\sqrt{2})}{4} + \frac{\sqrt{2}(2+\sqrt{2})^2}{4}}{2} = 145.7 \text{ per cent.}$$

(*c*) *Three-phase, three-wire system. All wires of the same cross-section.*

$$\text{Current in each wire} = \frac{i}{\sqrt{3}}.$$

$$\text{Total loss} = 3\left(\frac{i}{\sqrt{3}}\right)^2 r_1.$$

$$\text{Copper required} = \frac{3(\frac{1}{2})}{2} = 75 \text{ per cent.}$$

For the same efficiency of transmission the three-phase line requires less copper and for this reason is most desirable for long-distance transmission lines.

CHAPTER XXVII

CERTAIN FORMS OF NOTATION

TOPOGRAPHIC METHOD. ROOTS OF MINUS ONE AS OPERATORS. HYPERBOLIC SINES AND COSINES

It is often desirable to show in a single diagram the simultaneous time-phase relations of the currents and voltages at several points in an alternating-current system. It is especially advantageous to show both the time-phase and space-phase relations when discussing the electrical phenomena of long-distance transmission lines. Since the form of representation is somewhat like a topographic map it has been called the topographic method.

(*a*) **Topographic Method.**—For illustrative purposes, consider a single-phase circuit having resistances, inductances, condensers and leakage conductances as shown in Fig. 1.27. Let the following quantities be given, using the notation indicated in the figure: E_4; r_1, r_2, r_3, r_4; ${}_Lx_1$, ${}_Lx_2$, ${}_Lx_3$, ${}_Lx_4$; ${}_cx_1$, ${}_cx_2$, ${}_cx_3$, ${}_cx_4$; ${}_Lg_1$, ${}_Lg_2$, ${}_Lg_3$, ${}_Lg_4$.

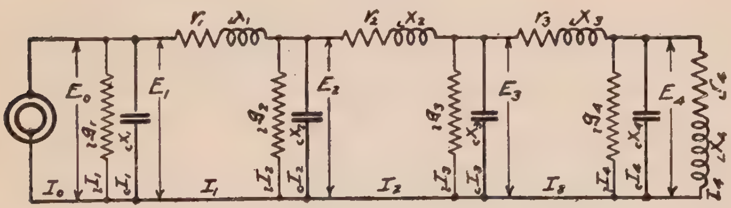


FIG. 1.27.

Starting at the fourth station, the point farthest from the generator, we have:

$$I_4 = \frac{E_4}{r_4 + j\,{}_Lx_4} \text{ and } \theta_4 = \tan^{-1}\frac{{}_Lx_4}{r_4} \qquad (1.27)$$

Draw E_4 along the X-axis as reference vector, Fig. 2.27, and lay off the current I_4, lagging $\theta°_4$.

The components of E_4, r_4I_4 and ${}_Lx_4I_4$, are drawn in phase and in quadrature, respectively, with I_4.

The charging current ${}_c I_4 = \dfrac{E_4}{{}_c x_4}$ is drawn in quadrature with E_4, and the leakage current ${}_l I_4 = {}_l g_4 E_4$ in phase with E_4.

$$\dot{I}_3 = \dot{I}_4 + {}_c\dot{I}_4 + {}_l\dot{I}_4 \tag{2.27}$$

At the third station:

Add to the vector E_4 the voltage drop, $r_3 I_3$ in phase and ${}_L x_3 I_3$ in quadrature with I_3.

$$\dot{E}_3 = \dot{E}_4 + r_3 \dot{I}_3 + {}_L x_3 \dot{I}_3 \tag{3.27}$$

The charging current ${}_c I_3 = \dfrac{E_3}{{}_c x_3}$ and is drawn in quadrature

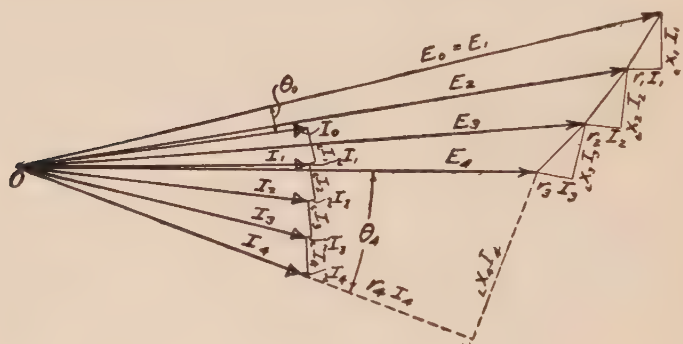


FIG. 2.27.

with E_3; the leakage current ${}_l I_3 = {}_l g_3 E_3$ is drawn in phase with E_3.

$$\dot{I}_2 = \dot{I}_3 + {}_c\dot{I}_3 + {}_l\dot{I}_3 \tag{4.27}$$

At the second station:

To the vector E_3 add the voltage drops $r_2 I_2$ and ${}_L x_2 I_2$ in phase and in quadrature, respectively, with I_2.

$$\dot{E}_2 = \dot{E}_3 + r_2 \dot{I}_2 + {}_L x_2 \dot{I}_2 \tag{5.27}$$

The charging current ${}_c\dot{I}_2$ is in time quadrature and the leakage current ${}_l\dot{I}_2$ in time phase with $\dot{E}_2$.

$$\dot{I}_1 = \dot{I}_2 + {}_c\dot{I}_2 + {}_l\dot{I}_2 \tag{6.27}$$

At the first station:

To the vector E_2 add the voltage drops $r_1 I_1$ and ${}_L x_1 I_1$ in phase and in quadrature, respectively, with I_1.

$$\dot{E}_1 = \dot{E}_2 + r_1 \dot{I}_1 + {}_L x_1 \dot{I}_1 \tag{7.27}$$

The charging current ${}_cI_1 = \dfrac{E_1}{{}_cx_1}$ is drawn in quadrature with E_1; the leakage current ${}_lI_1 = {}_lg_1 E_1$ drawn in phase with E_1.

$$\dot{I}_0 = \dot{I}_1 + {}_c\dot{I}_1 + {}_l\dot{I}_1 \tag{8.27}$$

$$\dot{E}_0 = \dot{E}_1. \tag{9.27}$$

From the diagram the magnitude and time-phase positions of currents and voltages may be found directly for all the points plotted. For example, the time-phase relation of E_3 with respect to E_0, E_1, E_2, E_4 or to any of the currents may be obtained by measuring the corresponding angles on the diagram. In transmission lines the resistance, inductance, condensance and leakage

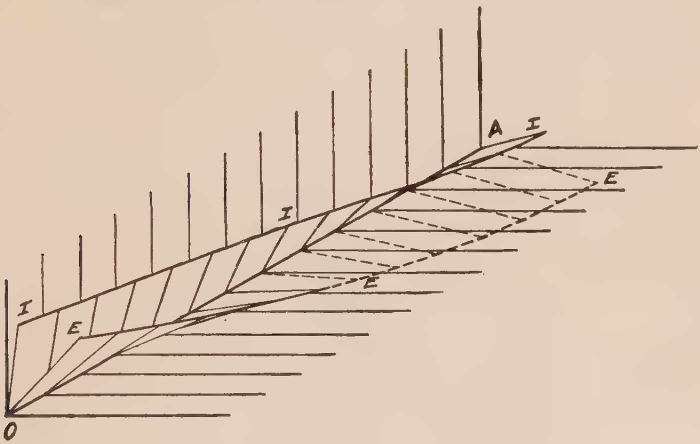


FIG. 3.27.

are uniformly distributed along the length of the line, and, therefore, the relative time-phase position of the current and voltage shifts continuously along the line. A typical perspective topographic diagram for a long line is shown in Fig. 3.27.

The time-phase relation of the current and voltage is given at any point on the line by the corresponding vectors at the given position on the line OA.

(*b*) **Roots of Minus One as Operators.**—In the preceding chapters the operator $j = \sqrt{-1}$ has been used as a coefficient to indicate a quadrature relation or a rotation of 90°. In the technical papers dealing with electrical phenomena the roots of minus one are frequently used as operators indicating time and

space-phase displacements, or a rotation of similar vectors. The operator j is used as both coefficient and exponent and an understanding of the physical concepts involved is necessary.

That the operator j indicates a rotation of 90° was shown in Chap. V.

Using the customary notation, and for a simple series circuit,

$$\begin{aligned}\dot{E} &= r\dot{I} + jx\dot{I}\\ &= z\dot{I}\cos\theta + jz\dot{I}\sin\theta\\ &= z\dot{I}\cos(\omega t) + jz\dot{I}\sin(\omega t)\\ &= z\dot{I}[\cos(\omega t) + j\sin(\omega t)] \qquad (10.27)\end{aligned}$$

$$\begin{aligned}\dot{E} &= \dot{I}\sqrt{r^2 + x^2} = z\dot{I}\\ &= z\dot{I}\sqrt{\cos^2(\omega t) + \sin^2(\omega t)} = z\dot{I} \qquad (11.27)\end{aligned}$$

Other roots of the factor -1 may also be used as operators indicating vector rotation.

Let

$$u = \sqrt[3]{-1}, \text{ or } u^3 + 1 = 0 \qquad (12.27)$$

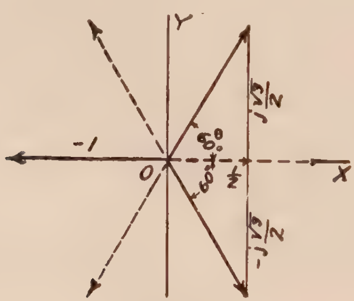


FIG. 4.27.

The three roots of the equation are:

$$\begin{aligned}u_1 &= \tfrac{1}{2} + \tfrac{1}{2}\sqrt{3}\sqrt{-1} = \tfrac{1}{2} + j\frac{\sqrt{3}}{2}\\ u_2 &= -1\\ u_3 &= \tfrac{1}{2} - \tfrac{1}{2}\sqrt{3}\sqrt{-1} = \tfrac{1}{2} - j\frac{\sqrt{3}}{2} \qquad (13.27)\end{aligned}$$

Letting j indicate a rotation of 90°, as before, the three roots may be shown graphically as in Fig. 4.27, in which the first root indicates a rotation of 60°, the second 180° and the third 300°. However, by the successive application of the first root the vector will rotate in 60° steps in the positive (counterclockwise) direction.

$$u = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) = u_1 = 60° \tag{14.27}$$

$$u^2 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^2 = -\left(\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -u_3 = 120° \tag{15.27}$$

$$u^3 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^3 = -1 = u_2 = 180° \tag{16.27}$$

$$u^4 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^4 = -\left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) = -u_1 = 240° \tag{17.27}$$

$$u^5 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^5 = \left(\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) = u_3 = 300° \tag{18.27}$$

$$u^6 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^6 = 1 = 360° \tag{19.27}$$

In a similar manner it may be shown that $\sqrt[4]{-1}$ indicates a rotation of 45°; $\sqrt[5]{-1}$ a rotation of 36°; and, in general, $\sqrt[n]{-1}$ indicates a rotation in the positive direction of $\frac{\pi}{n}$ radians.

The negative sign combined with these operators indicates a rotation of like amount in the negative direction. Thus:

$$-\sqrt{-1} = -90° \tag{20.27}$$

$$-\sqrt[3]{-1} = -60° \tag{21.27}$$

$$-\sqrt[n]{-1} = -\frac{\pi}{n} \tag{22.27}$$

From Fig. 4.27 it is evident that the operator (considered as a vector of unit length) may be expressed in terms of j and the sine and cosine functions of the angle of rotation.

$$-1 = j^2 = \cos \pi + j \sin \pi \tag{23.27}$$

$$\sqrt[3]{-1} = j^{\frac{2}{3}} = \cos\frac{\pi}{3} + j \sin\frac{\pi}{3} \tag{24.27}$$

$$\sqrt[n]{-1} = j^{\frac{2}{n}} = \cos\frac{\pi}{n} + j \sin\frac{\pi}{n} \tag{25.27}$$

or, in general,

$$-1 = j^2 = \cos\,(2m + 1)\pi + j \sin\,(2m + 1)\pi \tag{26.27}$$

$$\sqrt[n]{-1} = j^{\frac{2}{n}} = \cos\frac{(2m + 1)\pi}{n} + j \sin\frac{(2m + 1)\pi}{n} \tag{27.27}$$

Where m may be any positive or negative whole number, and n may be any positive or negative number, integral or fractional, constant or variable.

For given values let $\frac{\pi}{n} = \theta$.

$$\sqrt[\frac{\pi}{\theta}]{-1} = j^{\frac{2\theta}{\pi}} = \cos\theta + j\sin\theta \tag{28.27}$$

As a function of time let $\frac{\pi}{n} = \omega t$.

$$\sqrt[\frac{\pi}{\omega t}]{-1} = j^{\frac{2\omega t}{\pi}} = \cos(\omega t) + j\sin(\omega t) \tag{29.27}$$

Similarly, for $\frac{\pi}{n} = -\omega t$:

$$\sqrt[-\frac{\pi}{\omega t}]{-1} = j^{-\frac{2\omega t}{\pi}} = \cos(\omega t) - j\sin(\omega t) \tag{30.27}$$

The *product of two operators* indicates a rotation equal to the sum of the separate rotations.

Let

$$Q = \cos a + j\sin a, \tag{31.27}$$

and

$$V = \cos b + j\sin b. \tag{32.27}$$

$$\begin{aligned} QV &= (\cos a + j\sin a)(\cos b + j\sin b) \\ &= \cos a\cos b - \sin a\sin b + j(\sin a\cos b + \cos a\sin b) \\ &= \cos(a + b) + j\sin(a + b) \end{aligned} \tag{33.27}$$

The *quotient of two operators* indicates a rotation equal to the difference of the separate rotations.

$$\frac{Q}{V} = \frac{\cos a + j\sin a}{\cos b + j\sin b} \tag{34.27}$$

$$\begin{aligned} &= \frac{\cos a\cos b + \sin a\sin b + j(\sin a\cos b - \cos a\sin b)}{\cos^2 b + \sin^2 b} \\ &= \cos(a - b) + j\sin(a - b) \end{aligned} \tag{35.27}$$

When $Q = V$, their product becomes

$$(\cos a + j\sin a)^2 = \cos 2a + j\sin 2a \tag{36.27}$$

Stating De Moivre's theorem for n factors,

$$(\cos a + j\sin a)^n = \cos na + j\sin na \tag{37.27}$$

The exponential form is often a more convenient notation for expressing rotation of vectors than the trigonometric. By

expanding sin u, cos u and ϵ^u into series of the powers of u the relation between the two forms of notation is readily seen.

$$\sin u = \frac{u}{\underline{|1}} - \frac{u^3}{\underline{|3}} + \frac{u^5}{\underline{|5}} - \frac{u^7}{\underline{|7}} + \cdots (-1)^n \frac{u^{2n+1}}{\underline{|2n+1}} + \text{etc.} \qquad (38.27)$$

$$\cos u = 1 - \frac{u^2}{\underline{|2}} + \frac{u^4}{\underline{|4}} - \frac{u^6}{\underline{|6}} + \cdots (-1)^n \frac{u^{2n}}{\underline{|2n}} + \text{etc.} \qquad (39.27)$$

$$\epsilon^u = 1 + \frac{u}{\underline{|1}} + \frac{u^2}{\underline{|2}} + \frac{u^3}{\underline{|3}} + \frac{u^4}{\underline{|4}} + \cdots + \frac{u^n}{\underline{|n}} + \text{etc.} \qquad (40.27)$$

Substituting ju for u,

$$\epsilon^{ju} = 1 + j\frac{u}{\underline{|1}} - \frac{u^2}{\underline{|2}} - j\frac{u^3}{\underline{|3}} + \frac{u^4}{\underline{|4}} + \cdots + (j)^n \frac{u^n}{\underline{|n}} + \text{etc.} \qquad (41.27)$$

Therefore,

$$\epsilon^{ju} = \cos u + j \sin u \qquad (42.27)$$

Likewise,

$$\epsilon^{-ju} = \cos u - j \sin u \qquad (43.27)$$

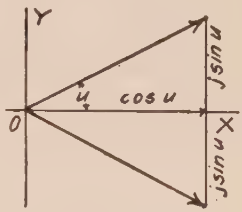


FIG. 5.27.

The factor ϵ^{ju}, therefore, indicates a rotation of $u°$ in the positive direction and ϵ^{-ju} a similar rotation of $u°$ in the negative direction. By adding and subtracting equations 42.27 and 43.27 or from the diagram in Fig. 5.27, we may obtain directly Euler's expressions for the sine and cosine.

$$\cos u = \frac{\epsilon^{ju} + \epsilon^{-ju}}{2} \text{ and } \sin u = \frac{\epsilon^{ju} - \epsilon^{-ju}}{2j} \qquad (44.27)$$

Vector rotation can, therefore, be indicated by several symbols or operators. A rotation of $\theta°$ in the positive direction is indicated by any one of the following equal operators:

$$\cos \theta + j \sin \theta = \epsilon^{j\theta} = \sqrt[\frac{\pi}{\theta}]{-1} = j^{\frac{2\theta}{\pi}} \qquad (45.27)$$

Similarly a rotation of $\theta°$ in the negative direction:

$$\cos \theta - j \sin \theta = \epsilon^{-j\theta} = \sqrt[-\frac{\pi}{\theta}]{-1} = j^{-\frac{2\theta}{\pi}} \qquad (46.27)$$

For a continuous rotation with ωt as the variable,

$$\cos(\omega t) + j\sin(\omega t) = \epsilon^{j(\omega t)} = \sqrt[\frac{\pi}{\omega t}]{-1} = j^{\frac{2\omega t}{\pi}} \tag{47.27}$$

$$\cos(\omega t) - j\sin(\omega t) = \epsilon^{-j(\omega t)} = \sqrt[-\frac{\pi}{\omega t}]{-1} = j^{-\frac{2\omega t}{\pi}} \tag{48.27}$$

(*c*) **Hyperbolic Sines and Cosines.**—If the problem involves vectors that vary in magnitude as well as in phase position the notation must be adjusted to comply with all the conditions. In long transmission lines, as explained in Chap. XXVIII, the voltage and current vectors differ both in magnitude and in time phase along the line. The quantitative relations may be expressed by exponential equations having both "real and imaginary terms" in the exponents. As stated in the preceding paragraph, the "imaginary" part of the exponent (containing the j factor) indicates angular displacement or relative angular position and may be expressed in trigonometric terms. The real part of the exponent represents a variation in the length of the vector and may be expressed in similar form by means of hyperbolic sines and cosines.

By comparing equations (44.27) and (49.27) it is seen that the exponential expressions for the sinh and cosh are strikingly similar to those of the sine and cosine.

$$\sinh u = \frac{\epsilon^{u} - \epsilon^{-u}}{2}; \quad \cosh u = \frac{\epsilon^{u} + \epsilon^{-u}}{2} \tag{49.27}$$

Solving for the exponential terms in equation (49.27),

$$\epsilon^{+u} = \cosh u + \sinh u \tag{50.27}$$
$$\epsilon^{-u} = \cosh u - \sinh u \tag{51.27}$$

It is often more convenient to develop the equations in the exponential form and then convert into trigonometric form when the numerical values must be found. Thus in Chap. XXVIII the expressions for the current and voltage are derived in the exponential form and then changed into trigonometric terms in order to use the tables of trigonometric functions when calculating the numerical results for any given transmission line.

(*d*) **Polar Expressions for Complex Quantities.**—As is noted in the preceding paragraph, the product of two operators indicates

a rotation equal to the sum of the separate rotations and the quotient indicates the difference of these rotations, etc. When dealing with products, quotients, powers and roots of complex quantities it is convenient to keep the expressions in polar form indicating the scalar length and the angle of rotation.

Let

$$\dot{C} = A + jB \tag{52.27}$$
$$= C\ (\cos\alpha + j\sin\alpha) \tag{53.27}$$
$$= C\ (\cos\tan^{-1} B/A + j\sin\tan^{-1} B/A) \tag{54.27}$$
$$= C/\underline{\tan^{-1} B/A} \tag{55.27}$$
$$= C/\underline{\alpha} \tag{56.27}$$

where C is the scalar value and α or $\tan^{-1} B/A$ is the angle of rotation.

Likewise let

$$\dot{F} = D + jE \tag{57.27}$$
$$= F\ (\cos\beta + j\sin\beta) \tag{58.27}$$
$$= F\ (\cos\tan^{-1} E/D + j\sin\tan^{-1} E/D) \tag{59.27}$$
$$= F/\underline{\tan^{-1} E/D} \tag{60.27}$$
$$= F/\underline{\beta} \tag{61.27}$$

The following are some of the operations easily performed with complex quantities in this form:

$$\dot{C}\dot{F} = CF/\underline{\alpha + \beta} \tag{62.27}$$
$$\dot{C}/\dot{F} = C/F/\underline{\alpha - \beta} \tag{63.27}$$
$$\dot{C}^2 = C^2/\underline{2\alpha} \tag{64.27}$$
$$\sqrt{\dot{C}} = \sqrt{C}/\underline{\alpha/2} \tag{65.27}$$
$$\dot{C}^n = C^n/\underline{n\alpha} \tag{66.27}$$
$$1/\dot{C} = 1/C/\underline{-\alpha} = 1/C/\underline{2\pi - \alpha} \tag{67.27}$$
$$\sqrt[n]{\dot{C}/\dot{F}} = \sqrt[n]{C/F}/\underline{(\alpha - \beta)/n} \tag{68.27}$$
$$(\dot{C}\dot{F})^n = C^nF^n/\underline{n(\alpha + \beta)} \tag{69.27}$$

A numerical example is given in the calculations for Line C, Chap. XXVIII.

PROBLEMS

1.27. Let eight units of the artificial transmission line described on page 671 be so adjusted that each one has 2.6-ohm resistance, 0.021-henry inductance and 0.5-mf. condensance. Let E_R = 1,500 volts; I_R = 10 amp.; f = 60 cycles; θ_R = 14° inductive load; circuit similar to Fig. 1.27. Find $\dot{E}_g$, $\dot{I}_g$ and θ_g graphically by plotting a topographic vector diagram.

2.27. Given the same circuit constants as in problem 1.27. Let $\dot{E}_R$ = 1,500 volts; $\dot{I}_R = 8 + j6$ amp.; f = 300 cycles. Compute θ_R. Find $\dot{E}_g$, $\dot{I}_g$ and θ_g graphically as in problem 1.27.

3.27. Rewrite in the notation shown in equations (45.27) and (46.27) the values of $\dot{I}_1$, $\dot{I}_2$ and $\dot{I}_0$ in problems 17.7, 22.7 and 24.7, Chap. VII.

4.27. Write in polar notation the equivalent values of $\dot{E}$, $\dot{I}$ and $\dot{E}\dot{I}$ for

(*a*) $\dot{E} = 20 + j15$ volts.
$\dot{I} = 5 + j18$ amp.
(*b*) $\dot{E} = 120 - j20$ volts.
$\dot{I} = 22 + j32$ amp.
(*c*) $\dot{E} = 84 + j12$ volts.
$\dot{I} = 7 - j3$ amp.

CHAPTER XXVIII

LONG TRANSMISSION LINES[1]

In transmission lines the condensance, inductance, resistance and leakage are uniformly distributed along the line; or the line may be considered as equivalent to an infinite number of infinitesimal series and parallel circuits uniformly distributed along its whole length. Each infinitesimal length of conductor may be represented by one of the series circuits in Fig. 1.28 and the corresponding infinitesimal portion of the dielectric between the line elements by a parallel circuit. The resistance and the inductance of the conductor element affect the voltage in proportion to the current flowing at that point. Similarly, the

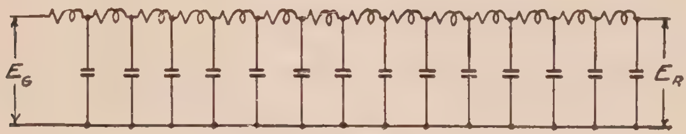


FIG. 1.28.

conductance and the susceptance in the elemental dielectric circuit affect the current in proportion to the voltage at the point selected. The relative phase positions of the voltages at successive points along the line differ in proportion to the resistance and the inductive reactance in each elemental series circuit. Likewise, the phase positions of the currents at successive points along the line vary in proportion to the conductance and susceptance of the parallel circuits. As a result, both the current and the voltage change continuously in magnitude and phase position along the line.

[1] A full discussion of transmission-line design is found in the following bulletins of the Engineering Experiment Station, University of Washington: KIRSTEN, F. K., "Mechanical Features. Supports at Equal Elevation," *Bull.* 17; SMITH, G. S., "Mechanical Features. Supports at Unequal Elevation," *Bull.* 29; KIRSTEN, F. K. and LOEW, E. A., "Electrical Features. The Line of Maximum Economy," *Bull.* 32; KIRSTEN, F. K., and BRIGGS, C. M., "A 500-mile Transmission Line," *Bull.* 44; LOEW, E. A., "Choice of line voltage and conductor size," *Bull.* 50.

(*a*) **Development of Transmission-line Equations.**—In Fig. 2.28 is shown an elemental vector diagram of the currents and voltages for two points at a distance *dl* apart, along the line, with *OX* and *OY* as reference axes. In the original development of the general equations, Dr. C. P. Steinmetz used the polar-vector-diagram notation, and the same system has generally been used in the technical literature on long transmission lines. To facilitate comparison and to give the student an illustration of the reversal of signs in the complex equations, the polar vector diagram and the corresponding complex equations are used in this chapter. Therefore, the symbol for inductive reactance is $-jx$ and for condensive susceptance $-jb$.

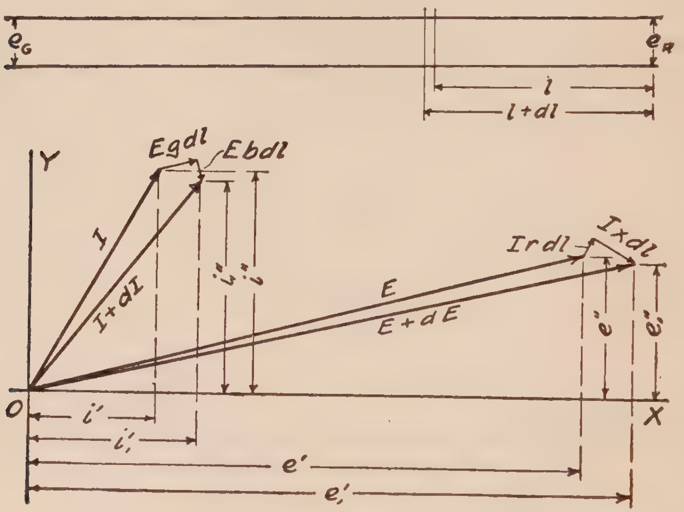


FIG. 2.28.

For unit length of line, let:

r = resistance, x = reactance } for electric and magnetic (series) circuits.
g = conductance, b = susceptance } for leakage and dielectric (parallel or shunted) circuits.

At the distance l from the receiver end of the line:

$$\dot{E} = e' + je''; \; \dot{I} = i' + ji'' \tag{1.28}$$

At the distance $l + dl$ from the receiver end of the line:

$$\dot{E} + d\dot{E} = e_1' + je_1''; \; \dot{I} + d\dot{I} = i_1' + ji_1'' \tag{2.28}$$

From the vector diagram in Fig. 2.28,

$$\dot{E} + d\dot{E} = e' + i'rdl + i''xdl + j(e'' + i''rdl - i'xdl) \quad (3.28)$$

$$d\dot{E} = i'rdl + i''xdl + j(i''rdl - i'xdl) \quad (4.28)$$

$$\frac{d\dot{E}}{dl} = i'r + i''x + ji''r - ji'x$$

$$= (i' + ji'')(r - jx) \quad (5.28)$$

$$\frac{d\dot{E}}{dl} = \dot{I}Z \quad (6.28)$$

Similarly,

$$\dot{I} + d\dot{I} = i' + e'gdl + e''bdl + j(i'' + e''gdl - e'bdl) \quad (7.28)$$

$$\frac{d\dot{I}}{dl} = \dot{E}Y \quad (8.28)$$

Differentiating equation (8.28) with respect to l:

$$\frac{d^2\dot{I}}{dl^2} = Y\frac{d\dot{E}}{dl} \quad (9.28)$$

Substituting equation (6.28) in equation (9.28):

$$\frac{d^2\dot{I}}{dl^2} = YZ\dot{I} \quad (10.28)$$

Multiplying through by $2\frac{d\dot{I}}{dl}$:

$$2\frac{d\dot{I}}{dl}\frac{d^2\dot{I}}{dl^2} = 2YZ\dot{I}\frac{d\dot{I}}{dl} \quad (11.28)$$

Integrating:

$$\left(\frac{d\dot{I}}{dl}\right)^2 = YZ\dot{I}^2 + \dot{K}' \text{ or} \quad (12.28)$$

$$\left(\frac{d\dot{I}}{dl}\right)^2 = YZ(\dot{I}^2 + \dot{K}_1^2) \quad (13.28)$$

$$\frac{d\dot{I}}{dl} = (YZ)^{1/2}(\dot{I}^2 + \dot{K}_1^2)^{1/2} \quad (14.28)$$

$$\frac{d\dot{I}}{(\dot{I}^2 + \dot{K}_1^2)^{1/2}} = (YZ)^{1/2}dl \quad (15.28)$$

Integrating:

$$\log_\epsilon[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] + \dot{K}'' = (YZ)^{1/2}l$$

or

$$\log_\epsilon[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] + \log_\epsilon \dot{K}_2 = (YZ)^{1/2}l \quad (16.28)$$

$$\log_\epsilon \dot{K}_2[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] = (YZ)^{1/2}l \quad (17.28)$$

$$\epsilon^{(YZ)^{1/2}l} = \dot{K}_2[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] \tag{18.28}$$

$$\frac{\epsilon^{(YZ)^{1/2}l}}{\dot{K}_2} - \dot{I} = (\dot{I}^2 + \dot{K}_1^2)^{1/2} \tag{19.28}$$

$$\frac{\epsilon^{2(YZ)^{1/2}l}}{\dot{K}_2^2} - \frac{2\dot{I}\epsilon^{(YZ)^{1/2}l}}{\dot{K}_2} = \dot{K}_1^2 \tag{20.28}$$

$$\dot{I} = \frac{1}{2}\left[\frac{\epsilon^{(YZ)^{1/2}l}}{\dot{K}_2} - \frac{\dot{K}_1^2\dot{K}_2}{\epsilon^{(YZ)^{1/2}l}}\right] \tag{21.28}$$

$$\dot{I} = \dot{A}_1\epsilon^{(YZ)^{1/2}l} - \dot{A}_2\epsilon^{-(YZ)^{1/2}l} \tag{22.28}$$

Where

$$\dot{A}_1 = \frac{1}{2\dot{K}_2} \text{ and } \dot{A}_2 = \frac{K_1^2K_2}{2}$$

From equation (8.28):

$$\dot{E} = \frac{1}{Y}\frac{d\dot{I}}{dl} \tag{23.28}$$

Differentiating equation (22.28) with respect to l,

$$\frac{d\dot{I}}{dl} = (YZ)^{1/2}(\dot{A}_1\epsilon^{(YZ)^{1/2}l} + \dot{A}_2\epsilon^{-(YZ)^{1/2}l}) \tag{24.28}$$

Substituting equation (24.28) in (23.28):

$$\dot{E} = \frac{(YZ)^{1/2}}{Y}(\dot{A}_1\epsilon^{(YZ)^{1/2}l} + \dot{A}_2\epsilon^{-(YZ)^{1/2}l}) \tag{25.28}$$

Equations (22.28) and (25.28) give the current and voltage at any point on the line. $\dot{A}_1$ and $\dot{A}_2$ are integration constants.

Equations (22.28) and (25.28) may now be made useful for actual computation of electrical line performances as follows:

The square root of the product of two vectors is also a vector:

$$(YZ)^{1/2} = \alpha - j\beta = [(r - jx)(g - jb)]^{1/2} \tag{26.28}$$

$$(\alpha - j\beta)^2 = (r - jx)(g - jb) \tag{27.28}$$

$$\alpha^2 - \beta^2 - j2\alpha\beta = rg - xb - j(rb + xg) \tag{28.28}$$

From equation (28.28) it is evident that:

$$\alpha^2 - \beta^2 = rg - xb, \text{ and } 2\alpha\beta = rb + xg \tag{29.28}$$

Squaring equations (29.28):

$$\alpha^4 + \beta^4 - 2\alpha^2\beta^2 = r^2g^2 + x^2b^2 - 2rxgb \tag{30.28}$$

$$4\alpha^2\beta^2 = r^2b^2 + x^2g^2 + 2rxgb \tag{31.28}$$

Adding equations (30.28) and (31.28):

$$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = r^2g^2 + x^2b^2 + r^2b^2 + x^2g^2 \qquad (32.28)$$
$$(\alpha^2 + \beta^2)^2 = (r^2 + x^2)(g^2 + b^2) \qquad (33.28)$$
$$= z^2y^2$$
$$\alpha^2 + \beta^2 = zy \qquad (34.28)$$

From equations (29.28) and (34.28):

$$\alpha = [\tfrac{1}{2}(zy + rg - xb)]^{\frac{1}{2}} \qquad (35.28)$$
$$\beta = [\tfrac{1}{2}(zy - rg + xb)]^{\frac{1}{2}} \qquad (36.28)$$

Equations (22.28) and (25.28) now assume the forms:

$$\dot{I} = \dot{A}_1\epsilon^{+(\alpha-j\beta)l} - \dot{A}_2\epsilon^{-(\alpha-j\beta)l} \qquad (37.28)$$
$$\dot{E} = \frac{\alpha - j\beta}{Y}[\dot{A}_1\epsilon^{+(\alpha-j\beta)l} + \dot{A}_2\epsilon^{-(\alpha-j\beta)l}] \qquad (38.28)$$

Substituting for the exponential function with the j factor exponent the trigonometric expression:

$$\epsilon^{\pm j\beta l} = \cos \beta l \pm j \sin \beta l \qquad (39.28)$$

Equations (37.28) and (38.28) change in form to:

$$\dot{I} = \dot{A}_1\epsilon^{+\alpha l}(\cos \beta l - j \sin \beta l) - \dot{A}_2\epsilon^{-\alpha l}(\cos \beta l + j \sin \beta l) \qquad (40.28)$$
$$\dot{E} = \frac{\alpha - j\beta}{Y}[\dot{A}_1\epsilon^{+\alpha l}(\cos \beta l - j \sin \beta l)$$
$$+ \dot{A}_2\epsilon^{-\alpha l}(\cos \beta l + j \sin \beta l)] \qquad (41.28)$$

Choosing as unit length the entire line $l = 0$, gives $\dot{I}_R$ and $\dot{E}_R$ at the receiver end; and $l = 1$ gives $\dot{I}_G$ and $\dot{E}_G$ at the generator end of the line, and Z is the total impedance, Y the total shunted admittance of the entire line.

Neglecting the shunted conductance, that is, assuming the power consumed by leakage and dielectric losses in the line as negligible compared with that consumed by the effective line resistance r—as is always done for low frequencies—gives from equations (35.28) and (36.28):

$$\alpha = [\tfrac{1}{2}b(z - x)]^{\frac{1}{2}} \qquad (42.28)$$
$$\beta = [\tfrac{1}{2}b(z + x)]^{\frac{1}{2}} \qquad (43.28)$$

and

$$Y = -jb \qquad (44.28)$$

Substituting $l = 1$ in equations (40.28) and (41.28):

$$\dot{I}_G = \dot{A}_1\epsilon^{+\alpha}(\cos\beta - j\sin\beta) - \dot{A}_2\epsilon^{-\alpha}(\cos\beta + j\sin\beta)$$
$$\dot{E}_G = \frac{\alpha - j\beta}{-jb}[\dot{A}_1\epsilon^{+\alpha}(\cos\beta - j\sin\beta) + \dot{A}_2\epsilon^{-\alpha}(\cos\beta + j\sin\beta)] \tag{45.28}$$

Substituting $l = 0$ in equations (40.28) and (41.28):

$$\dot{I}_R = \dot{A}_1 - \dot{A}_2$$
$$\dot{E}_R = \frac{\alpha - j\beta}{-jb}(\dot{A}_1 + \dot{A}_2) \tag{46.28}$$

Solving equations (46.28) for $\dot{A}_1$ and $\dot{A}_2$:

$$\dot{A}_1 = \frac{1}{2}\left(\dot{I}_R - \frac{jb}{\alpha - j\beta}\dot{E}_R\right)$$
$$\dot{A}_2 = -\frac{1}{2}\left(\dot{I}_R + \frac{jb}{\alpha - j\beta}\dot{E}_R\right) \tag{47.28}$$

Substituting equations (47.28) in equations (45.28):

$$\dot{I}_G = \frac{1}{2}\left[\left(\dot{I}_R - \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{+\alpha}(\cos\beta - j\sin\beta) + \left(\dot{I}_R + \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{-\alpha}(\cos\beta + j\sin\beta)\right]$$
$$\dot{E}_G = \frac{\alpha - j\beta}{-jb}\frac{1}{2}\left[\left(\dot{I}_R - \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{+\alpha}(\cos\beta - j\sin\beta) - \left(\dot{I}_R + \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{-\alpha}(\cos\beta + j\sin\beta)\right] \tag{48.28}$$

Since α is a small quantity even in very long lines, because in equations (42.28) and (43.28) b is small at 25 cycles, while $z - x$ is small at 60 cycles, $\epsilon^{\pm\alpha}$ may be replaced by the first terms of its series:

$$\epsilon^{\pm\alpha} = 1 \pm \alpha + \frac{\alpha^2}{2} \pm \frac{\alpha^3}{6} + \frac{\alpha^4}{24} \pm \cdots \tag{49.28}$$

that is, by

$$\epsilon^{\pm\alpha} = 1 \pm \alpha \tag{50.28}$$

or, if greater accuracy is desired, by

$$\epsilon^{\pm\alpha} = 1 \pm \alpha + \frac{\alpha^2}{2} \tag{51.28}$$

with a possible error of $\frac{\alpha^3}{6}$.

Substituting equation (51.28) in equations (48.28) and separating $\dot{I}_R$ and $\dot{E}_R$ terms:

$$\dot{I}_G = \frac{1}{2}\left\{\dot{I}_R\left(\left[1 + \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right] + \left[1 - \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta + j\sin\beta\right]\right) + \dot{E}_R\frac{jb}{\alpha - j\beta}\left(\left[1 - \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta + j\sin\beta\right] - \left[1 + \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right]\right)\right\}$$

$$\dot{E}_G = \frac{1}{2}\left\{\dot{E}_R\left(\left[1 + \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right] + \left[1 - \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta + j\sin\beta\right]\right) + \dot{I}_R\frac{\alpha - j\beta}{-jb}\left(\left[1 + \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right] - \left[1 - \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta + j\sin\beta\right]\right)\right\} \quad (52.28)$$

Simplifying coefficients of $\dot{I}_R$ and $\dot{E}_R$:

$$\left[1 + \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right] = \cos\beta + \alpha\cos\beta + \frac{\alpha^2}{2}\cos\beta - j\sin\beta - j\alpha\sin\beta - j\frac{\alpha^2}{2}\sin\beta \quad (53.28)$$

$$\left[1 - \alpha + \frac{\alpha^2}{2}\right]\left[\cos\beta + j\sin\beta\right] = \cos\beta - \alpha\cos\beta + \frac{\alpha^2}{2}\cos\beta + j\sin\beta - j\alpha\sin\beta + j\frac{\alpha^2}{2}\sin\beta \quad (54.28)$$

Adding equations (53.28) and (54.28) gives:

$$2\cos\beta + \alpha^2\cos\beta - j2\alpha\sin\beta$$

or

$$2\left[\left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta\right] \quad (55.28)$$

Subtracting equation (54.28) from equation (53.28) gives:

$$-2\alpha\cos\beta + j2\sin\beta + j2\frac{\alpha^2}{2}\sin\beta$$

or

$$2\left[-\alpha\cos\beta + j\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] \quad (56.28)$$

Subtracting equation (54.28) from equation (53.28) gives:

$$2\alpha\cos\beta - j2\sin\beta - j2\frac{\alpha^2}{2}\sin\beta$$

or

$$2\left[\alpha\cos\beta - j\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] \quad (57.28)$$

Substituting equations (55.28), (56.28) and (57.28) in equations (52.28).

$$\dot{I}_G = \dot{I}_R\left[\left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta\right] + \dot{E}_R\frac{jb}{\alpha - j\beta}\left[-\alpha\cos\beta + j\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] \quad (58.28)$$

$$\dot{E}_G = \dot{E}_R\left[\left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta\right] + \dot{I}_R\frac{\alpha - j\beta}{-jb}\left[\alpha\cos\beta - j\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] \quad (59.28)$$

Rationalizing the denominators of the fractions:

$$\frac{jb}{\alpha - j\beta} = \frac{(\alpha + j\beta)jb}{\alpha^2 + \beta^2} = \frac{(\alpha + j\beta)jb}{bz} = -\frac{\beta - j\alpha}{z}$$

and

$$\frac{\alpha - j\beta}{-jb} = \frac{j(\alpha - j\beta)}{b} = \frac{\beta + j\alpha}{b} \quad (60.28)$$

Substituting equations (60.28) in equations (58.28) and (59.28) and separating j terms:

$$\dot{I}_G = \dot{I}_R\left[\left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta\right] + \dot{E}_R\left\{\frac{1}{z}\left[\alpha\beta\cos\beta - \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{z}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right]\right\} \quad (61.28)$$

$$\dot{E}_G = \dot{E}_R\left[\left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta\right] + \dot{I}_R\left\{\frac{1}{b}\left[\alpha\beta\cos\beta + \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{b}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta - \alpha^2\cos\beta\right]\right\} \quad (62.28)$$

The coefficients of $\dot{I}_R$ in equation (61.28) and of $\dot{E}_R$ in equation (62.28) may be expressed by $a_1 - ja_2$, the coefficient of $\dot{E}_R$ in equation (61.28) as $c_1 - jc_2$ and the coefficient of $\dot{I}_R$ in equation (62.28) by $d_1 - jd_2$, and, finally, expressing $\dot{I}_R$ by $i'_R + ji''_R$ and $\dot{E}_R$ by $e'_R + je''_R$, equations (61.28) and (62.28) become:

$$\dot{I}_G = (i'_R + ji''_R)(a_1 - ja_2) + (e'_R + je''_R)(c_1 - jc_2)$$
$$\dot{E}_G = (e'_R + je''_R)(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2) \quad (63.28)$$

If in equations (40.28) and (41.28) the sign of l is reversed, the distance along the line is measured in the opposite direction; that is, $l = 0$ gives generator voltage and current $\dot{E}_G$ and $\dot{I}_G$, $l = -1$ gives receiver voltage and current $\dot{E}_R$ and $\dot{I}_R$. Remembering that $\cos(-\beta) = +\cos\beta$ and $\sin(-\beta) = -\sin\beta$, the only change

resulting from this substitution is a reversal of the sign between the two products in equations (63.28):

$$\begin{aligned}\dot{I}_R &= (i'_G + ji''_R)(a_1 - ja_2) - (e'_G + je''_G)(c_1 - jc_2)\\ \dot{E}_R &= (e'_G + je''_G)(a_1 - ja_2) - (i'_G + ji''_G)(d_1 - jd_2) \quad (64.28)\end{aligned}$$

Summary.—With known current, voltage and power factor at the receiver end of the line and choosing the time when $e''_R = 0$, the current and voltage vectors at the generator end are found from equations:

$$\begin{aligned}\dot{I}_G &= i'_G + ji''_G = (i'_R + ji''_R)(a_1 - ja_2) + e'_R(c_1 - jc_2)\\ \dot{E}_G &= e'_G + je''_G = e'_R(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2) \quad (65.28)\end{aligned}$$

where

$$\left.\begin{aligned} e'_R &= \text{voltage}\\ i'_R &= \text{power current}\\ i''_R &= \text{reactive current}\end{aligned}\right\} \text{at receiver end of line.}$$

With known current, voltage and power factor at the generator end of the line, the current and voltage vectors at the receiver end are found from equations:

$$\begin{aligned}\dot{I}_R &= i'_R + ji''_R = (i'_G + ji''_G)(a_1 - ja_2) - e'_G(c_1 - jc_2)\\ \dot{E}_R &= e'_R + je''_R = e'_G(a_1 - ja_2) - (i'_G + i''_G)(d_1 - jd_2) \quad (66.28)\end{aligned}$$

where

$$\left.\begin{aligned} e'_G &= \text{voltage}\\ i'_G &= \text{power current}\\ i''_G &= \text{reactive current}\end{aligned}\right\} \text{at generator end of line.}$$

The factors $a_1 - ja_2$, $c_1 - jc_2$ and $d_1 - jd_2$ are line constants, which, when once computed, give the performance of the line, in equations (65.28) for all load conditions at the receiver end and in equations (66.28) for all conditions under which power may be sent into the line at the generator end. These constants are:

$$a_1 - ja_2 = \left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta \quad (67.28)$$

$$\begin{aligned}c_1 - jc_2 = \frac{1}{z}&\left[\alpha\beta\cos\beta - \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right]\\ &- j\frac{1}{z}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right] \quad (68.28)\end{aligned}$$

$$\begin{aligned}d_1 - jd_2 = \frac{1}{b}&\left[\alpha\beta\cos\beta + \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right]\\ &- j\frac{1}{b}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta - \alpha^2\cos\beta\right] \quad (69.28)\end{aligned}$$

where

$\alpha = [\frac{1}{2}b(z - x)]^{\frac{1}{2}}$.

$\beta = [\frac{1}{2}b(z + x)]^{\frac{1}{2}}$, in radians.

β in degrees $= 57.3\beta$.

$$\left.\begin{aligned} z &= [r^2 + x^2]^{\frac{1}{2}} = \text{impedance} \\ b &= 2\pi fC = \text{shunted susceptance} \\ x &= 2\pi fL = \text{reactance} \\ r &= \text{resistance} \\ L &= \text{inductance} \\ C &= \text{condensance} \\ f &= \text{frequency} \end{aligned}\right\} \text{of entire line.}$$

Possible error of calculation is $\frac{\alpha^3}{6}$.

After the line constants have been computed and with the current, voltage and the power factor given at one end of the line, the corresponding values at the other end of the line may be calculated by means of equations (65.28) and (66.28). For convenience, the order for making the computations is given in tabular form.

GIVEN DATA

Let the current, voltage and power factor be given at the:		
Receiver end	Generator end	
For computing data at generator end	For computing data at receiver end	
$i_G' + ji_G''$	$i_R' + ji_R''$	Current vector
$e_G' + je_G''$	$e_R' + je_R''$	Voltage vector
$I_G = [(i_G')^2 + (i_G'')^2]^{\frac{1}{2}}$	$I_R = [(i_R')^2 + (i_R'')^2]^{\frac{1}{2}}$	Current
$E_G = [(e_G')^2 + (e_G'')^2]^{\frac{1}{2}}$	$E_R = [(e_R')^2 + (e_R'')^2]^{\frac{1}{2}}$	Voltage
$P_G = e_G'i_G' + e_G''i_G''$	$P_R = e_R'i_R' + e_R''i_R''$	Actual power
$Q_G = E_GI_G$	$Q_R = E_RI_R$	Volt-amperes
$\cos\theta_G = P_G \div Q_G$	$\cos\theta_R = P_R \div Q_R$	Power factor
$P_G - P_R$	$P_G - P_R$	Line loss

(b) **Transmission-line Equations with Hyperbolic Functions.**—Using hyperbolic functions, the approximation resulting from

the substitution of the first terms of its series for $\epsilon^{\pm\alpha}$ may be avoided.

$$\sinh \alpha = \frac{\epsilon^{\alpha} - \epsilon^{-\alpha}}{2}; \cosh \alpha = \frac{\epsilon^{\alpha} + \epsilon^{-\alpha}}{2} \tag{70.28}$$

Solving equation (70.28) for ϵ^{α} and $\epsilon^{-\alpha}$ gives:

$$\begin{aligned} \epsilon^{+\alpha} &= \cosh \alpha + \sinh \alpha \\ \epsilon^{-\alpha} &= \cosh \alpha - \sinh \alpha \end{aligned} \tag{71.28}$$

Substituting equations (71.28) in equations (48.28),

$$\begin{aligned} \dot{I}_G = \tfrac{1}{2}\{&\dot{I}_R([\cosh \alpha + \sinh \alpha][\cos \beta - j \sin \beta] - [\cosh \alpha - \sinh \alpha] \\ &[\cos \beta + j \sin \beta]) + \dot{E}_R \frac{jb}{\alpha - j\beta}([\cosh \alpha - \sinh \alpha][\cos \beta + j \sin \beta] \\ &- [\cosh \alpha + \sinh \alpha][\cos \beta - j \sin \beta])\} \end{aligned} \tag{72.28}$$

$$\begin{aligned} \dot{E}_G = \tfrac{1}{2}\{&\dot{E}_R([\cosh \alpha + \sinh \alpha][\cos \beta - j \sin \beta] + [\cosh \alpha - \sinh \alpha] \\ &[\cos \beta + j \sin \beta]) + \dot{I}_R \frac{\alpha - j\beta}{-jb}([\cosh \alpha + \sinh \alpha][\cos \beta - j \sin \beta] \\ &- [\cosh \alpha \sinh \alpha][\cos \beta + j \sin \beta])\} \end{aligned} \tag{73.28}$$

Simplifying:

$$\begin{aligned} \dot{I}_G &= i_G' + ji_G'' = (i_R' + ji_R'')(a_1' - ja_2') + (e_R' + je_R'')(c_1 - jc_2') \\ \dot{E}_G &= e_G' + je_G'' = (e_R' + jc_R'')(a_1' - ja_2') + (i_R' + ji_R'') \\ &\qquad (d_1' - jd_2') \end{aligned} \tag{74.28}$$

Following the same reasoning as that for equations (64.28):

$$\begin{aligned} \dot{I}_R &= i_R' + ji_R'' = (i_G' + ji_G'')(a_1' - ja_2') - (e_G' + je_G'')(c_1' - jc_2') \\ \dot{E}_R &= e_R' + je_R'' = (e_G' + je_G'')(a_1' - ja_2') - (i_G' + ji_G'') \\ &\qquad (d_1' - jd_2') \end{aligned} \tag{75.28}$$

where in both equations (74.28) and (75.28):

$$a_1' - ja_2' = \cosh \alpha \cos \beta - j \sinh \alpha \sin \beta, \tag{76.28}$$

$$c_1' - jc_2' = \frac{-jb}{\alpha - j\beta}[\sinh \alpha \cos \beta - j \cosh \alpha \sin \beta] \tag{77.28}$$

$$d_1' - jd_2' = \frac{\alpha - j\beta}{-jb}[\sinh \alpha \cos \beta - j \cosh \alpha \sin \beta] \tag{78.28}$$

These constants correspond to those used in equations (65.28) and (66.28) and should be used in their stead if greatest possible accuracy is desired.

Constants α and β are the same as for equations (65.28) and (66.28).

These equations are equivalent to the following expressions and may be used in this form if tables of complex hyperbolic functions are available.

$$a_1' - ja_2' = \cosh(\alpha - j\beta) \tag{79.28}$$

$$c_1' - jc_2' = \frac{-jb}{\alpha - j\beta}\sinh(\alpha - j\beta) = \sqrt{\frac{Y}{Z}}\sinh(\alpha - j\beta) \tag{80.28}$$

$$d_1' - jd_2' = \frac{\alpha - j\beta}{-jb}\sinh(\alpha - j\beta) = \sqrt{\frac{Z}{Y}}\sinh(\alpha - j\beta) \tag{81.28}$$

It will be found convenient to keep the components of these expressions in the form illustrated in the problem on line C.

(c) **Transmission-line Equations Approximated for Preliminary Computation.**—Substituting $l = 1$ in equations (22.28) and (25.28):

$$\begin{aligned} \dot{I}_G &= \dot{A}_1\epsilon^{(YZ)^{1/2}} - \dot{A}_2\epsilon^{-(YZ)^{1/2}} \\ \dot{E}_G &= \frac{(YZ)^{1/2}}{Y}[\dot{A}_1\epsilon^{(ZY)^{1/2}} + \dot{A}_2\epsilon^{-(YZ)^{1/2}}] \end{aligned} \tag{82.28}$$

Substituting $l = 0$ in equations (22.28) and (25.28):

$$\begin{aligned} \dot{I}_R &= \dot{A}_1 - \dot{A}_2 \\ \dot{E}_R &= \frac{(YZ)^{1/2}}{Y}[\dot{A}_1 + \dot{A}_2] \end{aligned} \tag{83.28}$$

Solving equations (25.28) for $\dot{A}_1$ and $\dot{A}_2$:

$$\begin{aligned} \dot{A}_1 &= \frac{1}{2}\left[\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right] \\ \dot{A}_2 &= -\frac{1}{2}\left[\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right] \end{aligned} \tag{84.28}$$

Substituting equations (84.28) in equations (82.28):

$$\begin{aligned} \dot{I}_G &= \frac{1}{2}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{(YZ)^{1/2}} + \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{-(YZ)^{1/2}}\right] \\ \dot{E}_G &= \frac{1}{2}\frac{(YZ)^{1/2}}{Y}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{(YZ)^{1/2}} \right. \\ &\qquad \left. - \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{-(YZ)^{1/2}}\right] \end{aligned} \tag{85.28}$$

In the converging series of:

$$\epsilon^{\pm(YZ)^{1/2}} = 1 \pm (YZ)^{1/2} + \frac{YZ}{2} \pm \frac{(YZ)^{3/2}}{6} + \frac{(YZ)^2}{24} \pm \cdots \tag{86.28}$$

the first four terms may be used to represent $\epsilon^{\pm(YZ)^{1/2}}$ with sufficient accuracy:

$$\begin{aligned} \dot{I}_G = \frac{1}{2}\Bigg[&\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\left(1 + (YZ)^{1/2} + \frac{YZ}{2} + \frac{(YZ)^{3/2}}{6}\right) \\ &+ \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\left(1 - (YZ)^{1/2} + \frac{YZ}{2} - \frac{(YZ)^{3/2}}{6}\right)\Bigg] \end{aligned}$$

$$\dot{E}_G = \frac{1}{2}\frac{(YZ)^{1/2}}{Y}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\left(1 + (YZ)^{1/2} + \frac{YZ}{2} + \frac{(YZ)^{3/2}}{6}\right) - \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\left(1 - (YZ)^{1/2} + \frac{YZ}{2} - \frac{(YZ)^{3/2}}{6}\right)\right] \quad (87.28)$$

Simplifying:

$$\dot{I}_G = \dot{I}_R\left(1 + \frac{YZ}{2}\right) + \dot{E}_R Y\left(1 + \frac{YZ}{6}\right)$$
$$\dot{E}_G = \dot{E}_R\left(1 + \frac{YZ}{2}\right) + \dot{I}_R Z\left(1 + \frac{YZ}{6}\right) \quad (88.28)$$

Since $Y = -jb$, and $Z = r - jx$,

$$\dot{I}_G = \dot{I}_R\left[1 - \frac{b}{2}x - j\frac{b}{2}r\right] + \dot{E}_R\left[-\frac{b^2}{6}r - j\left(b - \frac{b^2}{6}x\right)\right]$$
$$\dot{E}_G = \dot{E}_R\left[1 - \frac{b}{2}x - j\frac{b}{2}r\right] + \dot{I}_R\left[r\left(1 - \frac{b}{3}x\right) - j\left(x + \frac{b}{6}[r^2 - x^2]\right)\right] \quad (89.28)$$

Final equations:

$$\dot{I}_G = i_G' + ji_G'' = (i_R' + ji_R'')(a_1'' - ja_2'') + e_R'(c_1'' - jc_2'')$$
$$\dot{E}_G = e_G' + je_G'' = e_R'(a_1'' - ja_2'') + (i_R' - ji_R'')(d_1'' - jd_2'') \quad (90.28)$$

Measuring distance from generator end:

$$\dot{I}_R = i_R' + ji_R'' = (i_G' + ji_G'')(a_1'' - ja_2'') - e_G'(c_1'' - jc_2'') \quad (91.28)$$
$$\dot{E}_R = e_R' + je_R'' = e_G'(a_1'' - ja_2'') - (i_G' + ji_G'')(d_1'' - jd_2'')$$

where for both equations (90.28) and (91.28):

$$a_1'' - ja_2'' = 1 - \frac{b}{2}x - j\frac{b}{2}r$$
$$c_1'' - jc_2'' = -\frac{b^2}{6}r - j\left(b - \frac{b^2}{6}x\right)$$
$$d_1'' = jd_2'' = r\left(1 - \frac{b}{3}x\right) - j\left(x + \frac{b}{2}[r^2 - x^2]\right)$$

In the above constants

$$\left.\begin{aligned} b &= 2\pi fC = \text{shunted susceptance} \\ x &= 2\pi fL = \text{reactance} \\ C &= \text{condensance} \\ L &= \text{inductance} \\ f &= \text{frequency} \end{aligned}\right\} \text{of entire line.}$$

In case it is desired to find the operating conditions at the low-tension busses on both ends of the line, the equations must include the constants of the transformers. Sufficiently accurate results may be obtained by adding the equivalent reactance and resistance of the transformers to the reactance and resistance, respectively, of the line. This procedure would, of course, distribute the transformer reactance and resistance uniformly along the whole line, whereas for greatest accuracy they should be treated as concentrated on both ends.

The actual low bus potential and current are obtained from the known ratios of the transformers.

(*d*) **Arrangement of Conductors.**—The economy in copper of the three-phase system for long transmission lines was shown in

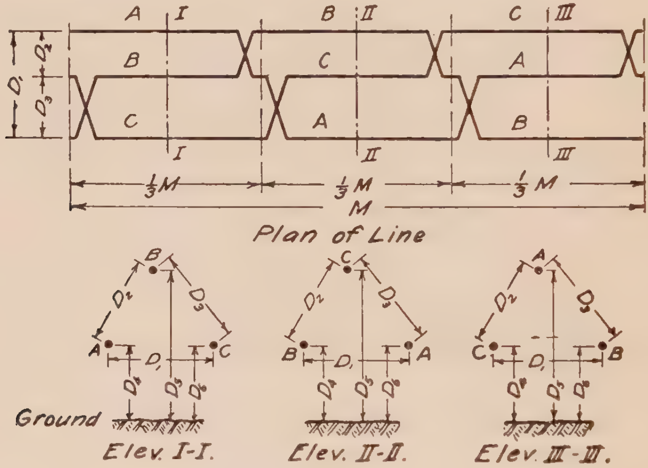


Fig. 3.28.

Chap. XXVI. In America this system is so generally used that no other need be considered. Assuming a balanced, three-phase system, with the corresponding neutral as explained in Chap. XXVI, letting e_G and e_R in the line equations represent the voltages to neutral and using in the line constants the inductance and condensance to neutral, the transmission line may be considered as three single-phase circuits. The complete performance curves for the line may be found by analyzing one phase to neutral since the system is balanced.

In order that the three phases shall have identical performance curves the electrical constants must be the same; that is,

1. The three conductors must be of the same length and cross-section and be of the same material so that the resistances shall be equal.

2. The inductance per conductor to neutral must be the same for the three phases.

3. The condensance per conductor to neutral must be the same for the three phases.

The first requirement is always met in long-distance lines. The inductance and condensance of the three phases are equal only if the wires are properly transposed. Two complete transpositions dividing the line into three sections of equal length give a balanced arrangement. In Fig. 3.28 the interaxial spacing for the one-third of the line for any pair of conductors is D_1; for the second one-third, D_2; and the last one-third, D_3.

In order to apply the equation (15.22) for inductance derived in Chap. XXII, it is desirable to find the equivalent equidistant spacing which gives the same inductance per conductor for the whole length of the line as that given by the sum of the inductances, each for one-third of the length, of the spacings D_1, D_2 and D_3. Let

D = the equivalent equidistant spacing.
d = the diameter of the line wire in the same units as D.
L = the inductance of one conductor in henrys.
l = the length of the line conductor in miles.
$k_1 = 74.1 \times 10^{-5}$
$k_2 = 8.05 \times 10^{-5}$

$$L = l\left(k_1 \log_{10} \frac{2D}{d} + k_2\right) \text{ henrys (from equation [15.22])}$$

$$= \frac{1}{3} l\left(k_1 \log_{10} \frac{2D_1}{d} + k_2\right) + \frac{1}{3} l\left(k_1 \log_{10} \frac{2D_2}{d} + k_2\right) + \frac{1}{3} l\left(k_1 \log_{10} \frac{2D_3}{d} + k_2\right) \quad (92.28)$$

Eliminating common factors and solving for D,

$$3 \log_{10} 2D = \log_{10} 2D_1 + \log_{10} 2D_2 + \log_{10} 2D_3$$
$$\log_{10} (2D)^3 = \log_{10} (2^3 D_1 \times D_2 \times D_3)$$
$$D^3 = (D_1 \times D_2 \times D_3)$$
$$D = (D_1 \times D_2 \times D_3)^{1/3} \quad (93.28)$$

Therefore, if the conductors are properly transposed and a spacing D is used equal to the cube root of the product of the three unequal spacings, the equation for the inductance (92.28) may be

used on all practical transmission lines whatever be the arrangement of the conductors.

Similarly, the condensance of one wire to neutral for lines having unequal spacings may be expressed by equation (26.22) (derived in Chap. XXII) provided the line conductors are properly transposed and an equivalent spacing D is used, equal to the cube root of the product of the three unequal spacings.

C = the condensance of one conductor to neutral in farads.
$k = 0.0388 \times 10^{-6}$
D, D_1, D_2, D_3, d and l as already defined.

$$C = l\frac{k}{\log_{10}\frac{2D}{d}} = \frac{l}{3}\frac{k}{\log_{10}\frac{2D_1}{d}} + \frac{l}{3}\frac{k}{\log_{10}\frac{2D_2}{d}} + \frac{l}{3}\frac{k}{\log_{10}\frac{2D_3}{d}} \tag{94.28}$$

Eliminating the common factors l and k,

$$\frac{1}{\log_{10}\frac{2D}{d}} = \frac{1}{3\log_{10}\frac{2D_1}{d}} + \frac{1}{3\log_{10}\frac{2D_2}{d}} + \frac{1}{3\log_{10}\frac{2D_3}{d}}$$

Hence:

$$\log_{10}\frac{2D}{d} = \frac{3\log_{10}\frac{2D_1}{d}\log_{10}\frac{2D_2}{d}\log_{10}\frac{2D_3}{d}}{\log_{10}\frac{2D_2}{d}\log_{10}\frac{2D_3}{d}+\log_{10}\frac{2D_1}{d}\log_{10}\frac{2D_3}{d}+\log_{10}\frac{2D_1}{d}\log_{10}\frac{2D_2}{d}}$$

$= \frac{1}{3}\log_{10} 2D_1 + \frac{1}{3}\log_{10} 2D_2 + \frac{1}{3}\log_{10} 2D_3 - \log_{10} d$, plus a quantity which is zero for equal spacings and so small for any unsymmetrical spacing used in practical transmission lines that for numerical calculations it may be neglected.

Therefore:

$$\log_{10}\frac{2D}{d} = \frac{1}{3}\log_{10} 2D_1 + \frac{1}{3}\log_{10} 2D_2 + \frac{1}{3}\log_{10} 2D_3 - \log_{10} d$$

$$\log_{10} 2D = \frac{1}{3}\log_{10}(2^3 D_1 \times D_2 \times D_3).$$

$$D = (D_1 \times D_2 \times D_3)^{1/3} \tag{95.28}$$

Likewise the condensance to ground, Fig. 3.28, is

$$C = l\frac{0.0388 \times 10^{-6}}{\log_{10}\frac{4D'}{d}} \tag{96.28}$$

where

$$D' = (D_4 \times D_5 \times D_6)^{1/3}.$$

(*e*) **Voltage Regulation by Synchronous Condenser (Power-factor Regulator.)**—The most important problem in the operation of long-distance transmission lines is the voltage regulation, which must be automatic to give satisfactory results. The ratio between the generator and receiver voltages is determined by the line constants and the nature of the load, but the equations, already derived, are complicated. As in most alternating-current phenomena, it is essential first to understand the energy changes involved. The amount of energy stored inductively in the magnetic field along the line depends on the current in the line, while the energy stored condensively in the dielectric depends on the voltage. Hence the energy stored magnetically is very small at no load and increases up to a maximum for the maximum current in the line. The energy stored dielectrically, on the other hand, is approximately constant for all loads for systems operating at practically constant potentials. In long transmission lines operating at high voltage the condensance as well as the inductance is of considerable magnitude. The energy stored dielectrically is of such quantitative value in comparison to the amount stored magnetically that they balance or offset each other at some load between no load and full load. At no load the condensive action predominates, while at full load, and particularly for the maximum current in the line, the inductance becomes the controlling factor. Variations in the receiver load during the daily cycle combined with the ever-changing power factor of the commercial load continually change the relative effect on the voltage regulation of the inductance and condensance of the line.

In constant-potential systems it is imperative that the receiver voltage e_R be kept constant at all loads. This eliminates one possible variable in each voltage equation. It is also highly desirable that the generator voltage e_G should be a constant for all loads. To secure satisfactory voltage regulation it is, therefore, necessary to provide some means by which the voltage at both ends of the line can be automatically kept constant for all variations in the load, from no load to the maximum value for which the line is designed. As mentioned in Chap. XV, all the conditions required for automatic voltage regulation are met by

the use of synchronous condensers placed at the receiver end of the line. The field excitation of the synchronous condenser is adjusted by some automatic device, as the *TA* regulator described in Chap. XIV, so that the current in the machine is lagging at no load and leading at maximum load. When properly designed and installed, this arrangement gives the required automatic voltage regulation for all loads. It is not necessary that the receiver and generator voltages be equal but only that they shall remain constant at all loads. With a given generator voltage the receiver voltage should be selected so as to require a synchronous condenser of minimum capacity.

As the function of the synchronous condenser is to provide the required reactive kv.a. for all loads, the ratio of the generator and receiver voltages should be so selected that the maximum lagging current (at no load) in the synchronous condenser should be equal to the maximum leading current (at maximum load) in order that the capacity or size of the machine may be a minimum. By solving the general voltage equation, equation (63.28), for the reactive current i''_R, the amount of reactive power (lagging or leading current) that will be required to give constant receiver voltage may be calculated. For $e''_R = 0$, $e'_R = e_R$ from equation (63.28):

$$\begin{aligned}
\dot{E}_R &= e_R(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2) \\
&= e_R a_1 + i'_R d_1 + i''_R d_2 + j(-e_R a_2 + i''_R d_1 - i'_R d_2) \\
e_G^2 &= (i''_R)^2(d_1^2 + d_2^2) + 2i''_R(e_R a_1 d_2 - e_R a_2 d_1) + (i'_R)^2 \\
&\qquad (d_1^2 + d_2^2) + 2i'_R(e_R a_1 d_1 + e_R a_2 d_2) + e_R^2(a_1^2 + a_2^2) \\
i''_R &= \frac{e_R(-a_1 d_2 + a_2 d_1)}{d_1^2 + d_2^2} + \left[\frac{e_G^2}{d_1^2 + d_2^2} - \frac{e_R^2(a_1^2 + a_2^2)}{d_1^2 + d_2^2}\right. \\
&\qquad \left. + \frac{e_R^2(a_1 d_2 - a_2 d_1)^2}{d_1^2 + d_2^2} - (i'_R)^2 - \frac{2e_R i'_R(a_1 d_1 + a_2 d_2)}{d_1^2 + d_2^2}\right]^{1/2} \\
&= \frac{e_R(-a_1 d_2 + a_2 d_1)}{d_1^2 + d_2^2} + \left[\frac{e_G^2}{d_1^2 + d_2^2} - \left(\frac{e_R(a_1 d_1 + a_2 d_2)}{d_1^2 + d_2^2} + i'_R\right)^2\right]^{1/2}
\end{aligned} \tag{97.28}$$

By referring to Figs. 4.28 and 5.28 for quantitative values, the method for determining the most desirable receiver voltage may be more easily understood. For known line constants and given constant generator voltage, plot curves for the reactive power, equation (97.28), as ordinates and the kw. receiver load as abscissæ for three values of the receiver voltage: of $e_R = 90$ per cent of e_G; $e_R = e_G$; and $e_R = 110$ per cent of e_G. On the same diagram,

Figs. 4.28 and 5.28, draw the line *OD* representing the kv.a. load for the specified power factors, 85 per cent and 90 per cent, respectively. For any given load the corresponding ordinate to the line *OD* represents the reactive power in the receiver load. Likewise, the ordinate to the curves for the receiver voltage selected gives the amount of reactive power that must be provided in order to give the desired regulation. The function of the synchronous condenser is therefore to supply the difference.

By interpolation, that receiver voltage may be found which will require an amount of inductive reactive power at no load, ordinate *OA*, equal in magnitude to the condensive reactive power at maximum load, ordinate *BD*.

The receiver voltage thus determined can be maintained automatically by a synchronous condenser of minimum capacity for the given line, load and generator voltage. Thus for line *A*, Fig. 4.28, for the given generator voltage, e_G = 135,000 volts and power factor of load = 85 per cent, the capacity of the synchronous condenser is a minimum if the receiver voltage e_R = 123,000 volts. Similarly for line *B*, Fig. 5.28, for e_G = 160,000 volts and load power factor = 90 per cent, the required capacity of the synchronous condenser is a minimum if the receiver voltage e_R = 155,500 volts.

The effect of varying the power factor of the load is apparent from reactive power curves, Figs. 4.28 and 5.28. For loads having power factors nearer unity the receiver voltage would be correspondingly increased while the reverse would be the case for loads with lower power factors.

In order to illustrate the application of the theoretical equations to practical transmission lines the complete solution is given for two typical high-tension systems. The complete operating characteristics of the two lines are shown in Figs. 6.28 and 7.28.

(*f*) **Performance Computations of Long-distance Transmission Lines.**—Below are given the performance computations of two transmission lines called *A* and *B*. The computations are given in parallel columns for the two lines in order to facilitate comparisons; the pages are divided so that the treatment of line *A* appears on the left-hand side and of line *B* on the right. Statements which hold true for both lines are carried across both divisions.

Line A	Given Data	Line B
120 miles	Distance of transmission	240 miles
30,000 kw.	Normal load per line	30,000 kw.
45,000 kw.	Maximum load per line	45,000 kw.
3,000 ft.	Elevation of power house	5,000 ft.
2,000 ft.	Elevation of receiving station	200 ft.
Eastern Washington	Location of line	Central California
75°F.	Average operating temperature	100°F.
135,000 volts	Transmission voltage at generator end	160,000 volts
500,000 cir. mil. hard-drawn copper stranded cable, diameter 0.819 in.	Conductor used	850,000 cir. mil. hard-drawn aluminum stranded cable, diameter 1.062 in.
14 ft. 6 in.	Spacing of conductors	16 ft. 9 in.
60 cycles	Frequency	50 cycles
0.85	Power factor at receiver end	0.90

—LINE *A*—	—LINE *B*—

Conductors arranged in a horizontal plane.

Line star-connected at both ends with the neutral grounded, but means provided to disconnect neutral from ground whenever necessary.

Voltage at generator and receiver ends to be kept constant for all loads by the installation of synchronous condensers, of adequate capacity, at the receiver end.

Equations (65.28) have been used in the computations for both lines.

—LINE CONSTANTS—

Resistance

For Copper	For Aluminum
$\rho = 9.35[1 + 0.0042(t - 32)0.556]$	$\rho = 15.4[1 + 0.00435(t - 32)0.556]$
$t = 75°\text{F.}$	$t = 100°\text{F.}$
$\rho = 9.35[1 + 0.0042(75 - 32)0.556] = 10.29$ ohms	$\rho = 15.4[1 + 0.00435(100 - 32)0.556] = 17.933$ ohms

where ρ = resistance per mil-foot of conductor.
t = temperature, at which resistance is desired, in degrees Fahrenheit.

Total resistance of conductor

$r = \dfrac{120 \times 5{,}280 \times 10.29}{500{,}000} = 13.04$ ohms	$r = \dfrac{240 \times 5{,}280 \times 17.933}{850{,}000} = 26.74$ ohms

—LINE *A*— —LINE *B*—

Inductance

$$L = l\left(0.00074 \log_{10} \frac{2D}{d} + 0.0000805\right).$$

where

L = inductance in henrys.

l = length of the line in miles.

D = equivalent interaxial spacing in inches between conductors symmetrically arranged in triangle.

$D = (D_1 \times D_2 \times D_3)^{1/3}$.

d = diameter of conductor in inches.

See equations (92.28) and (93.28).

Line A:

$$L = 120\left(0.00074 \log_{10} \frac{2 \times 174 \times 2^{1/3}}{0.819} + 0.0000805\right)$$
$$= 120(0.00074 \times 2.7285 + 0.0000805)$$
$$= 0.252 \text{ henry}$$

Line B:

$$L = 240\left(0.00074 \log_{10} \frac{2 \times 201 \times 2^{1/3}}{1.062} + 0.0000805\right)$$
$$= 240(0.00074 \times 2.6782 + 0.0000805)$$
$$= 0.495 \text{ henry}$$

Reactance

$$x = 2\pi fL$$

Line A: $x = 2 \times 3.1416 \times 60 \times 0.252 = 95.0$ ohms

Line B: $x = 2 \times 3.1416 \times 50 \times 0.495 = 155.5$ ohms

Impedance

$$z = (r^2 + x^2)^{1/2}$$

Line A: $z = (\overline{13.04}^2 + \overline{95.0}^2)^{1/2} = 95.885$ ohms

Line B: $z = (\overline{26.74}^2 + \overline{155.5}^2)^{1/2} = 157.78$ ohms

Condensance

$$C = l \times \frac{0.0388 \times 10^{-6}}{\log_{10} \frac{2D}{d}}$$

where C = condensance in farads.

Other units same as for inductance.

See equations (94.28) and (95.28).

—LINE A—

$$C = 120 \times \frac{0.0388 \times 10^{-6}}{\log_{10} \dfrac{2 \times 174 \times 2^{1/3}}{0.819}} = 1.7065 \times 10^{-6} \text{ farads}$$

—LINE B—

$$C = 240 \times \frac{0.0388 \times 10^{-6}}{\log_{10} \dfrac{2 \times 201 \times 2^{1/3}}{1.062}} = 3.4772 \times 10^{-6} \text{ farads}$$

Shunted line susceptance

$$b = 2\pi fC$$

Line A:

$$b = 2 \times 3.1416 \times 60 \times 1.7065 \times 10^{-6} = 0.6433 \times 10^{-3} \text{ mhos}$$

Line B:

$$b = 2 \times 3.1416 \times 50 \times 3.4772 \times 10^{-6} = 1.1068 \times 10^{-3} \text{ mhos}$$

$$\alpha = [\tfrac{1}{2}b(z - x)]^{1/2}$$

Line A:

$$\alpha = [\tfrac{1}{2} \times 0.6433 \times 10^{-3}(95.885 - 95.0)]^{1/2} = 0.016872 \text{ radian}$$

$$\alpha^2 = 0.00028465$$

$$1 + \frac{\alpha^2}{2} = 1.00014233$$

Line B:

$$\alpha = [\tfrac{1}{2} \times 1.1068 \times 10^{-3}(157.78 - 155.5)]^{1/2} = 0.03552 \text{ radian}$$

$$\alpha^2 = 0.0012616$$

$$1 + \frac{\alpha^2}{2} = 1.0006308$$

$$\beta = [\tfrac{1}{2}b(z + x)]^{1/2}$$

Line A:

$$\beta = [\tfrac{1}{2} \times 0.6433 \times 10^{-3}(95.885 + 95.0)]^{1/2} = 0.2478 \text{ radian}$$

$$\beta^\circ = 0.2478 \times 57.3 = 14^\circ\ 11'\ 53''$$

$$\sin\beta = 0.2453$$

$$\cos\beta = 0.96945$$

$$\beta \sin\beta = 0.06078$$

$$\beta \cos\beta = 0.2404$$

Line B:

$$\beta = [\tfrac{1}{2} \times 1.1068 \times 10^{-3}(157.78 + 155.5)]^{1/2} = 0.41638 \text{ radian}$$

$$\beta^\circ = 0.41638 \times 57.3 = 23^\circ\ 51'\ 22''$$

$$\sin\beta = 0.40443$$

$$\cos\beta = 0.91456$$

$$\beta \sin\beta = 0.16838$$

$$\beta \cos\beta = 0.38076$$

—Constants of Line Equations—

$$a_1 - ja_2 = \left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta$$

—LINE A—

$$a_1 - ja_2 = 1.00014233 \times 0.96945 - j0.016872 \times 0.2453$$
$$= 0.96958 - j0.0041387$$

—LINE B—

$$a_1 - ja_2 = 1.0012616 \times 0.91456 - j0.03552 \times 0.40443$$
$$= 0.91571 - j0.014365$$

$$c_1 - jc_2 = \frac{1}{z}\left[\alpha\beta\cos\beta - \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{z}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right]$$

LINE A:

$$c_1 - jc_2 = \frac{1}{95.885}[0.016872(0.2404 - 1.00014233 \times 0.2453)] - \frac{j}{95.885} \times [1.00014233 \times 0.06078 + 0.00028465 \times 0.96945] = -0.000000868 - j0.0006369$$

LINE B:

$$c_1 - jc_2 = \frac{1}{157.78}[0.03552(0.38076 - 1.0006388 \times 0.40443)] - \frac{j}{157.78}[1.0006308 \times 0.16838 + 0.0012616 \times 0.91456]$$
$$= -0.000005385 - j0.0010752$$

$$d_1 - jd_2 = \frac{1}{b}\left[\alpha\beta\cos\beta + \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{b}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta - \alpha^2\cos\beta\right]$$

LINE A:

$$d_1 - jd_2 = \frac{1}{0.6433 \times 10^{-3}}[0.016872(0.2404 + 1.00014233 \times 0.2453)] - j\frac{1}{0.6433 \times 10^{-3}}[1.00014233 \times 0.06078 - 0.00028465 \times 0.96945]$$
$$= 12.74 - j94.07$$

LINE B:

$$d_1 - jd_2 = \frac{1}{1.1068 \times 10^{-3}}[0.03552(0.38076 + 1.0006308 \times 0.40443)] - j\frac{1}{1.1068 \times 10^{-3}}[1.0006308 \times 0.16838 - 0.0012616 \times 0.91456]$$
$$= 25.206 - j151.2.$$

The above constants are independent of voltage or current in the line and are the numerical coefficients of the physical properties of the line. The performance of the line depends upon the magnitude and fre-

—LINE *A*— —LINE *B*—

quency of the voltage impressed at the generator end, the above line constants, and the nature of the load at the receiver end. The generator voltage is stipulated in the given data, but the nature of the load is only partly defined by the given power factor and magnitude of the load. The total receiver load also includes the synchronous condenser or power-factor regulator, the function of which has been described above.

The next step in the computations is the determination of the required capacity of the synchronous condenser (power-factor regulator) necessary to maintain constant potential at the receiver end for a range in load from zero to maximum, and the determination of the best voltage relation between E_G and E_R which makes the required capacity of the synchronous condenser a minimum in comparison to all other possible relations of E_G and E_R.

Reactive current required for constant voltage at both ends of the line.

$$i''_R = \frac{e_R(-a_1d_2 + a_2d_1)}{d_1^2 + d_2^2} + \left[\frac{e_G^2}{d_1^2 + d_2^2} - \left(\frac{e_R(a_1d_1 + a_2d_2)}{d_1^2 + d_2^2} + i'_R\right)^2\right]^{1/2} \quad \text{(see equations [97.28])}$$

Line A

$$e_G = \frac{135,000}{\sqrt{3}} = 77,940$$

Substituting constants for $e_G = 77,940$ and

$e_R = 90$ per cent of $e_G = 70,146$

$$i''_R = -709.6 + [674,150 - (99.18 + i'_R)^2]^{1/2}$$

$e_R = e_G = 77,940$

$$i''_R = -788.45 + [674,150 - (110.21 + i'_R)^2]^{1/2}$$

$e_R = 110$ per cent of $e_G = 84,834$

$$i''_R = -858.20 + [674,150 - (119.95 + i'_R)^2]^{1/2}$$

Line B

$$e_G = \frac{160,000}{\sqrt{3}} = 92,380$$

Substituting constants for $e_G = 92,380$ and

$e_R = 90$ per cent of $e_G = 83,142$

$$i''_R = -488.6 + [363,198 - (89.35 + i'_R)^2]^{1/2}$$

$e_R = e_G = 92,380$

$$i''_R = -542.90 + [363,198 - (99.27 + i'_R)^2]^{1/2}$$

$e_R = 110$ per cent of $e_G = 101,618$

$$i''_R = -597.22 + [363,198 - (109.20 + i'_R)^2]^{1/2}$$

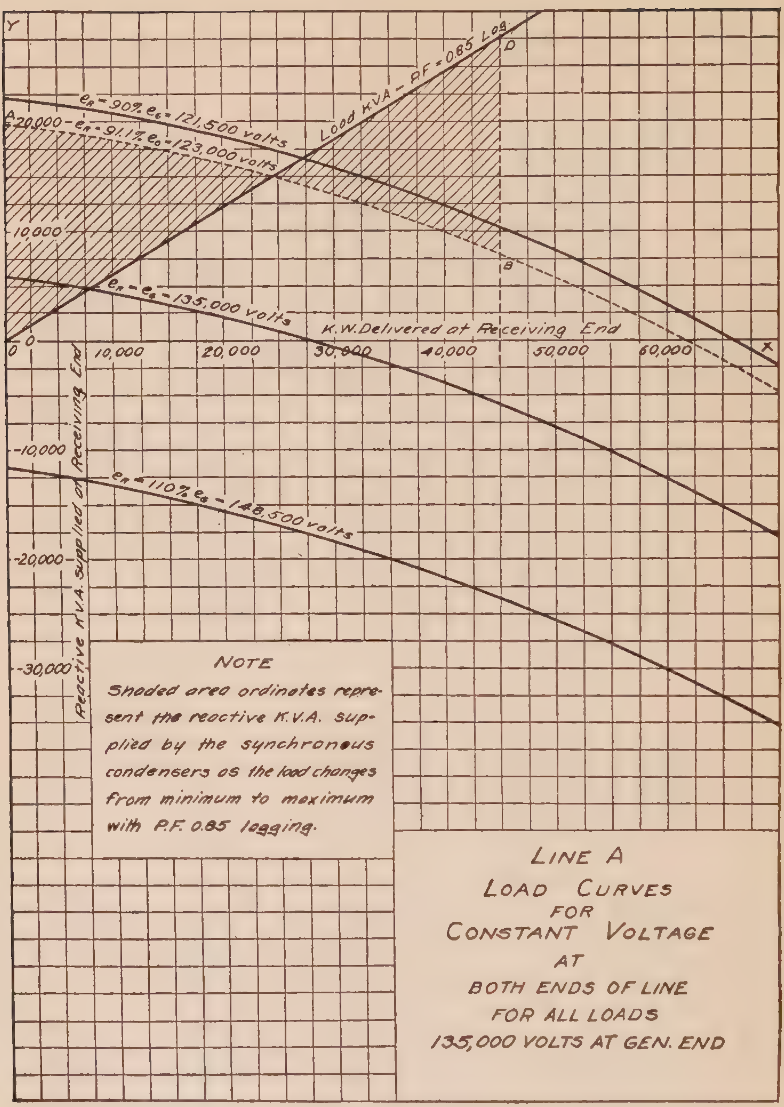

Y
e_R = 90% e_S = 121,500 volts
20,000 – e_R = 91.1% e_0 = 123,000 volts
Load KVA – PF = 0.85 Lag.
A
D
B
10,000
e_R = e_S = 135,000 volts
K.W. Delivered at Receiving End
0
10,000
20,000
30,000
40,000
50,000
60,000
X
-10,000
-20,000
-30,000
e_R = 110% e_S = 148,500 volts
Reactive K.V.A. Supplied at Receiving End
NOTE
Shaded area ordinates represent the reactive K.V.A. supplied by the synchronous condensers as the load changes from minimum to maximum with P.F. 0.85 lagging.
LINE A
LOAD CURVES
FOR
CONSTANT VOLTAGE
AT
BOTH ENDS OF LINE
FOR ALL LOADS
135,000 VOLTS AT GEN. END

FIG. 4.28.

—LINE *A*—

—LINE *B*—

The values of i''_R for a range of load from 0 to 70,000 kw. are tabulated below.

3φ Load$_R$, Kw.	e_R = 70,146		e_R = 77,940		e_R = 84,834	
	i'_R	i''_R	i'_R	i''_R	i'_R	i''_R
0	0	105.4	0	25.2	0	−46.0
10,000	47.52	98.3	42.77	18.2	39.295	−52.7
20,000	95.04	88.2	85.54	9.0	78.590	−61.5
30,000	142.56	75.1	128.31	−2.8	117.885	−72.3
40,000	190.08	58.8	171.08	−17.1	157.180	−85.3
50,000	237.60	39.2	213.85	−34.1	196.475	−100.6
60,000	285.12	16.0	256.62	−53.9	235.770	−118.2
70,000	332.64	−11.3	299.39	−76.9	275.065	−138.4

3φ Load$_R$, Kw.	e_R = 83,142		e_R = 92,380		e_R = 101,618	
	i'_R	i''_R	i'_R	i''_R	i'_R	i''_R
0	0	107.4	0	53.5	0	−4.4
10,000	40.092	100.0	36.085	44.4	32.802	−11.5
20,000	80.184	89.7	72.170	34.9	65.604	−20.7
30,000	120.276	75.4	108.255	22.9	98.406	−31.5
40,000	160.368	59.9	144.340	8.3	131.208	−44.8
50,000	200.460	39.8	180.425	−9.1	164.010	−60.1
60,000	240.552	15.8	216.510	−29.6	196.812	−78.0
70,000	280.644	−12.9	252.595	−53.6	229.614	−98.8

$$\text{Reactive kw.a.} = \text{kv.}\frac{i''_R}{i'_R}$$

The reactive kv.a. for the different values of e_R are tabulated below.

Kw.	e_R = 70,146	e_R = 77,940	e_R = 84,834
0	22,180	5,892	−11,683
10,000	20,686	4,255	−13,411
20,000	18,560	2,107	−15,650
30,000	15,802	−655	−18,397
40,000	12,373	−3,998	−21,706
50,000	8,249	−7,973	−25,600
60,000	3,367	−12,602	−30,080
70,000	−2,378	−17,980	−35,220

Kw.	e_R = 83,142	e_R = 92,380	e_R = 101,618
0	26,790	14,826	−1,491
10,000	24,943	12,305	−3,506
20,000	22,371	9,672	−6,310
30,000	18,805	6,345	−9,603
40,000	14,940	2,300	−13,656
50,000	9,927	−2,522	−18,320
60,000	3,941	−8,202	−23,777
70,000	−3,218	−14,853	−30,120

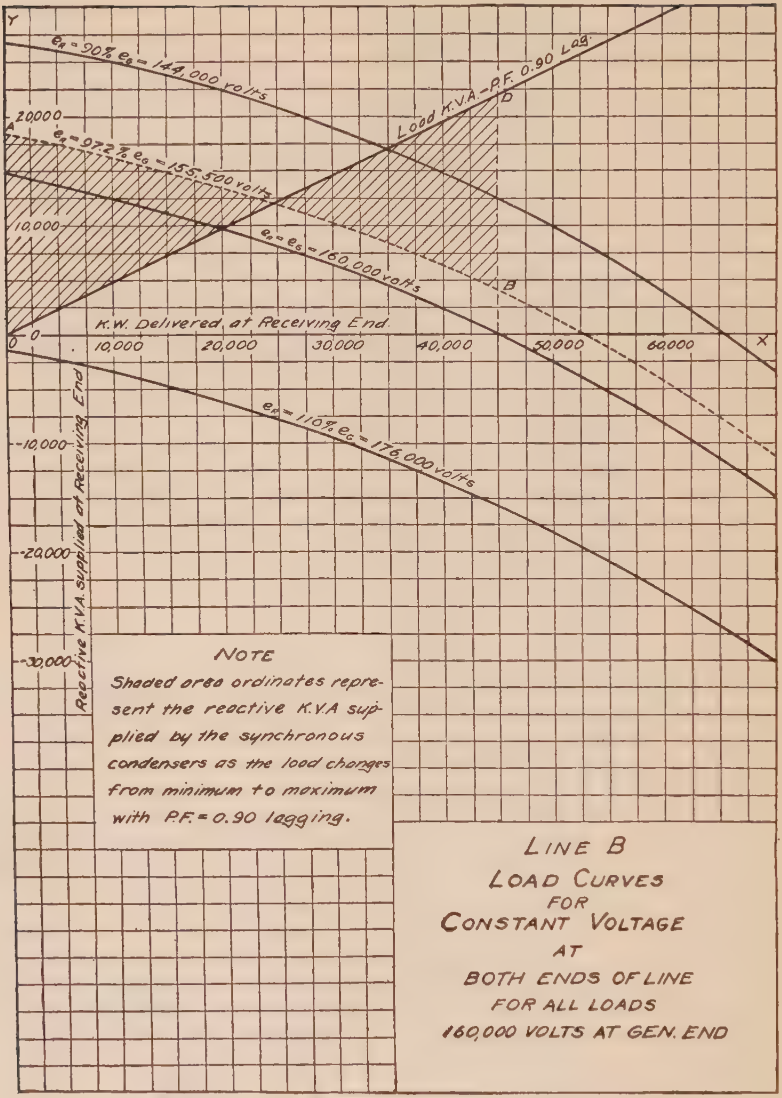

$e_R = 90\% e_G = 144,000$ volts
Load K.V.A.-P.F. 0.90 Lag
$e_R = 97.2\% e_G = 155,500$ volts
$e_R = e_G = 160,000$ volts
$e_R = 110\% e_G = 176,000$ volts
K.W. Delivered at Receiving End
Reactive K.V.A. supplied at Receiving End
20,000
10,000
0
-10,000
-20,000
-30,000
10,000
20,000
30,000
40,000
50,000
60,000
NOTE
Shaded area ordinates represent the reactive K.V.A supplied by the synchronous condensers as the load changes from minimum to maximum with P.F. = 0.90 lagging.
LINE B
LOAD CURVES
FOR
CONSTANT VOLTAGE
AT
BOTH ENDS OF LINE
FOR ALL LOADS
160,000 VOLTS AT GEN. END

FIG. 5.28.

—LINE *A*—

The constant-voltage curves drawn from the curves in Fig. 4.28 and the straight line giving the kv.a. of the load at 85 per cent power factor without the synchronous condenser show that, for equal voltages at both ends of the line, the reactive kv.a. must be 5,892 current lagging at no load, which must all be supplied by the synchronous condenser. At maximum load, the reactive kv.a. required are 6,000, current leading. The load supplies 27,500 kv.a., current lagging, therefore the load of the synchronous condenser would have to be 27,500 + 6,000 = 33,500 kv.a., current leading. This condition would be unsatisfactory. However, if the dotted curve, marked $e_R = 91.1$ per cent of e_G (constructed by interpolation) represents the actual relationship between the generator and receiver voltage, synchronous condensers of an aggregate capacity of 20,000 kv.a. would suffice for the total load range from zero to maximum. The machines would be running under full-load lagging current when the power load is zero, and under full-load leading current when the power load is at the maximum. Therefore, for this particular line, with the generator voltage assumed as 135,000, the receiver voltage should be 123,000 for all loads. The voltage to neutral, $e_G = 77{,}940$ volts; $e_R = 71{,}015$ volts.

—LINE *B*—

The constant-voltage curves drawn from the curves in Fig. 5.28 and the straight line giving the kv.a. of the load at 90 per cent power factor without the synchronous condenser show that, for equal voltages at both ends of the line, the reactive kv.a. must be 14,820, current lagging at no load, which must all be supplied by the synchronous condenser. At maximum load, the reactive kv.a. required are zero (power factor = 1.00). The load supplies 22,000 kv.a., current lagging, therefore the load of the synchronous condenser would have to be 22,000 + 0 = 22,000 kv.a., current leading. This condition is not the most desirable. However, if the dotted curve marked $e_R = 97.2$ per cent of e_G (constructed by interpolation) represents the actual relationship between the generator and receiver voltages, synchronous condensers of an aggregate capacity of 18,500 kv.a. would suffice for the total load range from zero to maximum. The machines would be running under full-load lagging current when the power load is zero, and under full-load leading current when the power load is at the maximum. Therefore, for this particular line, with the generator voltage assumed as 160,000, the receiver voltage should be 155,500 for all loads. The voltage to neutral is $e_G = 92{,}380$ volts; $e_R = 89{,}780$ volts.

Using again equation (97.28) for this new voltage relation and solving for i''_R:

$$i''_R = -718.3 + [674{,}150 - (100.42 + i'_R)^2]^{1/2} \qquad \Big| \qquad i''_R = -527.6 + [363{,}198 - (96.48 + i'_R)^2]^{1/2}$$

—LINE *A*—

3ϕ Load_R, kw.	i'_R	i''_R
0	0	96.5
10,000	46.94	89.35
20,000	93.88	79.33
30,000	140.82	66.42
40,000	187.76	50.45
50,000	234.70	31.18
60,000	281.64	8.35
70,000	328.58	−18.32

—LINE *B*—

3ϕ Load_R, kw.	i'_R	i''_R
0	0	67.3
10,000	37.13	60.1
20,000	74.26	50.4
30,000	111.39	38.1
40,000	148.52	23.0
50,000	185.65	4.9
60,000	222.78	−16.5
70,000	259.91	−41.6

With the knowledge of all load conditions at the receiver end, including the kv.a. of the synchronous condensers, the electrical conditions at the generator end may be computed. Of the conditions at the generator end, only the magnitude and frequency of the voltage are given. The computation carried out by the use of the voltage equation, therefore, is an effective check on the accuracy of the previous calculations.

Voltage at generator end with synchronous condensers operating at receiver end.

$$\dot{E}_G = e_R(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2)$$

Zero load

$$\dot{E}_G = 68{,}850 - j294 + j96.5(12.74 - j94.07) = 77{,}927 + j935$$

$$\dot{E}_G = 82{,}212 - j1{,}290 + j67.3(25.206 - j151.2) = 92{,}380 + j406$$

10,000 kw.

$$\dot{E}_G = 68{,}850 - j294 + (46.94 + j89.35)(12.74 - j94.07) = 77{,}853 - j3{,}571$$

$$\dot{E}_G = 82{,}212 - j1{,}290 + (37.13 + j60.1)(25.206 - j151.2) = 92{,}235 - j5{,}389$$

—LINE A—	—LINE B—
20,000 kw.	
$\dot{E}_G = 68{,}850 - j294 + (93.88 + j79.33)(12.74 - j94.07)$	$\dot{E}_G = 82{,}212 - j1{,}290 + (74.26 + j50.4)(25.206 - j151.2)$
$= 77{,}509 - j8{,}114$	$= 91{,}705 - j11{,}246$
30,000 kw.	
$\dot{E}_G = 68{,}850 - j294 + (140.82 + j66.42)(12.74 - j94.07)$	$\dot{E}_G = 82{,}212 - j1{,}290 + (111.39 + 38.1)(25.206 - j151.2)$
$= 76{,}893 - j12{,}696$	$= 90{,}781 - j17{,}172$
40,000 kw.	
$\dot{E}_G = 68{,}850 - j294 + (187.76 + j50.45)(12.74 - j94.07)$	$\dot{E}_G = 82{,}212 - j1{,}290 + (148.52 + j23.0)(25.206 - j151.2)$
$= 75{,}988 - j17{,}312$	$= 89{,}434 - j23{,}168$
50,000 kw.	
$\dot{E}_G = 68{,}850 - j294 + (234.70 + j31.18)(12.74 - j94.07)$	$\dot{E}_G = 82{,}212 - j1{,}290 + (185.65 + j4.9)(25.206 - j151.2)$
$= 74{,}773 - j21{,}975$	$= 87{,}632 - j29{,}236$
60,000 kw.	
$\dot{E}_G = 68{,}850 - j294 + (281.64 + j8.35)(12.74 - j94.07)$	$\dot{E}_G = 82{,}212 - j1{,}290 + (222.78 - j16.5)(25.206 - j151.2)$
$= 73{,}224 - j26{,}682$	$= 85{,}332 - j35{,}389$
70,000 kw.	
$\dot{E}_G = 68{,}850 - j294 + (328.58 - j18.32)(12.74 - j94.07)$	$\dot{E}_G = 82{,}212 - j1{,}290 + (259.91 - j41.6)(25.206 - j151.2)$
$= 71{,}313 - j31{,}436$	$= 82{,}473 - j41{,}639$

—LINE *A*—	—LINE *B*—
$$\dot{I}_G = e_R(c_1 - jc_2) + (i_R' + ji_R'')(a_1 - ja_2)$$	
Zero load	
$\dot{I}_G = -0.06 - j45.23 + j96.5(0.96959 - j0.0041387)$	$\dot{I}_G = -0.48 - j96.54 + j67.3(0.91571 - j0.014365)$
$= 0.34 + j48.33$	$= 0.49 - j34.92$
10,000 kw.	
$\dot{I}_G = -0.06 - j45.23 + (46.94 + j89.35)(0.96959 - j0.0041387)$	$\dot{I}_G = -0.48 - j96.54 + (37.13 + j60.1)(0.91571 - j0.014365)$
$= 45.84 + j41.21$	$= 34.38 - j42.04$
20,000 kw.	
$\dot{I}_G = -0.06 - j45.23 + (93.88 + j79.33)(0.96959 - j0.0041387)$	$\dot{I}_G = -0.48 - j96.54 + (74.26 + j50.4)(0.91571 - j0.014365)$
$= 91.29 + j31.30$	$= 68.24 - j51.46$
30,000 kw.	
$\dot{I}_G = -0.06 - j45.23 + (140.82 + j66.42)(0.96959 - j0.0041387)$	$\dot{I}_G = -0.48 - j96.54 + (111.39 + j38.1)(0.91571 - j0.014365)$
$= 136.75 + j18.59$	$= 102.07 - j63.25$
40,000 kw.	
$\dot{I}_G = -0.06 - j45.23 + (187.76 + j50.45)(0.96959 - j0.0041387)$	$\dot{I}_G = -0.48 - j96.54 + (148.52 + j23.0)(0.91571 - j0.014365)$
$= 182.18 + j2.91$	$= 135.85 - j77.61$
50,000 kw.	
$\dot{I}_G = -0.06 - j45.23 + (234.7 + j31.18)(0.96959 - j0.0041387)$	$\dot{I}_G = -0.48 - j96.54 + (185.65 + j4.9)(0.91571 - j0.014365)$
$= 227.62 - j15.97$	$= 169.59 - j94.72$

—LINE A—

—LINE B—

60,000 kw.

LINE A:

$$\dot{I}_G = -0.06 - j45.23 + (281.64 + j8.35)(0.96959 - j0.0041387)$$
$$= 273.05 - j38.30$$

LINE B:

$$\dot{I}_G = -0.48 - j96.54 + (222.78 - j16.5)(0.91571 - j0.014365$$
$$= 203.28 - j114.85$$

70,000 kw.

LINE A:

$$\dot{I}_G = -0.06 - j45.23 + (328.58 - j18.32)(0.96959 - j0.0041387)$$
$$= 318.43 - j64.35$$

LINE B:

$$\dot{I}_G = -0.48 - j96.54 + (259.91 - j41.6)(0.91571 - j0.014365)$$
$$= 236.92 - j138.36$$

—LINE *A*—

Generator voltage 135,000, receiver voltage 123,000, load $P.F._R$ = 0.85 lagging								
KW. receiver 3ϕ load $P_R \times 3$	0	10,000	20,000	30,000	40,000	50,000	60,000	70,000
$P_R = e_R i_R \times P.F._R$	0	3,333	6,667	10,000	13,333	16,667	20,000	23,333
i'_R	0	46.94	93.88	140.82	187.76	234.70	281.64	328.58
i''_R	96.50	89.35	79.33	66.42	50.45	31.18	8.35	−18.32
$i_R = [(i'_R)^2 + (i''_R)^2]^{1/2}$	96.50	100.93	122.90	155.70	194.41	236.74	281.78	329.09
i''_{load}	0	29.11	58.22	87.33	116.44	145.55	174.67	203.77
$i''_{s.c.} = i''_R - i''_{load}$	96.50	60.24	21.11	−20.92	−65.99	−114.37	−166.32	−222.09
$\dot{E}_G = e'_G - je''_G$	77,927	77,853	77,509	76,893	75,988	74,773	73,224	71,313
	$+j935$	$-j3{,}571$	$-j8{,}114$	$-j12{,}696$	$-j17{,}312$	$-j21{,}975$	$-j26{,}682$	$-j31{,}436$
$e_G = [(e'_G)^2 + (e''_G)^2]^{1/2}$	77,940	77,940	77,940	77,940	77,940	77,940	77,940	77,940
$E_G = \sqrt{3}e_G$	135,000	135,000	135,000	135,000	135,000	135,000	135,000	135,000
$\dot{I}_G = i'_G - ji''_G$	0.34	45.84	91.29	136.75	182.18	227.62	273.05	318.43
	$+j48.33$	$+j41.21$	$+j31.30$	$+j18.59$	$+j2.91$	$-j15.97$	$-j38.30$	$-j64.35$
$i_G = [(i'_G)^2 + (i''_G)^2]^{1/2}$	48.33	61.68	96.50	138.00	182.20	228.19	275.72	324.87
$P_G = e'_G i'_G + e''_G i''_G$	72	3,421	6,822	10,278	13,783	17,371	21,014	24,731
$P_G \times 3$	216	10,263	20,466	30,834	41,349	52,113	63,042	74,193
$Q_G = e_G i_G$	3,767	4,807	7,521	10,755	14,200	17,783	21,490	25,320
$Q_G \times 3$	11,301	14,421	22,563	32,265	42,600	53,349	64,470	75,960
$P.F._G = P_G \div Q_G$	−0.019	−0.712	−0.907	−0.956	−0.971	−0.977	−0.978	+0.970
Synchronous condenser load $e_R \times i''_{s.c.} \times 3$ kv.a	20,560	12,833	4,497	−4,457	−14,058	−24,368	−35,435	−47,310
Resultant $P.F._R = \cos[\tan^{-1}(i''_R \div i'_R)]$	0.00	0 580	0 776	0.915	0.970	0.993	1.00	−0.964
Line loss, per cent	100.0	2 6	2 3	2.7	3.3	4.1	4.8	5.7

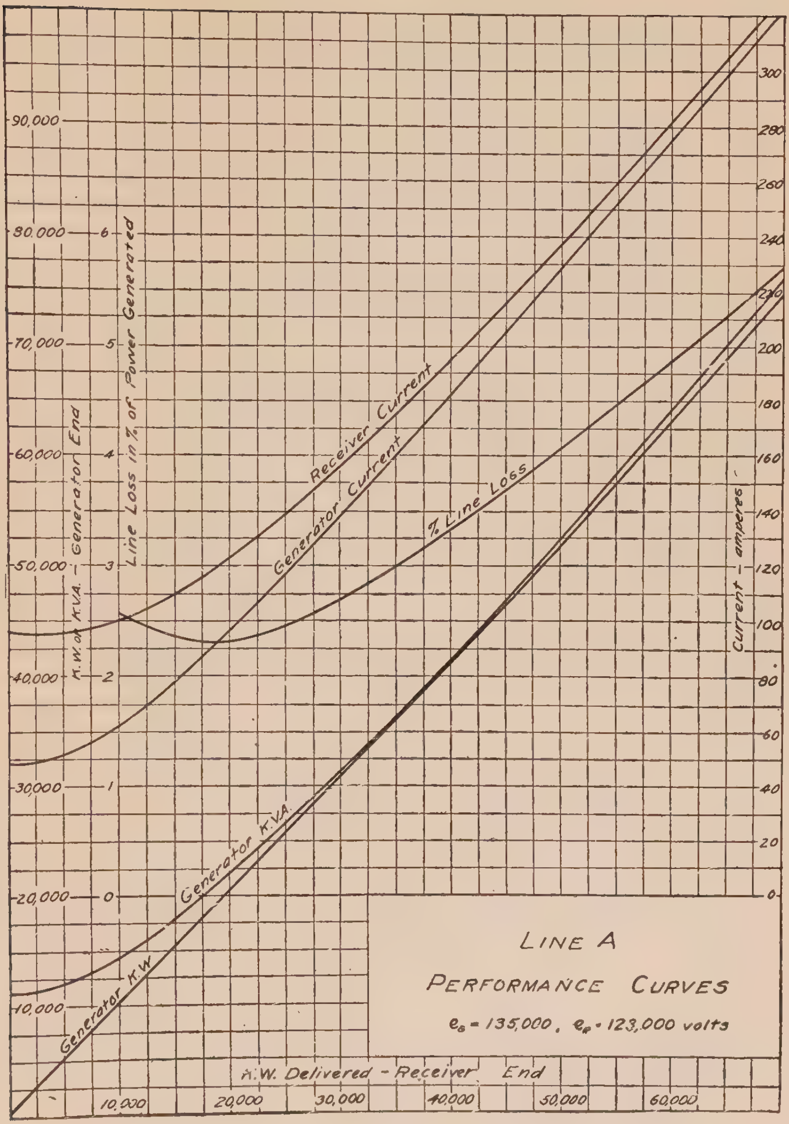

Receiver Current
Generator Current
% Line Loss
Generator K.V.A.
Generator K.W.
K.W. or K.V.A. - Generator End
Line Loss in % of Power Generated
Current - amperes
K.W. Delivered - Receiver End
LINE A
PERFORMANCE CURVES
e_s = 135,000, e_r = 123,000 volts
10,000
20,000
30,000
40,000
50,000
60,000
70,000
80,000
90,000
0
1
2
3
4
5
6
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300

FIG. 6.28.

—LINE *B*—

Generator voltage 160,000, receiver voltage 155,500 load $P.F._R$ = 0.90 lagging								
KW. receiver 3ϕ load $P_R \times 3$	0	10,000	20,000	30,000	40,000	50,000	60,000	70,000
$P_R = e_R i_R \times P.F._R$	0	3,333	6,667	10,000	13,333	16,667	20,000	23,333
i'_R	0	37.13	74.26	111.39	148.52	185.65	222.78	259.91
i''_R	+67.30	+60.10	+50.40	+38.10	+23.00	+4.90	−16.50	−41.60
$i_R = [(i'_R)^2 + (i''_R)^2]^{1/2}$	67.30	70.57	89.74	117.73	150.29	185.71	223.40	263.22
i''_{load}	0	+17.98	+35.96	53.95	71.93	89.91	107.88	125.86
$i''_{s.c.} = i''_R - i''_{load}$	+67.30	+42.12	+14.44	−15.85	−48.93	−85.01	−124.38	−167.46
$\dot{E}_G = e'_G - je''_G$	92,380	92,235	91,705	90,781	89,434	87,632	85,332	82,473
	$+j406$	$-j5,389$	$-j11,246$	$-j17,172$	$-j23,168$	$-j29,236$	$-j35,389$	$-j41,639$
$e_G = [(e'_G)^2 + (e''_G)^2]^{1/2}$	92,380	92,380	92,380	92,380	92,380	92,380	92,380	92,380
$E_G = e_G\sqrt{3}$	160,000	160,000	160,000	160,000	160,000	160,000	160,000	160,000
$I_G = i'_G - ji''_G$	0.49	34.38	68.24	102.07	135.85	169.59	203.28	236.92
	$-j34.92$	$-j42.04$	$-j51.46$	$-j63.25$	$-j77.61$	$-j94.72$	$-j114.85$	$-j138.36$
$i_G = [(i'_G)^2 + (i''_G)^2]^{1/2}$	34.93	54.31	85.47	120.08	156.45	194.25	233.48	274.36
$P_G = e'_G i'_G + e''_G i''_G$	31	3,398	6,836	10,352	13,948	17,629	21,408	25,301
$P_G \times 3$	93	10,194	20,508	31,056	41,844	52,887	64,224	75,903
$Q_G = e_G i_G$	3,227	5,017	7,895	11,091	14,451	17,944	21,568	25,344
$Q_G \times 3$	9,681	15,051	23,685	33,273	43,353	53,832	64,704	76,032
$P.F._G = P_G \div Q_G$	−0.0096	−0.677	−0.856	−0.933	−0.965	−0.983	−0.993	−0.999
Synchronous condenser load, $e_R \times i''_{s.c.} \times 3$ kv-a.	+18,125	+11,343	+3,889	−4,269	−13,178	−22,895	−33,500	−45,100
Resultant $P.F._R = \cos[\tan^{-1}(i''_R \div i'_R)]$	0.00	+0.525	+0.830	+0.946	+0.987	1.00	−0.997	−0.986
Line loss, per cent	100.0	1.93	2.48	3.40	4.41	5.46	6.58	7.78

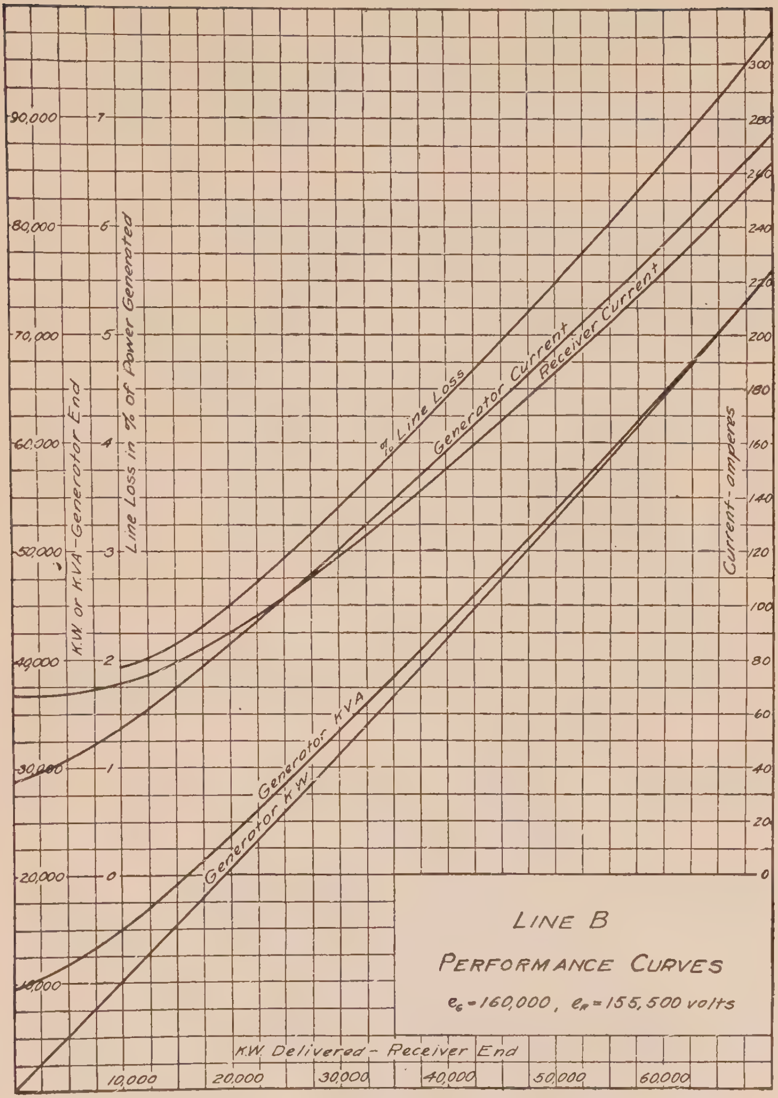

LINE B
PERFORMANCE CURVES
$e_G = 160{,}000$, $e_R = 155{,}500$ volts
% Line Loss
Generator Current
Receiver Current
Generator KVA
Generator KW
K.W. or K.V.A.-Generator End
Line Loss in % of Power Generated
Current-amperes
K.W. Delivered - Receiver End
10,000
20,000
30,000
40,000
50,000
60,000
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0

FIG. 7.28.

(*g*) **Computation of Performance Curves for Line C.**—The specifications for line C are the same as for line B, except that it is twice as long. The computation illustrates the use of the polar notation in the solution of transmission-line problems.

For line C:

$$
\begin{aligned}
r &= 2 \times 26.74 = 53.5 \text{ ohms.}\\
x &= 2 \times 155.5 = 311.0 \text{ ohms.}\\
z &= 2 \times 157.78 = 315.6 \text{ ohms.}\\
Z &= 315.6 \underline{/279.6^\circ}.\\
b &= 2 \times 1.1068 \times 10^{-3} = 2.214 \times 10^{-3} \text{ mho.}\\
Y &= 2.214 \times 10^{-3} \underline{/270.0^\circ}.\\
\alpha &= 2 \times 0.03552 = 0.07104 \text{ radian.}\\
\beta &= 2 \times 0.41638 = 0.8328 \text{ radian.}\\
&= 47.72^\circ.\\
\cosh \alpha &= 1.0025.\\
\sinh \alpha &= 0.0711.\\
\cos \beta &= 0.6727.\\
\sin \beta &= 0.7398.
\end{aligned}
$$

The slide rule is considered as sufficiently accurate for the following computations:

$$
\begin{aligned}
\cosh(\alpha - j\beta) &= \cosh \alpha \cos \beta - j \sinh \alpha \sin \beta\\
&= 0.674 - j0.0526\\
&\doteq 0.676 \underline{/355.5^\circ}\\
\sinh(\alpha - j\beta) &= \sinh \alpha \cos \beta - j \cosh \alpha \sin \beta\\
&= 0.0478 - j0.742\\
&= 0.743 \underline{/273.7^\circ}\\
\sqrt{Y/Z} &= \sqrt{\frac{2.214 \times 10^{-3}}{315.6}} \underline{\Big/ \frac{270.0^\circ - 279.6^\circ}{2}}\\
&= 2.645 \times 10^{-3} \underline{/355.2^\circ}\\
\sqrt{Z/Y} &= 378 \underline{/4.8^\circ}\\
a_1' - ja_2' &= 0.676 \underline{/355.5^\circ}\\
c_1' - jc_2' &= (2.645 \times 10^{-3} \underline{/355.2^\circ})(0.743 \underline{/273.7^\circ})\\
&= 1.967 \times 10^{-3} \underline{/268.9^\circ}\\
d_1' - jd_2' &= (378 \underline{/4.8^\circ})(0.743 \underline{/273.7^\circ})\\
&= 281 \underline{/278.5^\circ}
\end{aligned}
$$

The complete equations for the line are, therefore:

$$\dot{I}_G = (0.676 \underline{/355.5°})\dot{I}_R + (1.967 \times 10^{-3} \underline{/268.9°})\dot{E}_R.$$
$$\dot{E}_G = (0.676 \underline{/355.5°})\dot{E}_R + (281 \underline{/278.5°})\dot{I}_R.$$
$$\dot{I}_R = (0.676 \underline{/355.5°})\dot{I}_G - (1.967 \times 10^{-3} \underline{/268.9°})\dot{E}_G.$$
$$\dot{E}_R = (0.676 \underline{/355.5°})\dot{E}_G - (281 \underline{/278.5°})\dot{I}_G.$$

In order to show the method of evaluating with numeric I and E consider the following problem:

What are the vector values of current and voltage at each end of the line when power-factor control is used to maintain constant voltage (92,380 to neutral) at both ends of the line? What are the power factors, line efficiency, etc.?

Select $\dot{E}_G = 92{,}380$ as the reference vector.

Then $\dot{E}_R = 92{,}380\underline{/\psi}$ where ψ is the angle of phase displacement with respect to the reference axis.

Substitute in the equations for $\dot{E}_G$ and $\dot{E}_R$ above and

$$92{,}380 = (0.676 \underline{/355.5°})(92{,}380 \underline{/\psi}) + (281 \underline{/278.5°})\dot{I}_R$$

and

$$92{,}380 \underline{/\psi} = (0.676 \underline{/355.5°})(92{,}380) - (281 \underline{/278.5°})\dot{I}_G$$

From which

$$\dot{I}_R = 329 \underline{/81.5°} - 222 \underline{/77.0° + \psi}$$

and

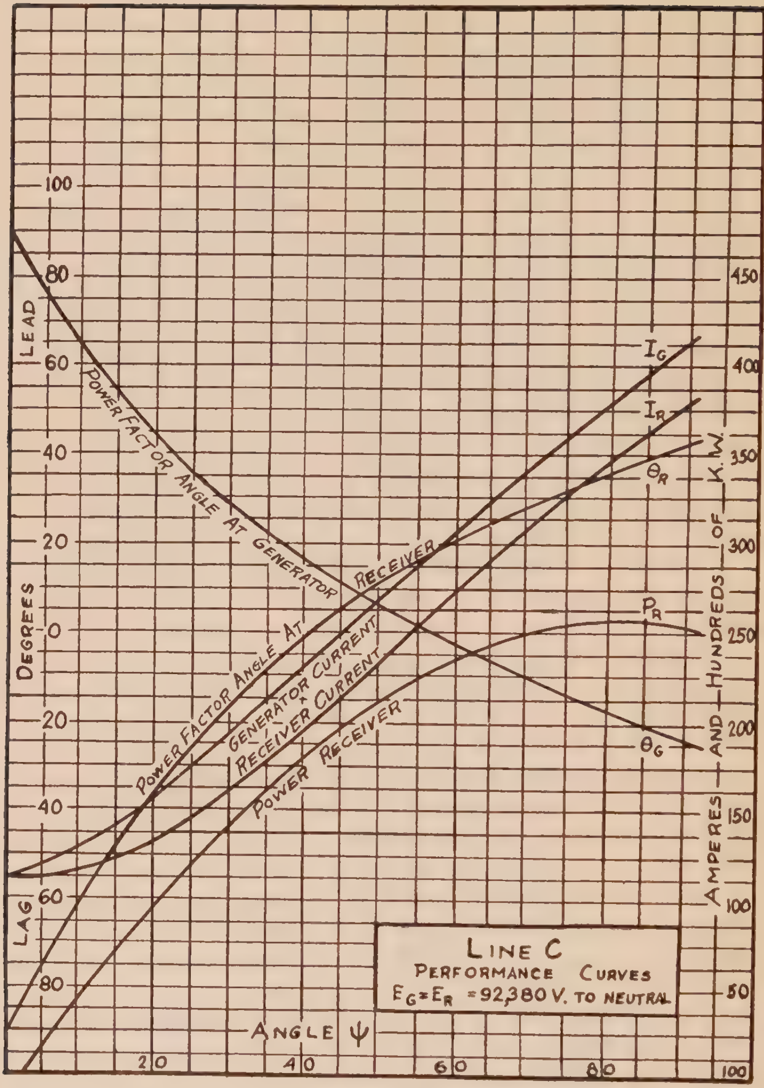
$$\dot{I}_G = 222 \underline{/77.0°} - 329 \underline{/81.5° + \psi}.$$

It will be noted that the loci of these currents are circles, that of $\dot{I}_R$ with its center at 329 $\underline{/81.5°}$ amp., and that of $\dot{I}_G$ with its center at 222 $\underline{/77.0°}$ amp. from the origin.

An easy method of solution is to assume values for ψ and solve for $\dot{I}_R$ and $\dot{I}_G$. When these are obtained, power, power factor, etc., may be readily calculated.

The figures for $\psi = 60.0°$ follow:

$$\begin{aligned} \dot{I}_R &= 329 \underline{/81.5°} - 222 \underline{/77.0° + 60.0°} \\ &= 48.6 + j325 + 162 - j151 \\ &= 210.6 + j174 = 274 \underline{/39.6°} \\ \dot{E}_R &= 92{,}380 \underline{/60.0°} \end{aligned}$$



Line C
Performance Curves
$E_G = E_R$ = 92,380 V. to neutral
Power Factor Angle at Generator
Power Factor Angle at Receiver
Generator Current
Receiver Current
Power Receiver
I_G
I_R
θ_R
P_R
θ_G
Degrees
Lead
Lag
Angle ψ
Amperes and Hundreds of K.W.
100
80
60
40
20
0
20
40
60
80
450
400
350
300
250
200
150
100
50
20
40
60
80
100

Fig. 8.28.

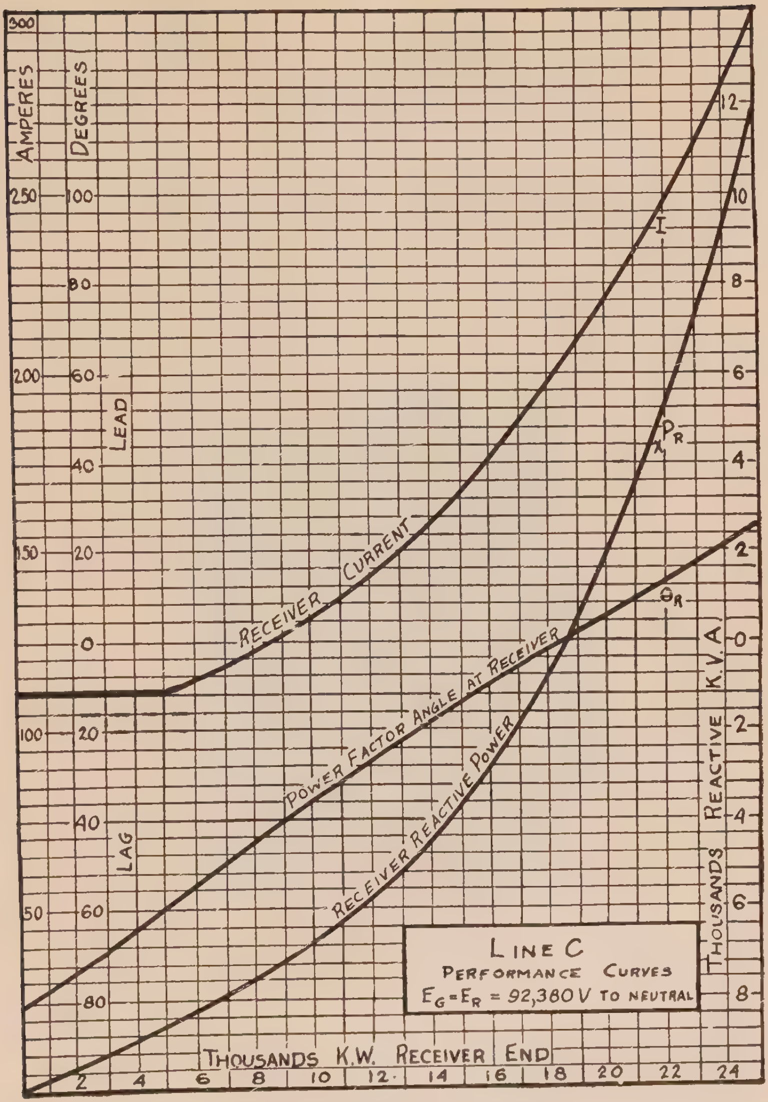

AMPERES
DEGREES
300
250
100
80
200
60
LEAD
40
150
20
0
100
20
40
LAG
50
60
80
RECEIVER CURRENT
I
P_R
θ_R
POWER FACTOR ANGLE AT RECEIVER
RECEIVER REACTIVE POWER
12
10
8
6
4
2
0
2
4
6
8
THOUSANDS REACTIVE K.V.A.
LINE C
PERFORMANCE CURVES
$E_G = E_R$ = 92,380 V TO NEUTRAL
THOUSANDS K.W. RECEIVER END
2 4 6 8 10 12 14 16 18 20 22 24

FIG. 9.28.

Therefore

$$\theta_R = 20.4^\circ \text{ leading}, \quad \cos\ \theta_R = 0.937 \quad \text{and} \quad \sin\ \theta_R = 0.349$$

$$P_R = \frac{(92{,}380 \times 274 \times 0.937)}{1{,}000}$$

$$= 23{,}700 \text{ kw. per phase}$$

$$\dot{I}_G = 222\ \underline{/77.0^\circ} - 329\ \underline{/81.5^\circ + 60.0^\circ}$$

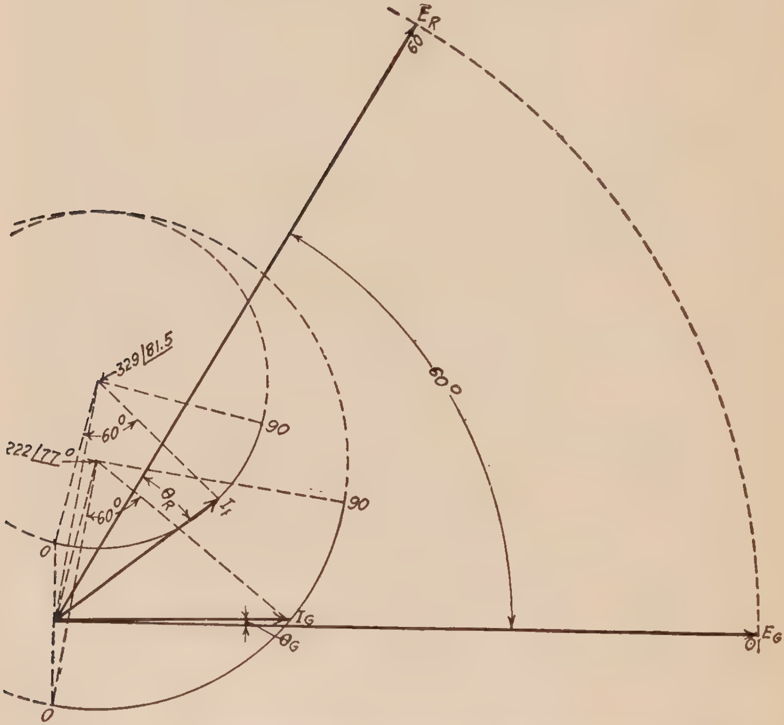


FIG. 10.28.—Vector diagram showing loci of currents and voltage for line *C*. Full lines for $\psi = 60°$.

$$= 50 + j216 + 258 - j205$$

$$= 308 + j9 = 308\ \underline{/1.7^\circ} \text{ (lagging)}$$

$$\cos\ \theta_G = 0.9999, \sin\ \theta_G = 0.0297$$

$$P_G = \frac{(92{,}380 \times 308 \times 0.9999)}{1{,}000}$$

$$= 28{,}450 \text{ kw. per phase}$$

$$\text{Line efficiency} = \frac{23{,}700}{28{,}450} = 0.832 = 83.2 \text{ per cent.}$$

Solutions for other points may be obtained in the same way. If complete performance curves are desired, it is convenient to plot curves of scalar values of current and their respective phase positions, and power as found above against ψ.

These are given in Fig. 8.28 for values of ψ from 0 to 90° for line *C*. Interpolated values from these curves may now be used in plotting curves such as those in Fig. 9.28 against receiver power.

Vector diagrams showing the loci of $\dot{I}_R\dot{E}_R$ and $\dot{I}_G$ are shown in Fig. 10.28. The full lines indicate the values for $\psi = 60°$.

(*h*) **Artificial Transmission Lines.**—It has been shown in Chaps. XXII and XXVIII that the operating characteristics of

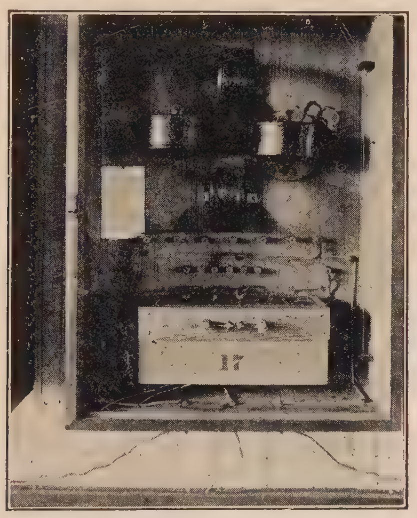


Fig. 11.28.—One unit of 400-mile artificial transmission line. (*University of Washington.*)

transmission lines are determined by the electrical constants of the line—the resistance, inductance and condensance. The length of the line and the size and spacing of the conductors are merely factors in the equations giving the magnitude of the electrical constants of the line. The distance or space feature, therefore, is not an essential factor in the electrical phenomena. The operating characteristics can be accurately reproduced in laboratory structures occupying comparatively small space. An artificial transmission line at Union College,[1] housed in a small

[1] *Trans. Am. Inst. Elec. Eng.*, Vol. 30, p. 245; Vol. 31, p. 887.

room, is electrically equivalent to 130 miles of an actual line of No. 1 A.w.g. copper with 72-in. spacing. The line has 400 units. Each unit consists of a glass cylinder 6¼ in. in diameter and 5 ft. long. The inside of the tube is lined with tinfoil, and on the outside is wound a helix of No. 8 A.w.g. copper wire. This arrangement gives uniformly distributed resistance, inductance and condensance. The total resistance for the 400 units is 93.6 ohms; the inductance, 0.394 henry; the condensance, 1.135 mf. Electrically the artificial line has the same constants as the actual 130-mile transmission line, and experiments have shown that the operating characteristics of the actual line can be accurately reproduced in the laboratory apparatus.

In the *lumpy* type, a less expensive and more compact design, the uniformly distributed inductance and condensance are represented by a number of small inductance coils and condensers, connected as indicated in Fig. 11.28.

Some of Dr. Kennelly's researches on long transmission lines were made on a lumpy, artificial line[1] equivalent to 1,500 miles of No. 3/0 A.w.g. aluminum cable, spaced 90.5 in. The line had 30 units, each consisting of four small air-core solenoids and a condenser. Although each unit was equivalent to 50 miles of line, it occupied less than 1 cu. ft. of space.

As the lumpy type only approximates a uniform distribution of the line resistance, inductance and condensance, the size of each unit must be small in comparison with the total length of the line.

In Fig. 11.28 is shown a 10-mile unit of the lumpy type having adjustable line constants. The artificial transmission line at the University of Washington[2] has 40 units similar to Fig. 11.28. The electrical constants of each unit are adjustable within the following limits.

Resistance, minimum value = 2.59 ohms.

Inductance, maximum value = 0.021 henry.

Condensance, from 0.1 to 1.0 mf.

The resistance may be increased to any desired amount by moving the clamp on the resistance loop or by inserting resistance elements between the units; the inductance may be decreased by turning the right-hand coil and by taps in the lower coil; and the

[1] Kennelly, A. E., "Electric Lines and Nets," 2d ed. of "Artificial Electric Lines," McGraw-Hill Book Company, Inc., 1928.

[2] *Trans. Am. Inst. Elec. Eng.*, Vol. 35, p. 1137.

condensance may be varied in steps by using ten or a less number of condensers in series. Adjustments can be made so as to give to the artificial line the electrical constants equivalent to an actual transmission line of any size of wire up to No. 4/0 A.w.g. hard-drawn copper and for any spacing up to 120 in.

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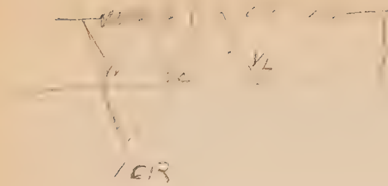
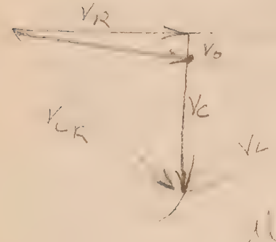
D





$= R + j0$

Building Transformer.

$$E = 4.44\, \phi N f 10^{-8}$$

volts / turn $= K \sqrt{\text{watts}}$

$K = \frac{1}{40}$ for shell type distribution

$K = \frac{1}{80}$ for core

$K = \frac{1}{25}$ " shell type power

$K = \frac{1}{50}$ " core " "

Normal density of transformer 70,000 lines / □"

shell type has generally twice the flux and half the number of turns as the core.

$$\frac{Ce^2}{2} = \frac{Li^2}{2}$$

$x = \frac{1}{2\pi f C}$ $\qquad \frac{1e^2}{2\pi f x_c^2} = \frac{x E_c^2}{2\pi f 2 (x_L)}$

$x = 2\pi f L$

Disconnect the potential lead which reads the larger of the common wire and connect it to the line W_1 wattmeter and if W_2 does not reverse sign the W_1 and W_2 are adding. If W_2 reverses it will be the difference.

Inductive curve is equal to derivative of $\frac{Li^2}{2}$. The first derivative of energy curve gives the power curve

arco sin loop.

$y = \sin\theta$

$A = \int y\, d\theta = \sin\theta\, d\theta = -\cos\theta \Big]_0^{\pi}$

$A = (-\cos\pi) - (-\cos 0) = -(-1) - (-1) = 2$

Power factor of Transformer.

It is the angle between the induced current and induced voltage

er. Three phase reactive power.
" " " actual " "
Method of measuring reactive power in, balanced circuit
" " " " " P.F.
der. of P. from complex quantities
determination of phase curves in two phase

347 - 29 ave
Melvin Abrams.

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